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Approximation Results of Nonlinear Operators via Power Series Statistical Convergence in Modular Space

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Abstract. In the setting of modular spaces, we present a Korovkin-type approximation theorem for a class of nonlinear operators by employing power series statistical convergence. An illustrative example is also provided to demonstrate the applicability of the main result.

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1. Introduction

The classical Korovkin theorem [15] occupies a central place in approximation theory and has inspired a wide array of generalizations over the years. Among these, nonlinear extensions are particularly noteworthy due to the inherent complexity of nonlinear operators, which do not adhere to the superposition and homogeneity principles that characterize linear operators. In recent years, significant attention has been directed toward the approximation behavior of nonlinear operators, with notable contributions found in [1–3, 10–13, 23].

Concurrently, modular spaces have emerged as a robust generalization of normed vector spaces by relaxing certain linearity conditions, thus offering greater flexibility in tackling a variety of problems in functional analysis. These spaces have proven particularly effective in contexts such as Orlicz spaces, Musielak-Orlicz spaces, and other generalized function spaces, where classical norm-based approaches fall short. The concept of modular convergence has played a significant role in both the theoretical advancement and practical applications within this framework [21, 22, 24]. A Korovkin-type theorem for nonlinear operators in modular spaces has been previously established in [20].

In the present study, we extend this line of inquiry by establishing a Korovkin-type approximation theorem for nonlinear operators within the framework of modular spaces, utilizing power series statistical convergence. To validate the applicability of the main result, a representative example is also provided.

We begin by recalling the notion of power series statistical convergence, which serves as the principal tool in our study.

Let (p_j) be a sequence of real numbers such that $p_0 > 0$ and $p_j \geq 0$ for all $j \geq 1$, and define

$$p(t) := \sum_{j=0}^{\infty} p_j t^j,$$

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with radius of convergence $R \in (0, \infty]$. If the limit

$$\lim_{t \to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} s_j p_j t^j = L$$

exists, then we say that the sequence $s = (s_j)$ converges to L in the power series sense [16, 18].

The regularity of the power series is determined by the condition [8]:

$$\lim_{t \to R^-} \frac{p_j t^j}{p(t)} = 0, \quad \text{for any } j \in \mathbb{N}_0.$$

Power series statistical convergence is a recent generalization of classical convergence. In fact, a sequence may be power series statistically convergent without being ordinarily convergent. The main idea is to combine statistical convergence with power series methods, thereby producing a strong and distinct notion of convergence.

Definition 1.1 ([19]). Let P be a regular power series method and $K \subset \mathbb{N}_0$. If the limit

$$\delta_P(K) := \lim_{t \to R^-} \frac{1}{p(t)} \sum_{j \in K} p_j t^j$$

exists, then $\delta_P(K)$ is called the P-density of K.

A sequence $s = (s_j)$ is said to be power series statistically convergent to L if for any given $\varepsilon > 0$,

$$\delta_P(K_\varepsilon) = 0,$$

where

$$K_\varepsilon := \{j \in \mathbb{N}_0 : |s_j - L| \geq \varepsilon\}.$$

This convergence is denoted by $st_P\text{-}\lim s_j = L$.

We now introduce the notions of power series statistical limit superior and inferior, following the approach in [9], which are important tools in summability theory.

Definition 1.2. Let $s = (s_j)$ be a real sequence and define

$$K_s := \left\{b \in \mathbb{R} : \delta_P\left(\{j \in \mathbb{N}_0 : s_j > b\}\right) \neq 0\right\}.$$

Then, the power series statistical limit superior is given by

$$st_P\text{-}\limsup s = \begin{cases} \sup K_s, & \text{if } K_s \neq \varnothing, \\ -\infty, & \text{if } K_s = \varnothing. \end{cases}$$

Similarly, define

$$L_s := \left\{a \in \mathbb{R} : \delta_P\left(\{j \in \mathbb{N}_0 : s_j < a\}\right) \neq 0\right\}.$$

Then, the power series statistical limit inferior is given by

$$st_P\text{-}\liminf s = \begin{cases} \inf L_s, & \text{if } L_s \neq \varnothing, \\ \infty, & \text{if } L_s = \varnothing. \end{cases}$$

We interpret the condition $\delta_P(K) \neq 0$ as indicating that either $\delta_P(K) > 0$ or that $\delta_P(K)$ does not exist. Throughout this paper, we also assume that the power series method P is regular.

Next, we introduce some fundamental definitions and properties pertaining to nonlinear operators and modular.

Let $I = [a, b]$ be a measurable subset of $\mathbb{R}$ equipped with the Lebesgue measure μ. We denote by $X(I)$ the space of all real-valued measurable functions on I, identified up to almost everywhere equality. Furthermore, let $C(I)$ represent the space of all continuous real-valued functions defined on I, and $C^\infty(I)$ denote the space of all infinitely differentiable functions on I.

Definition 1.3 ([5]). A functional $\rho : X(I) \to [0, \infty]$ is said to be modular if the followings are satisfied:
(i) $\rho[g] = 0 \iff g = 0$ a.e. on I.
(ii) $\rho[-g] = \rho[g]$, for all g *in* $X(I)$.
(iii) $\rho[\alpha_1 g + \alpha_2 h] \leq \rho[g] + \rho[h]$ for all g, h *in* $X(I)$ and for every $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$.

A modular ρ is called Q-quasi convex, if

$$\rho\left[\alpha_1 g + \alpha_2 h\right] \leq Q\alpha_1\rho\left[Qg\right] + Q\alpha_2\rho\left[Qh\right]$$

is satisfied for at least a constant $Q \geq 1$, for any g, h *in* $X(I)$ and $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$. In the case of $Q = 1$, ρ is called convex [5].

A modular ρ is called Q-quasi semiconvex whenever the inequality

$$\rho\left[\alpha g\right] \leq Q\alpha\rho\left[Qg\right]$$

holds at least a constant $Q \geq 1$ and for all g *in* $X(I)$, $g \geq 0$ and $\alpha \in (0, 1]$. Observe that Q-quasi convexity yields Q-quasi semiconvexity.

A modular ρ is called monotone if $\rho\left[g\right] \leq \rho\left[h\right]$ for all $g, h \in X(I)$ satisfying $|g| \leq |h|$ [5].

Now let us give some subspaces of $X(I)$ [6] (see also [7], [14])

$$L^{\rho}\left(I\right) := \left\{f \in X(I) : \lim_{\lambda \to 0^{+}} \rho\left[\lambda f\right] = 0\right\}$$

and

$$E^{\rho}\left(I\right) := \{f \in L^{\rho}\left(I\right) : \rho\left[\lambda f\right] < \infty \text{ for every } \lambda > 0\}.$$

Some definitions about modulars are as follows [6]:

a) ρ is finite, if $1 \in L^{\rho}\left(I\right)$ denoting by 1 the function $1\left(t\right) = 1$ for every $t \in I$.

b) ρ is strongly finite, if $1 \in E^{\rho}\left(I\right)$.

c) ρ is absolutely continuous, if there exists $\alpha > 0$ such that for every $g \in X\left(I\right)$, with $\rho\left[g\right] < +\infty$, satisfying that for every $\varepsilon > 0$ there is $\delta > 0$ such that $\rho\left[\alpha g\chi_B\right] < \varepsilon$, for every measurable subset $B \subset I$ with $\mu\left(B\right) < \delta$.

d) A modular ρ is named absolutely finite if ρ is finite and for every $\varepsilon > 0$, $r > 0$ there is $\delta > 0$ such that $\rho[r\chi_E] < \varepsilon$ for all measurable subset $E \subset I$ with $\mu(E) < \delta$.

Among the classical instances of modular spaces are Orlicz spaces, defined by means of a φ-function φ. Extending this framework, Musielak-Orlicz spaces are generated by a parameter-dependent φ-function that adheres to specific growth conditions relative to the parameter [4]. The modular defining these spaces comply with all the conditions previously specified.

A sequence $\left(f_j\right)_{j\in\mathbb{N}} \subset L^{\rho}\left(I\right)$ is said to be power series statistically modularly convergent to f in $L^{\rho}\left(I\right)$, whenever

$$st_P - \lim \rho\left[\lambda\left(f_j - f\right)\right] = 0$$

holds for some $\lambda > 0$. This expression is a generalization of the norm convergence in L^p spaces.

Also, a sequence $\left(f_j\right)_{j\in\mathbb{N}} \subset L^{\rho}\left(I\right)$ is said to be power series statistically strongly convergent to f in $L^{\rho}\left(I\right)$ whenever

$$st_P - \lim \rho\left[\lambda\left(f_j - f\right)\right] = 0$$

for all $\lambda > 0$.

Clearly, the concept of power series statistical modular convergence is weaker than that of power series statistical strong convergence in $L^{\rho}\left(I\right)$.

Now, we give some definitions about nonlinear operators in general form. Consider Hausdorff topological spaces X with vector sublattice E and Y with vector sublattice F, respectively belonging to $C(X)$ and $C(Y)$. Let V represent an operator mapping from E to F.

- If for every $g, h \in E$ and $a \geq 0$, the conditions

  $$V(g + h) \leq V(g) + V(h)$$

  and

  $$V(ag) = aV(g) \quad \text{(positively homogeneous)}$$

  hold, then the operator is classified as sublinear.
- If for every $g, h \in E$, $g \leq h$ implies $V(g) \leq V(h)$ then the operator is said to be monotonic.
- If for any pair of comonotone functions g, h in E, meaning

  $$\left(g(x) - g(s)\right).\left(h(x) - h(s)\right) \geq 0 \text{ for every } x, s \in X,$$

  the property

  $$V(g + h) = V(g) + V(h)$$

holds, then the operator is said to be comonotonic additive.

Let $T = \left(T_j\right)_{j\in\mathbb{N}}$ be a sequence of monotonic and sublinear operators being $T_j : K \to X(I)$, where $C(I) \subset K \subset X(I)$. Also, along this paper assume that $\left(T_j\right)_{j\in\mathbb{N}}$ satisfies the following property which will be called as (CS): there exists $X_T \subset K \cap L^\rho(I)$ that includes $C(I)$ such that for all functions $f \in X_T$ we get $T_j f \in L^\rho(I)$ and

$$st_P - \limsup \rho\left[\lambda\left(T_j f\right)\right] \leq D.\rho\left[\lambda f\right] \tag{1.1}$$

for any $\lambda > 0$ and absolute constant D.

2. Approximation Results for Nonlinear Operators on Modular Spaces

This section is devoted to the development of Korovkin-type approximation results for nonlinear operators on modular spaces, utilizing the framework of power series statistical convergence. We commence with the following lemma, as given in [20].

Lemma 2.1. *Consider a modular ρ being absolutely finite, monotone and absolutely continuous on $X(I)$. Hence, we get $\overline{C(I)} = L^\rho(I)$.*

We will use the notations h_j for the test functions x^j, $j = 0, 1, 2$.

Theorem 2.2. *Assume that modular ρ is strongly finite, monotone, absolutely continuous and Q-quasi semiconvex on $X(I)$. Consider $(T_j)_{j\in\mathbb{N}}$ being a sequence of sublinear and monotonic operators conforming to the property (CS). If $h \in \{h_0, h_1, -h_1, h_2\}$*

$$T_j h \to h$$

holds power series statistically strongly in $L^\rho(I)$, we get

$$T_j f \to f$$

holds power series statistically modularly in $L^\rho(I)$ for any nonnegative $f \in L^\rho(I) \cap K$ such that $f - C(I) \subset X_T$.

Proof. Consider any nonnegative function f in $C(I)$. Hence we write for every $\varepsilon > 0$, there exists $\delta > 0$ such that

$$|f(x) - f(y)| \leq \frac{2\|f\|_\infty}{\delta^2}.|x-y|^2$$

holds for any $x, y \in I$ with $|x - y| < \delta$. Therefore, we obtain

$$\begin{aligned}|f(x) - f(y)| &\leq \varepsilon + \frac{2\|f\|_\infty}{\delta^2}.|x-y|^2 \\ &= \varepsilon.1 + \frac{2\|f\|_\infty}{\delta^2}\left[h_2(x) + 2h_1(x).(M - h_1(y)) + 2M.(-h_1(x)) + h_2(y)\right]\end{aligned}$$

for any $x, y \in I$, where $M = \max\{h_1(y), h_2(y), 0\}$. Since T_j is positively homogeneous, for $f \geq 0$, we can write

$$\begin{aligned}\left|T_j(f)(y) - f(y)\right| &\leq \left|T_j(f)(y) - T_j(f(y).h_0)\right| + \left|f(y).\left(T_j(h_0) - h_0\right)\right| \\ &\leq \varepsilon.T_j(h_0) + \frac{2\|f\|_\infty.}{\delta^2}\left[\begin{array}{c} T_j(h_2) + 2(M - h_1(y)).T_j(h_1) + 2M.T_j(-h_1) \\ +h_2(y)\,T_j(h_0)\end{array}\right] \\ &\quad + f(y).\left|T_j(h_0) - h_0\right|.\end{aligned}$$

For all $\lambda > 0$, we obtain

$$\begin{aligned}\rho\left[\lambda.\left(T_j(f) - f\right)\right] &\leq \rho\left[3\lambda\varepsilon T_j(h_0)\right] + \rho\left[\frac{6\lambda\|f\|_\infty}{\delta^2}\left\{\begin{array}{c} T_j(h_2) + 2(M - h_1(y)).T_j(h_1) \\ +2M.T_j(-h_1) + h_2(y).T_j(h_0)\end{array}\right\}\right] \\ &\quad + \rho\left[3\lambda.f(y)\left|T_j(h_0) - h_0\right|\right].\end{aligned}$$

Let us denote

$$B := T_j(h_2) - h_2 + 2M\left(T_j(h_1) - h_1\right) - 2h_1\left(T_j(h_1) - h_1\right) + 2M.\left(T_j(-h_1) + h_1\right) + h_2(y).\left(T_j(h_0) - h_0\right).$$

Hence, we have

$$\begin{aligned}\rho\left[\frac{6\lambda.\|f\|_\infty}{\delta^2}.B\right] &\leq \rho\left[\frac{30\lambda.\|f\|_\infty}{\delta^2}.\left|T_j(h_2)-h_2\right|\right]\\ &+\rho\left[\frac{60.M.\lambda.\|f\|_\infty}{\delta^2}\left|T_j(h_1)-h_1\right|\right]+\rho\left[\frac{60.M.\lambda.\|f\|_\infty}{\delta^2}\left|T_j(-h_1)+h_1\right|\right]\\ &+\rho\left[\frac{60.M.\lambda.\|f\|_\infty}{\delta^2}\left|T_j(h_1)-h_1\right|\right]+\rho\left[\frac{30.M.\lambda.\|h_2\|_\infty.\|f\|_\infty}{\delta^2}\left|T_j(h_0)-h_0\right|\right].\end{aligned}$$

By using our assumptions, we have for any constant $\eta > 0$,

$$st_P - \limsup \rho\left[\eta\left(T_j(f)-f\right)\right] \leq P.\rho\left[3\eta\varepsilon h_0\right].$$

Then, by using Q-quasi semiconvexity and strongly finiteness of ρ, we obtain by assuming $\varepsilon < 1$ that

$$\rho\left[3\eta\varepsilon h_0\right] \leq Q\varepsilon\rho\left[3Q\eta h_0\right].$$

Hence, we get the power series statistical strong convergence to nonnegative $f \in C(I)$. Take into consideration nonnegative $f \in K \cap L_\rho(I)$. From Lemma 2.1, there exist a nonnegative sequence $(g_k)_{k\in\mathbb{N}} \subset C(I)$ and a $\lambda > 0$ such that $\rho[3\lambda f] < \infty$ and

$$\lim_{k\to\infty} \rho\left[3\lambda\left(g_k - f\right)\right] = 0. \tag{2.1}$$

Since T_j is subadditive for $j \in \mathbb{N}$, we find

$$\begin{aligned}\left|T_j(f)-f\right| &\leq \left|T_j(f)-T_j(g_k)\right|+\left|T_j(g_k)-g_k\right|+|g_k-f|\\ &\leq T_j(|f-g_k|)+\left|T_j(g_k)-g_k\right|+|g_k-f|.\end{aligned}$$

For a $\lambda > 0$, $f - g_k \in X_T$, we obtain

$$\rho\left[\lambda\left|T_jf-f\right|\right] \leq \rho\left[3\lambda.T_j(|f-g_k|)\right]+\rho\left[3\lambda.\left|T_j(g_k)-g_k\right|\right]+\rho\left[3\lambda.|g_k-f|\right].$$

If we take advantage of property (CS), we write

$$\rho\left[3\lambda.T_j(|f-g_k|)\right] \leq D.\rho\left[4\lambda.|f-g_k|\right].$$

If we apply power series statistical lim sup operator to each side of the above inequality, we get

$$st_P - \limsup \rho\left[\lambda.\left|T_jf-f\right|\right] \leq (D+1).\rho\left[3\lambda.(f-g_k)\right].$$

By using equality (2.1), for some $\lambda > 0$ we observe

$$st_P - \limsup \rho\left[\lambda\left|T_jf-f\right|\right] = 0.$$

The proof is completed. □

Theorem 2.3. *In addition to the hypotheses of Theorem 2.2, we further assume that the sequence of operators $(T_j)_{j\in\mathbb{N}}$ is comonotonic additive, i.e., each T_j preserves comonotonic additivity for all $j \in \mathbb{N}$. Then,*

$$T_jf \to f$$

holds power series statistically modularly in $L^\rho(I)$ for any f in $L^\rho(I) \cap K$ such that $f - C(I) \subset X_T$.

Proof. Let each T_j be monotonic, sublinear and comonotonic additive. Because of Theorem 2.2, we get

$$T_j\left(f+\|f\|_\infty\right) \to f+\|f\|_\infty \tag{2.2}$$

holds power series statistically modularly in $L^\rho(I)$ for each f in $L^\rho(I) \cap K$ such that $f - C(I) \subset X_T$. Each constant function consists of a comonotonic pair with any function f. Since T_j is positively homogeneous and comonotonic additive for any $j \in \mathbb{N}$, we get

$$T_j\left(f+\|f\|_\infty.h_0\right) = T_j(f)+\|f\|_\infty.T_j(h_0). \tag{2.3}$$

By using (2.2), (2.3) and our assumption

$$T_j(h_0) \to h_0, \text{ power series statistically strongly in } L^\rho(I),$$

we obtain

$$T_j(f) \to f$$

power series statistically modularly in $L^{\rho}(I)$ for each $f \in L^{\rho}(I) \cap K$ such that $f - C(I) \subset X_T$. □

We now proceed to characterize the class X_T in a particular setting. To this end, we first recall the definition of Orlicz spaces [17].

Let $\varphi : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ be a convex function such that $\varphi(0) = 0$, $\varphi(v) > 0$ for all $v > 0$, and $\lim_{v\to\infty} \varphi(v) = \infty$. The collection of all such functions is denoted by Φ.

For $\varphi \in \Phi$ and $f \in X(I)$, we define the modular functional

$$\rho_{\varphi}[f] = \int_I \varphi(|f(s)|)\, ds.$$

It is well known that ρ_{φ} is a convex functional on $X(I)$ [4].

$$L_{\varphi}(I) = \left\{ f \in X(I) : \exists \lambda > 0 \ \ni \ \rho_{\varphi}[\lambda f] < \infty \right\}.$$

The subspace $L_{\varphi}(I)$ generated by a function $\varphi \in \Phi$ is referred to as the Orlicz space [17]. The corresponding modular ρ_{φ} is finite, strongly finite, and absolutely continuous.

In the special case where $\varphi(v) = v^p$ for $v \geq 0$ and $p \geq 1$, the Orlicz space coincides with the classical Lebesgue space, that is,

$$L_{\varphi}(I) = L^p(I),$$

and the modular norm satisfies

$$\|f\|_{\varphi} = \|f\|_p.$$

Furthermore, let F_{φ} denote the subset of functions in $L_{\varphi}(I)$ that satisfy inequality (1.1) for a constant K independent of both λ and f, and for all values of λ.

We also consider the classical Bernstein operator $B_j : C[0,1] \to C[0,1]$ defined by

$$B_j(f)(x) = \sum_{k=0}^{j} p_{j,k}(x) f\left(\frac{k}{j}\right),$$

where

$$p_{j,k}(x) = \binom{j}{k} x^k (1-x)^{j-k}.$$

By using this operator we will define nonlinear operator $T_j : C[0,1] \to C[0,1]$ as follows:

$$T_j(f) = u_j \max\left\{B_j(f), B_{j+1}(f)\right\},$$

where (u_j) is defined by

$$u_j = \begin{cases} 0, & j = k^2 \\ j, & otherwise. \end{cases}$$

These operators are monotonic and sublinear. (These operators are obviously comonotonic additive). It is well known that

$$B_j(h_0)(x) = h_0,$$
$$B_j(-h_1)(x) = -x,$$

$$B_j(h_2)(x) = x^2 + \frac{x.(1-x)}{j}.$$

By the help of above equalities, we get $T_j(h_0)(x) = u_j h_0(x)$, $T_j(-h_1)(x) = -u_j x$ and $T_j(h_2)(x) = u_j \max\left\{B_j(h_2)(x), B_{j+1}(h_2)(x)\right\} = u_j\left(x^2 + \frac{x.(1-x)}{j}\right)$. Since φ is convex, for every $\lambda > 0$ we have

$$\begin{aligned}\rho_\varphi\left[\lambda B_j f\right] &= \int_0^1 \varphi\left(\lambda\left|\sum_{k=0}^{j} p_{j,k}(s) f\left(\frac{k}{j}\right)\right|\right) ds \\ &= \sum_{k=0}^{j} \varphi\left(\left|\lambda f\left(\frac{k}{j}\right)\right|\right)\int_0^1 p_{j,k}(s)\, ds \\ &= \sum_{k=0}^{j} \varphi\left(\left|\lambda f\left(\frac{k}{j}\right)\right|\right)\cdot\frac{1}{j+1} \\ &= \frac{1}{j+1}\rho_j^{\varphi}[\lambda f].\end{aligned}$$

By using this inequality, we obtain for every integrable function f:

$$\begin{aligned}\limsup_{j\to\infty}\frac{1}{j+1}\rho_j^{\varphi}[\lambda f] &= \limsup_{j\to\infty}\frac{1}{j+1}\cdot\frac{j}{j}\sum_{k=0}^{j}\varphi\left(\left|\lambda f\left(\frac{k}{j}\right)\right|\right) \\ &= \limsup_{j\to\infty}\frac{j}{j+1}\sum_{k=0}^{j}\varphi\left(\left|\lambda f\left(\frac{k}{j}\right)\right|\right)\left(\frac{k+1}{j}-\frac{k}{j}\right) \\ &= 1.\rho_\varphi[\lambda f].\end{aligned}$$

From the above equation, $f \in F_\varphi$, that is every integrable function in the Riemann sense, belongs to the class F_φ. So there exists a subset $X_T \subset K \cap L^\rho(I)$ including the space $C(I)$ for the operator (T_i) with

$$\begin{aligned}st_P - \limsup\rho_\varphi\left[\lambda\left(T_j f\right)\right] &= st_P - \limsup\rho_\varphi\left[\lambda u_j \max\{B_j f, B_{j+1} f\}\right] \\ &= st_P - \limsup u_j\rho_\varphi\left[\lambda\max\{B_j f, B_{j+1} f\}\right] \\ &\le D.\rho_\varphi\left[\lambda f\right],\end{aligned}$$

where P is any regular power series method. That is, the property (CS) is satisfied at least for the class of Riemann integrable functions. This shows that $F_\varphi \subset X_T$. Especially, if we consider following power series method given by

$$p_j = \begin{cases}1, & j = k^2 \\ 0, & otherwise,\end{cases}$$

then we get for the function h_0

$$\begin{aligned}\rho_\varphi\left[\lambda\left(T_j(h_0) - h_0\right)\right] &= \int_0^1 \varphi\left(\lambda.|u_j - 1|\right) dx \\ &= \begin{cases}\varphi(\lambda), & j = k^2 \\ \varphi\left(\lambda|j-1|\right), & otherwise\end{cases}, \; for\ all\ \lambda > 0\end{aligned}$$

for the function h_1

$$\begin{aligned}\rho_\varphi\left[\lambda\left(T_j(h_1) - h_1\right)\right] &= \int_0^1 \varphi\left(\lambda.|u_j x - x|\right) dx \\ &= \begin{cases}\int_0^1 \varphi(\lambda x) dx, & j = k^2 \\ \int_0^1 \varphi\left(\lambda|jx - x|\right) dx, & otherwise\end{cases}, \; for\ all\ \lambda > 0\end{aligned}$$

for the function $-h_1$

$$\rho_\varphi\left[\lambda\left(T_j\left(-h_1\right)+h_1\right)\right]=\int_0^1 \varphi\left(\lambda.|-u_j x+x|\right)dx$$
$$=\begin{cases}\int_0^1 \varphi(\lambda x)dx, & j=k^2\\ \int_0^1 \varphi\left(\lambda|-jx+x|\right)dx, & otherwise\end{cases}, for\ all\ \lambda>0$$

and for the function h_2

$$\rho_\varphi\left[\lambda\left(T_j\left(h_2\right)-h_2\right)\right]=\int_0^1 \varphi\left(\lambda|u_j x^2+\frac{u_j x\left(1-x\right)}{j}-x^2|\right)dx$$
$$=\begin{cases}\int_0^1 \varphi(\lambda x^2)dx, & j=k^2\\ \int_0^1 \varphi\left(\lambda|jx^2+\frac{jx(1-x)}{j}-x^2|\right)dx, & otherwise\end{cases}, for\ all\ \lambda>0.$$

As one can see, for $h\in\{h_0,h_1,-h_1,h_2\}$ we have

$$T_j h\to h$$

power series statistically strongly in $L^\rho(I)$. This sequence of operators T_j fulfills all the requirements of our theorem.

3. Concluding Remarks

In this study, we have established a Korovkin-type approximation theorem for a class of nonlinear operators within the framework of modular spaces by employing the concept of power series statistical convergence. The main theorem extends and generalizes several well-known results in the existing literature to a broader functional setting. Furthermore, an illustrative example has been provided to verify the applicability and effectiveness of the obtained result, thereby highlighting the relevance of the proposed approach in the theory of nonlinear approximation. The findings presented here contribute to the ongoing development of approximation theory in modular spaces and may serve as a foundation for further research on the interplay between nonlinear operator theory and generalized modes of convergence. Future investigations may focus on extending the present results to other types of convergence methods, such as λ-statistical convergence, and to more general classes of modular function spaces. Moreover, one could explore the approximation behavior of different subclasses of nonlinear operators, including sublinear and monotone operators, under similar or relaxed conditions.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

The authors has read and agreed the published version of the manuscript.

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