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Economic growth and human capital: an approach from dynamic stochastic general equilibrium and vector error correction modelling for Colombia

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Abstract
The analysis of economic growth determinants has allowed for the development of an extensive body of literature that has recognized the role of human capital as the main explanatory variable of countries' economic growth. However, the evidence shows that this relationship does not always occur in such a precise way; therefore, it is necessary to analyse the dynamics of each economy. In Colombia, although there is empirical evidence that correlates these two variables, economic growth and human capital, there is little research that explains the behaviour of growth and human capital expected in the long term. The objective of this research was to determine the interactions between human capital and economic growth in long-term dynamics in Colombia. To achieve this goal, a dynamic stochastic general equilibrium (DSGE) model was applied with parameters calibrated from empirical evidence in Colombia. On the other hand, a vector error correction model (VECM) was applied to the student–teacher ratio series and per capita GDP in Colombia for the 1970–2019 period, which is novelty to develop a calibration technique for the model proposed to determine the relationship between human capital and economic growth in an emerging economy. The results of the DSGE model indicate that economic growth has a positive effect in the long term. These results are reinforced through the VECM. Disturbances in the productivity of education captured through the student–teacher ratio in tertiary education show the existence of short- and long-term relationships of the mentioned variable with per capita GDP, as in the DSGE model, the variables perceive convergence to new equilibria. Both models indicate that long-term physical capital and human capital have positive relationships with economic growth, which generate theoretical and practical implications to represent a starting point in understanding the effects of economic policies associated with education on Colombian economic growth, which implies promoting policies related to human capital in the country to achieve development, social mobility, and overall societal well-being and to generate models that support and strength development theory in the context of emerging economy such as Colombia as an input to policy makers and decision makers.

Keywords Economic growth · Human capital · Colombia · Education · Dynamic stochastic general equilibrium · Vector error correction

Extended author information available on the last page of the article

JEL Classification I29 · J24 · N16 · O47 · D58 · C32

1 Introduction

At the end of the twentieth century, the frontier of knowledge in economics on economic growth was expanded by the massive emergence of various models that, compared with previous models, better explained the disparities in growth dynamics experienced between different countries, giving rise to the consolidation of the new growth theory, also known as endogenous growth theory. Neoclassical theory determines growth in the steady state from an exogenous parameter of technology and a rate of capital accumulation that does not influence the long-term growth rate. In contrast, endogenous growth theory postulates that growth is the result of processes and interactions of variables that are defined within the same models (Vergara 1991; Ahortor and Adenutsi 2009; Aslam 2020).

The new theory explained a state of continuous growth over time because of the returns on investments in physical capital and human capital. This was possible from the application of production functions with increasing or constant returns in the production factors susceptible to accumulation and the introduction of human capital and technological development to the traditional neoclassical model (Jiménez 2011; Managi et al. 2021). The analysis of the determinants of economic growth has made it possible to develop an extensive body of literature that recognizes the role of human capital as the main explanatory variable of the economic growth of countries, with human capital stock being one of the most included variables within the neoclassical analysis and the theory of endogenous growth (Mincer 1984; Galor and Tsiddon 1997; Robeyns 2005; Teixeira and Queirós, 2016; Rehman 2014).

In Colombia, studies on economic growth and human capital have been analysed from different perspectives, and some examples are as follows: i. investment in education is an essential strategy for optimizing human capital, promoting economic development, and contributing to overall well-being in a dynamic and challenging economic environment (Gómez 2024; Guarnizo 2019); and ii. the stock level of human capital impacts the effect of foreign direct investment (FDI) on gross domestic product (GDP) per capita growth, and education, as a component of human capital, plays a significant role in mediating the effects of FDI on economic growth (Plantin 2021). These studies show that in the Colombian context, studies are limited and that analysing the relationships between economic growth and human capital with more detail and other methods is important.

Intuitively, it could be assumed that human capital largely explains the performance in the growth rates of countries and the differences between them; however, empirical evidence has concluded that this relationship does not always occur in such a precise way, and as a consequence, it is necessary to analyse the dynamics of each economy in particular. The Colombian case is not alien to this particularity because although there is empirical evidence that correlates the variables economic growth and human capital, there are few investigations that explain the behaviour of growth and the expected human capital in the long term. In this context, the following research question arises: How do human capital and long-term economic growth interact in Colombia, and what are the dynamics of the interaction?

This research represents a proposal in the Colombian empirical literature that applies a DSGE model and a VECM simultaneously to compare impulse response functions. The first

describes the effects on production due to shocks in education productivity, and the second describes the long-term relationship of the student–teacher ratio variable as a proxy for the quality of education with economic growth which is novelty to develop a calibration technique for the model proposed to determine the relationship between human capital and economic growth in an emerging economy. The document is divided into five sections. The first consists of this introduction, and the second section is a brief review of the literature focused on the relationship between economic growth and human capital. The third section presents the stochastic dynamic general equilibrium model, the input parameters calibrated from the empirical evidence in Colombia, and the analysis of the long-term relationship between human capital and economic growth carried out on the basis of the impulse response functions. The fourth section applies a VECM to the series student–teacher ratio and GDP per capita and analyses the interactions of these variables in the long term. Finally, the final section presents the main conclusions of this work and the robustness of the methodology used and proposed, according to the results obtained.

2 Conceptual theoretical framework

At the end of the twentieth century, the barrier in the knowledge of economists about economic growth was the massive emergence of various models that sought to explain the disparities in growth dynamics experienced between different countries, which the first growth models were not capable of. To justify this, the consolidation of a new growth theory, also known as endogenous growth theory, took place. This theory postulates that growth is the result of processes and interactions of variables that are defined within the same models, in contrast to the neoclassical theory, which determines growth in the steady state from an exogenous parameter of technology and a rate of capital accumulation that does not influence the long-term growth rate (Vergara 1991; Acemoglu 2007; Angus 2020).

Starting from the application of production functions with increasing or constant returns on the factors of production susceptible to accumulation and the introduction of human capital and technological development to the traditional neoclassical model, the new theory managed to explain a state of continuous growth in time because of returns on investments in physical capital and human capital (Jiménez 2011; Fernandez-Arias 2017).

These models consider that technological progress is not the only generator of long-term economic growth. On the basis of this distinction, several categories are recognized in which endogenous growth models are framed (Shapirov 2015). The first category includes the studies by Lucas (1988) and Romer (1990). The second category includes the studies by Grossman and Helpman (1991) and Aghion and Howitt (1992).

The first category identifies the developments in which human capital emerges as a fundamental determinant of economic growth. For Romer (1990), knowledge or information is the key factor in growth theory; therefore, the growth rate depends directly on the value of human capital aimed at generating new knowledge. Research is a necessary but not sufficient condition for economic growth since it allows for the accumulation of human capital; consequently, countries with greater accumulation of capital present higher rates of economic growth. Lucas (1988) suggested that this growth rate considers the decision of individuals between participating in the production process or accumulating human capital. For Pernagallo (2024), scholarships and grants do not involve the problem that moral hazards

are socially efficient and are no more expensive than alternative financing schemes. Scholarships may pose a problem of overinvestment in education, but they are more cost-effective than grants are, whereas grants achieve the first best.

One of the main methodological problems facing any researcher wishing to empirically study endogenous growth models is the selection of the proxy variable as a measure of human capital stock (Pelinescu 2015). There is a broad branch of economic literature that has opted to use years of schooling as a variable (Romer 1990; Barro & Lee 1993; Bils & Klenow 2000; Krueger & Lindahl 2001; Barro and Lee 1993).

While this research shows that years of schooling can explain the accumulation of human capital, when quality measures are added, the effect of years of schooling on economic performance loses relevance. Therefore, the analysis by quantity loses strength, and the explanatory power of human capital is determined by quality (Hanushek & Woessmann 2015; Laverde 2015).

The role of higher education institutions enables growth because they enable technological change and the management of more advanced information, which reduces costs and increases competitiveness. Investment in higher education levels is a necessary condition for economies to experience growth. If the positive externalities of basic and higher education are considered, it becomes evident that the latter generates greater returns to growth (Lemus & Casas 2015). People with higher education contribute intensively to the accumulation of human capital (Laverde 2015).

In Colombia, higher education influences economic growth by promoting innovation and technical change, which largely explain the growth rates of the country's departments (Lemus and Casas 2015). Given the practical availability of information on a higher education quality indicator for the analysis period, the literature suggests that the student–teacher ratio may capture the quality effect (Arias and Walker 2004; Kokkelenberg 2008; Bandiera, et al. 2010). Likewise et al. described how variables such as the student–teacher ratio, included in education production functions, influence per capita income.

Class size influences the multidimensional perspective of the class, the ability to anticipate difficulties students may experience, monitoring and feedback, and students' ability to take responsibility for their own learning (Lemus and Casas 2015).

There is a negative effect between class size and academic performance in higher education (Arias and Walker 2004). Students with higher test scores are more affected by changes in class size, especially when classes are very large (Bandiera, et al. 2010). Similarly, the average grade decreases as class size increases, and evidence shows that diseconomies of scale are associated with deteriorating student performance as class size increases (Kokkelenberg 2008).

The second category includes those models in which R&D represents the determining factor of growth. The works of Grossman and Helpman (1991), Aghion and Howitt (1992) and the European Central Bank (2017), Aldieri (2019) describe the effect of innovation on rates of growth as a fundamental driver of economic progress that benefits consumers, businesses and the economy as a whole. Accordingly, this growth is the result of technological progress guaranteed by competition between firms since, in the long term, it allows a cycle of production and technological innovation to take place, and in this context, it is important that growth implies sociotechnical transitions that demand investment, infrastructure, and sociopolitical innovation to become successful (Prabhu 2017; Taalbi 2021; Özdemir 2023).

Since the eighties, the differences between the growth rates of the countries have led to the development of theories of economic growth with the emergence of various theoretical models that attempt to explain the cause of this behaviour through factors not included in the neoclassical growth models. The incorporation of human capital and the consideration of education as an element that affects the growth rates of an economy play important roles in economic theory, and nonlinearities in this relationship should be accounted for (Terrones & Calderón, 1993; Marquez-Ramos and Mourelle 2019; Klomp 2011).

DSGE models have been utilized by different studies. For example, Agentiero et al. (2021) analysed whether the accumulation of social capital supports economic performance coupled with human capital in the Italian context, determining that social capital may improve economic performance via its impact on total factor productivity and human capital in the model and increasing social capital's ability to explain the Solow residual. Mu et al. (2022) explained the simultaneous increases in economic growth and income inequality in China, establishing that workers with high initial human capital are central to the effectiveness of macroeconomic policies. These studies show the importance of analysing the dynamics of human capital and long-term economic growth from empirical models that can include this technique and VECM simultaneously to compare impulse response functions.

The economic literature has indicated the importance of the calibration of growth models that go beyond the medium run and act as a forecasting device of macroeconomic trends to maintain the richer model structure of economic models (Dawkins 2001; Karagedikli et al. 2010; Ciesielski and Anthof 2018), and calibration in models related to human capital has been used, for example, by Carrillo (2012) to analyse job search and general human capital accumulation and Collin and Weil (2019) to determine the dynamic responses of income and poverty to increased investment in human capital. Allub et al. (2019) quantified how a lack of human capital and financial frictions, as well as the joint effect of both restrictions, interact to explain cross-country differences in aggregate output per capita, productivity and average firm size. Therefore, this study seeks to develop a calibration technique for the model proposed to determine the relationship between human capital and economic growth in an emerging economy.

3 Methodological approach for the DSGE model

An economy with an infinite number of homogeneous agents and an infinite horizon is considered, where there is a representative agent that for each period t receives a predetermined endowment of physical capital K_t and human capital H_t, which depreciate at constant rates δ_K and δ_H, respectively. The model is composed of two productive sectors, a sector of homogeneous final goods and a sector that provides educational services; additionally, it is assumed that the labour force L_t is associated with H_t.

There is an infinite number of firms producing a single type of homogeneous final good, Y_t. under a Cobb–Douglas-type production function with constant yields at the scale in the following form:

$$Y_t = A_t K_t^{\alpha} (L_t H_t)^{1-\alpha}$$

where A_t is the technological productivity factor of the productive goods sector.

The benefits function of firms, $\prod_t$, is understood as the difference between income, Y_t, and the cost W_t of work, $L_t H_t$, and the cost, R_t, of physical capital, K_t, such that:

$$\prod_t = Y_t - R_t K_t - W_t L_t H_t$$

Firms must face the problem of maximizing their profit by allocating endowments of physical capital and labour, and the cost of the factors of production is obtained from the first-order conditions (F.O.C.)

$$\max \prod_t = Y_t - R_t K_t - W_t L_t H_t s.a. Y_t = A_t K_t^{\alpha} (L_t H_t)^{1-\alpha}$$

$$R_t = \alpha A_t K_t^{\alpha-1} (L_t H_t)^{1-\alpha}$$

$$W_t = (1-\alpha) A_t K_t^{\alpha} (L_t H_t)^{-\alpha}$$

Following the condition of macroeconomic equilibrium where the product, Y_t, is equal to household consumption, C_t, and investment in physical capital, I_{Kt}.

$$Y_t = C_t + I_{Kt}$$

Consequently, the accumulation process of physical capital is determined by the stock of physical capital of period t together with the investment in it.

$$K_{t+1} = (1-\delta_k) K_t + I_{Kt}$$

Investment in human capital, I_{Ht}, is based on a productivity factor, B_t, and the time allocated to training, u_t:

$$I_{Ht} = B_t u_t^{\phi}$$

The parameter ϕ determines the concavity if 0< ϕ <1 or the convexity if ϕ >1 of the human capital investment equation. On the other hand, the process of accumulation of human capital is determined by the stock of human capital of period t together with the investment in it.

$$H_{t+1} = (1-\delta_H) H_t + I_{Ht}$$

Households have a preference structure on the infinite horizon for a utility function dependent on consumption, C_t, and leisure, O_t. The restriction of household time, normalized to 1, is composed of leisure time, O_t, time spent working, L_t, and the time allocated to educational training, u_t. The functional form of utility can be transformed in terms of the time households spend working and training, where $\gamma \in (0,1)$ corresponds to the elasticity of substitution between consumption and leisure.

$$U(C_t, L_t, u_t) = \gamma Ln C_t + (1-\gamma) Ln(1 - L_t - u_t)$$

Consequently, households face the problem of choosing the path of consumption, work and training time that allows them to maximize their well-being. Thus, the problem of maximizing the intertemporal utility of households is defined as follows:

$$\max E_t \sum_{t=0}^{\infty} \beta^t (\gamma Ln C_t + (1-\gamma) Ln(1 - L_t - u_t))$$

$$s.a. K_{t+1} = R_t K_t + W_t L_t H_t - C_t + (1-\delta_k) K_t$$

$$H_{t+1} = (1-\delta_H) H_t + B_t u_t^{\phi}$$

$$K(0) > 0 H(0) > 0$$

The parameter $\beta \in (0,1)$ is the intertemporal discount factor of utility that depends on the subjective rate of intertemporal preference θ, whose value is strictly positive and determines the valuation of individuals of present utility against future utility, thus guaranteeing their stable behaviour over time. In this way, the intertemporal discount factor for utility is defined as follows:

$$\beta = \frac{1}{1+\theta}$$

The previous model describes a planned economy composed of a set of endogenous variables Y_t, C_t, K_t, I_{Kt}, H_t, I_{Ht}, L_t, u_t, W_t, and R_t, and exogenous variables A_t and Bt. To date, a deterministic system has been defined; however, in practice, the behaviour of the economy is characterized by more complex dynamics. For this reason, on the basis of general dynamic equilibrium (GDE) models (Torres 2010), the technological productivity factors of the productive goods sector A_t and the educational goods sector B_t are endogenized. For this, it is assumed that they are determined by a stochastic process such that they follow a first-order autoregressive process AR1 defined as follows:

$$A_t = A_{t-1}^{\rho_A} + \varepsilon_{A_t} \; B_t = B_{t-1}^{\rho_B} + \varepsilon_{B_t}$$

Now, the system of equations is made up of 12 endogenous variables Y_t, C_t, K_t, I_{Kt}, H_t, I_{Ht}, L_t, u_t, W_t, R_t, A_t, and B_t and two exogenous variables ε_{A_t} and ε_{B_t} so that the equilibrium is defined as follows:

$$Y_t = A_t K_t^{\alpha} (L_t H_t)^{1-\alpha}$$

$$C_t \left(\frac{1-\gamma}{\gamma} \right) = W_t H_t (1 - L_t - u_t)$$

$$\beta \left(\frac{C_{t-1}}{C_t} \right) (1 - \delta_k + R_t) = 1$$

$$K_{t+1} = (1 - \delta_k)K_t + I_{Kt}$$

$$Y_t = C_t + I_{Kt}$$

$$H_{t+1} = (1 - \delta_H)H_t + I_{Ht}$$

$$I_{Ht} = B_t u_t^{\phi}$$

$$\beta \left[\frac{\gamma W_t L_t}{C_t} + \frac{(1-\gamma)(1-\delta_H)}{(1-L_t-u_t)(\phi B_t u_t^{\phi-1})} \right] = \frac{(1-\gamma)}{(1-L_{t-1}-u_{t-1})(\phi B_{t-1} u_t^{\phi-1})}$$

$$R_t = \alpha A_t K_t^{\alpha-1} (L_t H_t)^{1-\alpha} \, W_t = (1-\alpha) A_t K_t^{\alpha} (L_t H_t)^{-\alpha}$$

$$A_t = {A_{t-1}}^{\rho_A} + \varepsilon_{A_t}$$

$$B_t = {B_{t-1}}^{\rho_B} + \varepsilon_{B_t}$$

4 Calibration

Relatively recent studies have supported the calibration of the parameters defined in the set Ω for the case of the Colombian economy with the following structure:

$$\Omega = \{\alpha, \beta, \delta_k, \delta_H, \gamma, \phi\rho_A, \rho_B\}$$

α: This parameter represents the product elasticity of capital, which measures, in the production function, the response of GDP to changes in the productive factor of physical capital. Consequently, 1-α corresponds to the elasticity product of human capital. Thus, the alpha parameter determines the participation of each factor in the production process. The value of this parameter is obtained from the Penn World Table (PWT) 10.0 series on the basis of the participation of physical capital in production as a proxy, and the calibrated value is 0.2910. This value is consistent with the value of various economies (Torres 2010) and coincides with the calculations obtained in Greco (2002).

β: This parameter corresponds to the intertemporal discount factor of the utility. For its calculation, the calibration is carried out according to Arango and Ramírez (2007), whose estimated value corresponds to 0.87.

δ_k : The parameter delta subindex *k* corresponds to the depreciation rate of physical capital, and the series average depreciation rate of the capital stock of PWT 10.0 is used. The calibrated value is 0.032.

δ_H : The parameter delta subindex *H* corresponds to the depreciation rate of human capital. The calibrated value of Castillo–Aroca (2016) is 0.0242.

γ : The gamma parameter represents the leisure-consumption elasticity and allows households to choose between consumption, work and training time. The parameter is obtained from Anzoátegui (2015) and Torres (2010) as 0.4.

ϕ : The phi parameter corresponds to the productivity of education, which determines its marginal returns on investment in human capital. Pardo (2007) assumed a value of 0.75 for this parameter.

ρ_A: Rho sub A corresponds to the autoregressive parameter that measures the persistence of the disturbance in the productivity of the productive goods sector such that $|\rho_A| < 1$ guarantees the stationarity of the process. In the literature, this parameter is arbitrarily selected at values close to 0.9. Using the Solow residual series for Colombia, the correlogram analysis indicates that the condition is satisfied, and a value of 0.967931 is calculated.

ρ_B: Rho sub-B corresponds to the autoregressive parameter that measures the persistence of the disturbance in the productivity of education such that $|\rho_B| < 1$ so that the stationarity of the process is guaranteed. When using the Professor Ratio series is used as a proxy for the productive factor of education, the correlogram analysis indicates that it satisfies the condition, and a value of 0.999659 is calculated. Table 1 shows the calibration results for the parameters of the set.

To determine the long-term path of the variables, the values close to the steady state are calculated. The steady state assumes that the variables converge to a constant value such that the following sequentially holds for all variables:

$$Y = Y_t = Y_{t+1} = Y_{t+2} = \ldots = Y_{t+n},\ C = C_t = C_{t+1} = C_{t+2} = \ldots = C_{t+n},\ K = K_t = K_{t+1} = K_{t+2} = \ldots = K_{t+n},\ I_K = I_{Kt} = I_{Kt+1} = I_{Kt+2} = \ldots = I_{Kt+n},\ H = H_t = H_{t+1} = H_{t+2} = \ldots = H_{t+n},\ I_H = I_{Ht} = I_{Ht+1} = I_{Ht+2} = \ldots = I_{Ht+n},\ L = L_t = L_{t+1} = L_{t+2} = \ldots = L_{t+n},\ u = u_t = u_{t+1} = u_{t+2} = \ldots = u_{t+n},\ W = W_t = W_{t+1} = W_{t+2} = \ldots = W_{t+n},\ R = R_t = R_{t+1} = R_{t+2} = \ldots = R_{t+n},\ A_t = A^*,\ B_t = B^*$$

Given the above, we carry out the computation of the model through MATLAB with the Dynare complement, and it is possible to obtain the deviations of the neighbourhoods of the steady state, maintaining the assumption of normalization of the time allocation to 1 (see Table 2).

By applying the Hodrick–Prescott filter for the historical series of the variables Y_t, C_t, K_t, I_{Kt}, H_t, and R_t for the period of 1970–2019 from data from Penn World Table 10.0, the correlation matrix of the detrended data is obtained (see Table 3).

Table 3 reveals key economic patterns for the period of 1970–2019. Notably, there are strong positive correlations between consumption (C) and GDP (Y) (0.8971) and between physical capital (K) and consumption (0.6819), confirming the central role of consumption in aggregate demand and its link to capital accumulation. Human capital (H) has a weak relationship with GDP (0.0479) but a moderate relationship with physical capital (0.4211), suggesting that its impact operates mainly through indirect complementarities. Interest rates (R) have a positive relationship with investment (0.3656), a pattern that merits further investigation. These findings reflect the complex dynamics between factor accumulation and macroeconomic outcomes.

These results are comparable with the correlation matrix produced for the computable model solution (see Table 4).

When comparing the results obtained from the correlation matrix constructed from the historical series and the correlation matrix for the solution of the model, it is evident that the correlations (positive or negative) of the variables are mostly similar between the two matrices. Consequently, the model roughly describes the reality of economic growth in Colombia.

Table 1 Parameter calibration for the set

Parameter	Calibrated value	Sample range	Description	Paper source	Source
α : Elasticity product of capital	0.2910	1950–2019	Geometric mean of capital share in GDP	The Next Generation of the Penn World Table	Author's calculations from (Torres 2010)
β : U tility intertemporal discount factor	0.87	1970–2003	Discount rate	Risk aversion and subjective discount rate: the Colombian case, 1970–2003	Arango y Ramírez (2007)
δ_k : Physical capital depreciation rate	0.032	1950–2019	Geometric mean of the average depreciation rate of capital stock	The Next Generation of the Penn World Table	Author's calculations from (Feenstra et al. 2015)
δ_H : Human capital depreciation rate	0.0242	2007–2014	Geometric mean of the human capital depreciation rate	Depreciation of human capital in the Pacific Alliance during 2007–2014	Castillo-Aroca (2016)
γ : Leisure-consumption elasticity	0.4	2000–2013	Preference parameter	A deterministic general equilibrium model with heterogeneous agents and fiscal policy: An analysis for Colombia/ Introduction to Dynamic General Equilibrium	Anzoátegui (2015)/Torres (2010)
ϕ : Education productivity	0.75	NA	Social returns to education; arbitrarily assumed	Increasing social returns to human capital accumulation and public financing of education	Pardo (2007)
ρ_A : Autoregressive parameter of technological factor A	0.9679	1950–1996	Persistence of productivity shocks in the goods sector ($\|\rho_A\|<1$ *ensures stationarity*); estimated using the Solow residual correlogram analysis	The revised Solow residual	Author's calculations from Bernal (2010)
ρ_B : Autoregressive parameter of technological factor B	0.9996	1970–2019	Persistence of productivity shocks in the education sector ($\|\rho_B\|<1$ ensures stationarity); estimated using the student–teacher ratio correlogram analysis	World bank open data	Author's calculations from World Bank (2021)

Source: Author's calculations

5 Impulse response functions

The results of the matrices reflect the expected signs for C, IK, K, IH, and H. Regarding the proportion of time spent working L, from a macroeconomic perspective, it is presumed that an increase in the labour force has an impact on greater production, so dedicating more time to work should reflect a positive correlation with Y (Schein and Haruvi 2017; Muñoz de Bustillo 1997).

Table 2 Steady-state values

Variable	Value	Deviations from Y^*
Y^*	3.3265	1
C^*	3.1558	0.9487
K^*	5.3356	1.6040
${I_K}^*$	0.1707	0.0513
H^*	9.6161	2.8908
${I_H}^*$	0.2327	0.07
L^*	0.2850	0.0857
u^*	0.1431	0.0430
W^*	0.8607	0.2587
R^*	0.1814	0.0545
A^*	1	
B^*	1	

Source: Author's calculations

Table 3 Correlation matrix from the historical series 1970–2019

Variables	Y	C	IK	K	H	R
Y	1					
C	0.8971	1				
IK	0.6100	0.6021	1			
K	0.4851	0.6819	0.2557	1		
H	0.0479	0.1135	0.0405	0.4211	1	
R	0.3948	0.3040	0.3656	−0.1063	−0.0946	1

Source: Author's calculations

Table 4 Correlation matrix for the model solution

Variables	Y	C	IK	K	IH	H	u	R	B
Y	1								
C	0.9999	1							
IK	0.9946	0.9940	1						
K	0.9999	0.9999	0.9929	1					
IH	0.9889	0.9885	0.9901	0.9866	1				
H	0.9996	0.9995	0.9956	0.9990	0.9927	1			
u	0.9883	0.9880	0.9898	0.9861	0.9999	0.9922	1		
R	0.0065	0.0014	0.1003	-0.0104	0.1217	0.0293	0.1241	1	
B	0.9900	0.9897	0.9910	0.9879	0.9999	0.9936	0.9999	0.1160	1

Source: Author's calculations

From an analysis with impulse response functions, the deviations of the variables with respect to their steady state in the face of an exogenous shock in the productivity factor of education are described. In particular, the effects on the decision of households to allocate time to training activities, the behaviour of the human capital index and the effect on national income are considered. Assuming an increase of 1% in the productivity of the education factor, that is, $\sigma_B = 0.01$, the stationary states move as described by the impulse response functions (see Fig. 1).

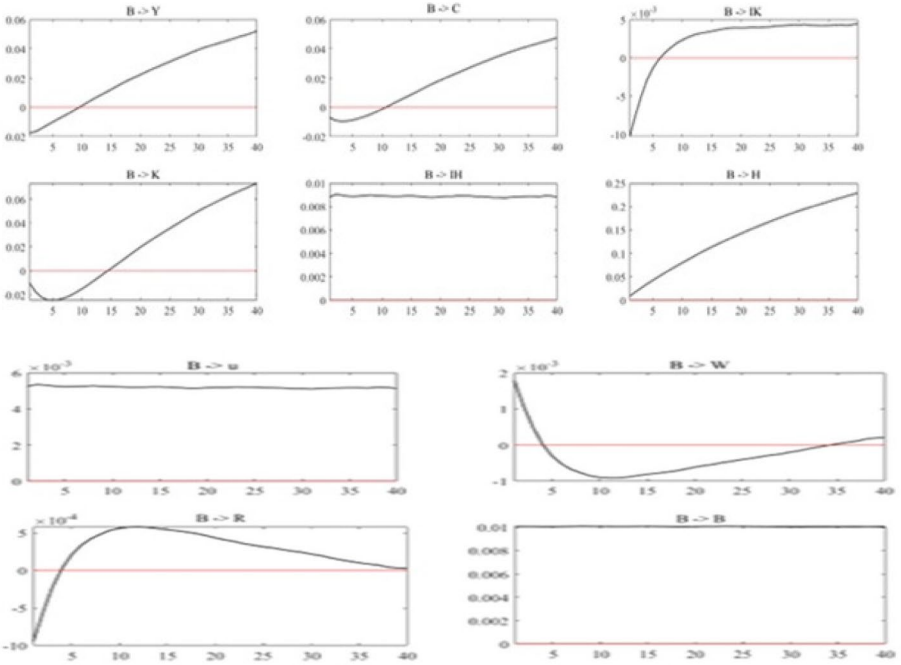


Fig. 1 Impulse response functions in the face of innovation in the productivity of education factor B of one standard deviation in the system variables. *Note* The figure displays the dynamic responses of key macroeconomic variables Y_t, C_t,K_t,I_{Kt},H_t, I_{Ht},u_t, andR_t to an exogenous shock in the productivity of education factor B_t. Each panel traces the deviation of a variable from its steady state over a 40-period horizon. The horizontal red line indicates the baseline (no effect) level. Positive and persistent responses in output, capital, and human capital suggest long-term growth effects triggered by improvements in educational productivity.
Source Author's calculations

A positive shock in the productivity of education generates a deviation of 2% below its steady state on the production of the economy; however, this effect is only maintained for an additional period. This deviation begins to present a positive increase until it reaches the initial value between Periods 9 and 10 and continues to increase until finally reaching a positive deviation of 5%. The above findings indicate that a shock initially generates a contraction in GDP but allows growth in the long term. This transition assumes that an immediate increase in the productivity of education is transferred with a transitory effect to the final goods production function.

The mechanism implicit in the model considers the changes in the decisions of the agents regarding the relationship between consumption and investment in physical capital and allows us to understand the way in which this effect operates since these variables have a greater share of aggregate demand.

Faced with a positive shock, consumption decreases by 0.7% and continues to decrease until Period 3, after which a recovery begins and reaches the initial level of the steady state in Period 11; already, in Period 40, it reflects a positive deviation of 4.7% with respect to the initial value. For its part, physical capital shows a negative deviation of 1% and continues to decrease until reaching a minimum in Period 5, after which it reflects a recovery and reaches

an increase of 7.3% in Period 40. Consequently, consumption and physical capital reflect a negative effect in the short term in the face of the shock; however, in the long term, there is growth above the steady state of these variables.

The investment in physical capital decreases by 1%; however, it quickly returns to the initial point in Period 6, which may be associated with the response of physical capital to the same disturbance. It finally reaches a new steady state with a deviation of approximately 0.4% once consumption and capital enter a transition phase. With respect to investment in human capital, a positive effect of 1% on the productivity of education generates a positive displacement of the steady state in a slightly lower proportion, that is, 0.8%. This seems to indicate that a shock in the variable will have an infinite persistence effect on investment in human capital.

The effect on human capital is persistent over time, which allows an increase of more than 22% in Period 40. As expected, this persistence reflects the enhancing effect of a positive variation in education productivity on human capital. The mechanism that operates within this component is the marginal ratio of substitution between consumption and leisure; education can be pure consumption or pure investment, or it can serve both purposes. For those cases in which activities related to the knowledge and skills of individuals are described as consumption, the latter is spoken of as an enhancer of the stock of human capital (Fuentes and Vatter 1991; Thakurata 2021). Likewise, changes in the productivity of this factor affect the cost of the productive factors, which in turn ultimately results in the deviation previously described.

Labour decreases by 0.2% with a persistent but very slight negative effect, reaching a negative deviation of 0.1% in Period 40. Consequently, the time allocated to work will not return to its initial stationary state in the face of an exogenous shock since a transitory state of the variable is reflected in subsequent periods. This effect may indicate how an improvement in the productivity of education is properly associated with an improvement in labour productivity and an increase in the payment of said factor (Hutter and Weber 2021) so that individuals perceive a greater incentive to train.

As shown in Fig. 1, a positive shock of 1% in the productivity of education generates a positive shift to a new steady state with a deviation of 0.5% in the time that individuals spend to acquire knowledge in the context of formal education. This response suggests that a shock in the variable has a permanent effect on the time spent on training. The opportunity cost that individuals face is understood as the sacrifice of not receiving wages at the present time, reducing their working time and increasing their study time, which will improve their productivity and consequently lead to better entry pay in the future and improved welfare and economic and social inclusion (Mincer 1974; Aslam et al. 2021; Reindl and Tyran 2021).

The effect of a 1% deviation in productivity generates an immediate positive effect on wages of 0.1%. However, the variable returns to its values close to its initial state for Period 37, and it continues to increase over time. This effect is expected, since the remuneration of work should reflect a persistent increase as a result of the potentiating effect that a positive variation in the productivity of education has (Tran et al. 2019).

The return on physical capital deviates negatively by 0.09%, although it initiates a positive transition that allows it to reach a maximum in Period 12, with a deviation of 0.05% from its initial steady state; it decreases slightly to reach Period 40, a deviation close to the starting point. The long-term return on physical capital reflects growth above the steady

state in the early periods. This behaviour is associated with the disturbance of physical capital because when it begins the growth path, profitability returns to its initial stationary state, and a growth transition begins.

6 Vector error correction model

To expand on the previous results and analyse the relationship between education quality and economic growth, a vector error correction model (VECM) is estimated considering that this model has the ability to address endogeneity among macroeconomic variables by capturing bidirectional relationships. Unlike other single-equation models, where the presence of endogenous regressors can bias the estimators, VECMs explicitly consider the system variables to be simultaneously determined. This feature allows for consistent modeling of both short-term dynamics and long-term equilibrium relationships among the variables included (Johansen & Juselius 1990; Lütkepohl 2005a, b).

To do so, GDP per capita is used as a proxy for population-adjusted economic growth. The variable of interest, the student–teacher ratio in tertiary education, is used as a proxy for education quality. The control variables include the human capital index, which measures years of schooling and educational returns; capital stock per capita, which is a proxy for physical capital; the investment share, which is used as a proxy for gross capital formation; and the net rate of depreciation, which captures capital depreciation rates. Government spending is also considered a measure of growth through the provision of public goods, and openness to trade is used as a proxy for the degree of integration into international trade.

The variables are taken for the period 1970–2019 in Colombia, primarily from the Penn World Table 10.0 database, except the student–teacher ratio, which is obtained from World Bank Data. Estimations were performed using RStudio software, which uses the URCA and VARS libraries.

To confirm the stationarity of the series, the Augmented Dickey–Fuller (1979) and Phillips–Perron (1988) unit root tests are applied. Both tests test H0, indicating that the series has at least one unit root. If H0 is not rejected, the series is considered nonstationary (see Table 5).

Since the series in levels are not stationary, except for the investment share, they must have an integration order of d: I(d). Consequently, they must be differentiated d times until stationarity is achieved. The series transformed into first differences are tested using the Augmented Dickey–Fuller, Phillips–Perron, and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) unit root tests (see Table 6).

According to Anaya et al. (2016), Colombia experienced two structural changes in economic growth during the period under analysis. The first occurred in 1980, associated with the external debt crisis in Latin America (Caballero 2019), and the second occurred in 1995, recognized as the beginning of the deterioration of the financial system that would culminate in the 1999 crisis (Torres 2011). Given the above, the Chow test was applied to detect the existence of possible structural breaks in the log (GDP per capita) series in the years 1980 and 1995 (see Table 7). This test allows us to contrast whether there are significant changes in the relationship between the dependent variable and time before and after a specific break point, which is essential since traditional tests such as Dickey–Fuller or Phillips–Perron

Table 5 Test of the unit roots of the Augmented Dickey–Fuller and Phillips–Perron series in levels

Test	Serie	t-Stat	*p* value	Conclusion
Augmented Dickey–Fuller (1979)	GDP per capita	−2.1834	0.5007	Nonstationary (fails to reject H0)
	Student–teacher ratio	−1.7052	0.6926	Nonstationary (fails to reject H0)
	Human capital	−2.6567	0.3109	Nonstationary (fails to reject H0)
	Capital stock per capita	−2.6132	0.3283	Nonstationary (fails to reject H0)
	Investment share	−3.9150	0.0204	Stationary (rejects H0)
	Net rate of depreciation	−2.3569	0.4311	Nonstationary (fails to reject H0)
	Government spending	0.7310	0.9617	Nonstationary (fails to reject H0)
	Openness to trade	−3.1647	0.1071	Nonstationary (fails to reject H0)
Phillips–Perron (1988)	GDP per capita	−3.7782	0.8957	Nonstationary (fails to reject H0)
	Student–teacher ratio	−7.6562	0.6579	Nonstationary (fails to reject H0)
	Human capital	−2.0431	0.9647	Nonstationary (fails to reject H0)
	Capital stock per capita	−5.0393	0.8184	Nonstationary (fails to reject H0)
	Investment share	−19.4434	0.0547	Stationary (rejects H0)
	Net rate of depreciation	−3.4441	0.9119	Nonstationary (fails to reject H0)
	Government spending	0.9661	0.9900	Nonstationary (fails to reject H0)
	Openness to trade	−15.8106	0.1577	Nonstationary (fails to reject H0)

Source: Author's calculations

may not reject the null hypothesis of the unit root when there is an unconsidered structural break (Bai and Perron 1998; Goméz, et al. 2015).

The results obtained show statistically significant evidence of structural changes in both years analysed. For 1980, the F statistic was 7.18, with a *p* value of 0.0019, whereas for 1995, the F statistic was 19.45, with a *p* value of 0.00000076. These results indicate the presence of structural breaks in the aforementioned years, which is why they are considered break points to be included in the estimation of the VECM model.

On the basis of the unit root tests applied, the log (GDP per capita), log (capital stock per capita), and openness to trade (in first differences) series are strongly stationary, as all three tests (ADF, PP, and KPSS) agree on this conclusion, or at least two of them do so with sufficient statistical evidence. Likewise, the log (student–teacher ratio) series is also strongly stationary in first differences. In the case of the net rate of depreciation, although one of the tests (PP) does not show evidence of stationarity, the other two do, so it can be considered stationary.

In contrast, the log (human capital) and government spending series in first differences present consistent evidence of nonstationarity, as two or more tests agree in rejecting the

Table 6 Test of the unit roots of the Augmented Dickey–Fuller, Phillips–Perron and KPSS series in first differences

Serie	Test	t-Stat	p value	Conclusion	Composite conclusion
GDP per capita (log first differences)	Augmented Dickey–Fuller (1979)	−2.6690	0.3064	Nonstationary (fails to reject H0 at all levels)	Stationary (PP and KPSS agree)
	Phillips–Perron (1988)	−18.8580	0.0634	Stationary (rejects H0 at 10%)	
	KPSS (1992)	0.1224	0.0845	Stationary (fails to reject H0 at all levels)	
Student–teacher ratio (log first differences)	Augmented Dickey–Fuller (1979)	−3.5351	0.0478	Stationary (rejects H0 at 5% and 10%)	Strongly stationary (ADF/PP and KPSS agree)
	Phillips–Perron (1988)	−58.4571	0.0100	Stationary (rejects H0 at 5% and 10%)	
	KPSS (1992)	0.1009	>0.1	Stationary (fails to reject H0 at all levels)	
Human capital (log first differences)	Augmented Dickey–Fuller (1979)	−0.9401	0.9382	Nonstationary (fails to reject H0 at all levels)	Strongly nonstationary (ADF/PP and KPSS agree)
	Phillips–Perron (1988)	−5.0047	0.8201	Nonstationary (fails to reject H0 at all levels)	
	KPSS (1992)	0.6803	0.0153	Nonstationary (rejects H0 at 5% and 10%)	
Capital stock per capita (log first differences)	Augmented Dickey–Fuller (1979)	−3.2667	0.0873	Stationary (rejects H0 at 10%)	Strongly stationary (ADF/PP and KPSS agree)
	Phillips–Perron (1988)	−17.8777	0.0801	Stationary (rejects H0 at 10%)	
	KPSS (1992)	0.3054	>0.1	Stationary (fails to reject H0 at all levels)	
Net rate of depreciation (raw first differences)	Augmented Dickey–Fuller (1979)	−3.3123	0.0803	Stationary (rejects H0 at 10%)	Stationary (ADF and KPSS agree)
	Phillips–Perron (1988)	−12.8426	0.3377	Nonstationary (fails to reject H0 at all levels)	
	KPSS (1992)	0.1763	>0.1	Stationary (fails to reject H0 at all levels)	
Government spending (raw first differences)	Augmented Dickey–Fuller (1979)	−3.0754	0.1438	Nonstationary (fails to reject H0 at all levels)	Nonstationary (ADF and KPSS agree)
	Phillips–Perron (1988)	−45.6608	0.0100	Stationary (rejects H0 at 5% and 10%)	
	KPSS (1992)	0.5395	0.0328	Nonstationary (rejects H0 at 5% and 10%)	
Openness to trade (raw first differences)	Augmented Dickey–Fuller (1979)	−4.0670	0.0143	Stationary (rejects H0 at 5% and 10%)	Strongly stationary (ADF/PP and KPSS agree)
	Phillips–Perron (1988)	−33.5367	0.0100	Stationary (rejects H0 at 5% and 10%)	
	KPSS (1992)	0.0539	>0.1	Stationary (fails to reject H0 at all levels)	

Source: Author's calculations

null hypothesis of stationarity. In the case of log (human capital), all tests (ADF, PP, and KPSS) agree that the series remains nonstationary in first differences, suggesting a possible integration of order two: I (2). The same is true for government spending, whose evidence also points to integration greater than one. Since Johansen's cointegration analysis requires

Table 7 Chow test for structural break testing for Colombia: log (GDP per capita)

Test	Serie	Breakpoint date	f-Stat	p value
Chow test	Log (GDP per capita)	1980	7.1825	0.0019
	Ln (GDP per capita)	1995	19.4463	0.0000

Source: Author's calculations

Table 8 Information criteria for determining the length of the model.
Source Author's calculations

Lag	AIC	HQ	SC	FPE
1	−3.6596	−3.2152	−2.4787	0.0259
2	−4.7205	−3.9057*	−2.5554*	0.0093*
3	−4.7996*	−3.6146	−1.6505	0.0095

* Indicates the optimal lag selected by the information criterion. AIC (Akaike information criterion), HQ (Hannan–Quinn criterion), SC (Schwarz criterion) and FPE (final prediction error)

all variables to be I (1), these two variables were excluded from the cointegration analysis to preserve the validity of the procedure.

Therefore, the variables considered in the cointegration model (except for log (human capital) and government spending) are integrated into order one: I (1), which is a prerequisite for cointegration analysis.

A set of time series is said to be cointegrated, that is, the vector X_t is cointegrated of order CI(d,b) if all the components of X_t are I(d) and if there exists a vector of coefficients β, called the cointegration vector, such that the linear combination $\beta\prime X_t = e_t$ is I(d-b), b>0 (Engle and Granger 1987).

According to the results of the unit root tests, the vector X_t, which is composed of the variables log (GDP per capita), log (capital stock per capita), log (student–teacher ratio), net rate of depreciation, and openness to trade, is integrated of order one: I(1). Once this assumption was met, the existence of the cointegration vector β was determined using the Johansen multivariate cointegration test.

To select the optimal number of lags, information criteria were used, considering a constant (intercept) with no linear trend. These results allow the length of the model to be estimated (see Table 8).

The HQ, SC, and FPE criteria indicate that the optimal number of lags for the model is two, which is common in models with annual data. Although the AIC suggests that three lags could minimize information loss, the selection was made while considering a balance between parsimony and fit, choosing two lags as the optimal value (see Table 9).

The results of the Johansen cointegration test differ depending on the statistic considered. While the trace test suggests the existence of up to four cointegrating vectors Γ(4), rejecting the null hypothesis from Γ(0) to Γ(3) at the 1% significance level, the maximum eigenvalue test provides statistically significant evidence only for two cointegrating vectors Γ(2) at the same significance level. In this case, it was decided to use Γ(4) in the model estimation since the trace test tends to be more conservative and robust when evaluating systems with possible multiple long-run relationships. Johansen and Juselius (1990) suggest that the trace test may be preferable if there are reasons to believe that more than one cointegrating relationship may be present. Since the model includes multiple interrelated macroeconomic

Table 9 Johansen multivariate cointegration test

Hypothesized No. of CE(s)	Unrestricted Cointegration Rank Test (Trace)			
	Eigenvalue	Trace Statistic	0.05 Critical Value	Conclusion
None	0.7696	151.07	76.07	Rejects H_0 at 5%: Significant evidence of at least 1 cointegrating vector
At most 1	0.5779	82.09	53.12	Rejects H_0 at 5%: Significant evidence of at least 2 cointegrating vectors
At most 2	0.3576	41.55	34.91	Rejects H_0 at 5%: Significant evidence of at least 3 cointegrating vectors
At most 3	0.3218	21.02	19.96	Rejects H_0 at 5%: Significant evidence of at least 4 cointegrating vectors
At most 4	0.0517	1.62	9.24	Fails to reject H_0: No evidence beyond 5 cointegrating vectors
Hypothesized No. of CE(s)	Unrestricted cointegration rank test (Maximum eigenvalue)			
	Eigen value	Max-eigen statistic	0.01 critical value	Conclusion
None	0.7696	68.99	34.40	Rejects H_0 at 1%: Significant evidence of at least 1 cointegrating vector
At most 1	0.5779	40.54	28.14	Rejects H_0 at 1%: Significant evidence of at least 2 cointegrating vectors
At most 2	0.3576	20.80	22.00	Fails to reject H_0: No evidence of ≥ 3 cointegrating vectors
At most 3	0.3218	18.25	15.67	Rejects H_0 at 5%: Significant evidence of at least 4 cointegrating vectors
At most 4	0,0517	2.50	9.24	Fail to reject H_0: No evidence of ≥ 5 cointegrating vectors

Source: Author's calculations

Table 10 Normalized cointegration vectors from the VECM Γ(4).
Source Author's calculations

Variable	ECT1 (Vector 1)	ECT2 (Vector 2)	ECT3 (Vector 3)	ECT3 (Vector 4)
Log(GDP per capita)	1.0000	0.0000	0.0000	0.0000
Log (capital stock per capita)	~0.0000	1.0000	0.0000	~0.0000
Net rate of depreciation	~0.0000	0.0000	1.0000	~0.0000
Openness to trade	~0.0000	~0.0000	~0.0000	1.0000
Log(student–teacher ratio)	−1.2022	−0.6429	−0.0159	0.0566
Constant	−4.9968	−7.3829	0.0232	−0.3051

"~0.0000" represents values that are numerically close to zero due to rounding

variables, it is theoretically plausible to expect more than one equilibrium relationship (see Table 10).

Since the investment share variable was found to be stationary at different levels, it cannot be included as an endogenous variable in the cointegration model. However, given its important role as a determinant of growth, it was incorporated as an exogenous variable in the VECM. Similarly, the structural break dummy 1980 and structural break dummy 1995, which capture potential structural breaks identified using the Chow test, were also included

as exogenous variables because they are stationary dummies. This inclusion allows controlling for structural effects without distorting the cointegration relationship between the endogenous variables.

The cointegration vectors presented have been normalized with respect to a different variable in each case, which allows for the identification of long-term structural relationships between the system's endogenous variables (see Table 11).

In Vector 1, normalized with respect to log (GDP per capita), the negative coefficient of log (student–teacher ratio) suggests that a lower student–teacher ratio (higher educational quality) is positively correlated with higher per capita income levels in the long run.

Vector 2, normalized with respect to log (capital stock per capita), presents an even more moderate negative coefficient for log (student–teacher ratio), which reinforces the interpretation that higher educational quality is also positively correlated with capital accumulation.

Vector 3, normalized with respect to the net rate of depreciation, shows a weak negative relationship with log (student–teacher ratio), which indicates that higher educational quality has a weak positive correlation with the average rate of depreciation of the capital stock.

Table 11 VECM estimates for the short-run equation of GDP per capita

Variable	Estimate	Std. Error	t value	Pr(>|t|)
Cointegration term 1 (ECT1)	−0.0384	0.0936	−0.4110	0.6842
Cointegration term 2 (ECT2)	0.3184	0.0707	4.5030	0.0001
Cointegration term 3 (ECT3)	−30.0225	10.3914	−2.8890	0.0071
Cointegration term 4 (ECT4)	−0.5674	0.4496	−1.2620	0.2166
Structural break dummy 1980	−0.0279	0.0197	−1.4150	0.1673
Structural break dummy 1995	−0.0375	0.0162	−2.3250	0.0270
Investment share	0.7412	0.1752	4.2310	0.0002
GDP per capita (1st diff log)	−0.4167	0.2102	−1.9830	0.0566
Capital stock per capita (1st diff log)	0.5034	0.3509	1.4350	0.1618
Student–teacher ratio (1st diff)	0.0803	0.0617	1.3020	0.2029
Net rate of depreciation (1st diff)	−25.6491	38.7113	−0.6630	0.5127
Openness to trade (1st diff)	0.9946	0.4731	2.1020	0.0440
GDP per capita (2nd diff log)	−0.0743	0.2138	−0.3480	0.7305
Capital stock per capita (2nd diff log)	0.0017	0.3116	0.0060	0.9956
Student–teacher ratio (2nd diff)	0.1210	0.0701	1.7250	0.0948
Net rate of depreciation (2nd diff)	−52.4268	27.2376	−1.9250	0.0638
Openness to trade (2nd diff)	0.2025	0.4471	0.4530	0.6539

Source: Author's calculations

Finally, Vector 4, normalized with respect to the openness to trade variable, indicates a slightly positive relationship with log (student–teacher ratio), which could be interpreted as an inverse relationship between the degree of economic integration and the quality of human resources.

Below are the estimated coefficients for the short-term equation for GDP per capita in logarithms, derived from the VECM model with four cointegration terms. The equation also includes differentiated first- and second-order terms for the explanatory variables, as well as structural dummy variables for the years 1980 and 1995.

The estimated model shows the short-term adjustment dynamics of GDP per capita towards its long-term equilibrium relationship.

The cointegration terms ECT2 and ECT3 are highly significant, indicating that long-term deviations from these relationships have a significant effect on the dynamics of GDP per capita. The positive sign of ECT2 and the negative sign of ECT3 suggest different long-term adjustment mechanisms. The terms ECT1 and ECT4 were not statistically significant ($p > 0.05$).

With respect to the structural variables, investment share reflects a positive and statistically significant association with short-term GDP per capita growth. The 1995 structural break dummy was significant, revealing a persistent negative effect of the economic crisis that began with the deterioration of the Colombian financial system.

The trade openness variable (openness to trade) is also significant in first differences and has a positive sign, suggesting that a higher degree of openness is associated with increases in economic growth.

Although other variables in first differences, such as capital stock per capita, student–teacher ratio, and net rate of depreciation, show the expected signs, they do not reach statistical significance at the 5% level. However, the student–teacher ratio and net rate of depreciation in second differences reach significance at the 10% level, suggesting possible lagged effects of educational quality.

Overall, these results provide evidence of robust long-term relationships in the system and adjustment dynamics that are consistent with economic theory.

Once the VECM model is estimated, compliance with the classic assumptions associated with residual analysis is evaluated to validate the robustness of the results obtained.

First, the multivariate normality of the residuals is analysed using Mardia's tests for skewness and kurtosis. In both cases, the p values (0.1900 and 0.1968) exceed the conventional 10% threshold, thus failing to reject the null hypothesis of multivariate normality, supporting the statistical validity of the inferences made from the model (see Table 12).

Second, the Portmanteau test, with a lag of 330, yielded a p value of 0.4153, indicating the absence of serial autocorrelation in the residuals. This result suggests that there is no systematic information not captured by the model, reinforcing the suitability of the adopted specification.

Regarding heteroskedasticity, the multivariate ARCH test showed a p value of 1, leading to the conclusion that the residuals are homoskedastic and, therefore, exhibit constant variance over time.

Finally, the stability analysis of the model shows that all characteristic roots of the associated VAR system have a modulus less than one (ranging between 0.181 and 0.902), which confirms that the dynamic system is stable in the sense that disturbances tend to dissipate in the long term.

Table 12 VECM residue analysis

Assumption	Test	*p* value	Conclusion
Normality	Mardia skewness test/ Mardia kurtosis test	0.1900/0.1968	Fails to reject H_0 (*p* > 10%): Residuals are multivariate normal
Serial correlation	Portmanteau test (lag 330)	0.4153	Fails to reject H_0 (*p* > 10%): No serial correlation in residuals
Heteroskedasticity	Multivariate ARCH Test	1	Fails to reject H_0 (*p* > 10%): Residuals are homoskedastic
Stability	VAR characteristic roots	–	All 10 roots have modulus < 1 (range: 0.181 to 0.902): Model is stable

Source: Author's calculations

To illustrate the relationships between the variables, impulse response functions are used. The response of variables j to a shock related to variable k allows for a graphical observation of the dynamic interrelationships of the system. Impulse response functions are generally more appropriate in vector autoregressive (VAR) models since a cointegrated model lacks valid moving average (MA) processes. However, it is possible for a VECM to construct impulse response functions homologous to the VAR functions from the prediction errors but with a different scale (Lütkepohl 2005a, b).

Since the variables are integrated to order I(1), the functions will have permanent effects, so a shock will not necessarily decline asymptotically but will remain constant over time (see Fig. 2).

Orthogonal impulse–response functions allow us to analyse the dynamic effects of an exogenous shock in the log (student–teacher ratio) ln_STR on the other variables in the system. For each function, 100 bootstrap simulations were used with a 95% confidence interval, which provides robustness to the estimates.

The impact of an increase in ln_STR generates a sustained positive response in the log (GDP per capita) ln_GDPpc, which gradually increases until it stabilizes around Period 30, suggesting a permanent effect of the shock on economic growth. This could indicate that, in the context analysed, increases in educational coverage have had beneficial effects on economic growth, even when there are tensions in the quality of educational services. This finding suggests that, in phases of educational expansion, the aggregate benefits of access can outweigh the marginal efficiency losses, at least in the short and medium terms. However, the literature warns that the effect of human capital depends on other factors. For example, Aslam (2020) found that human capital alone often does not drive growth, except when strong institutions exist to amplify its impact.

In the case of log (capital stock per capita) ln_Capital_Stock_pc, a similar response is observed: the initial shock to ln_STR induces a persistent increase in capital, with an upwards trajectory that stabilizes towards the end of the analysed time horizon. The litera-

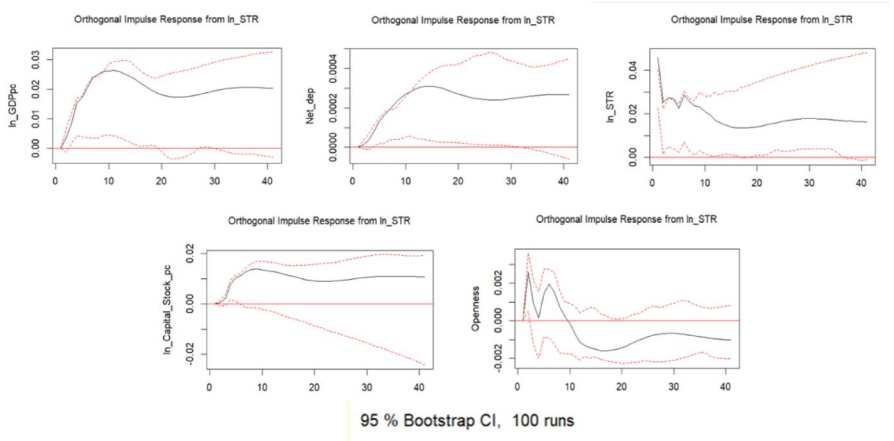


Fig. 2 The impulse response functions to an innovation in the student–teacher ratio of a standard deviation in the variables of the system. *Note* The figure shows the impulse response functions of the variables log (GDP per capita) *ln_GDPpc*, net rate of depreciation *Net-dep*, log (student–teacher ratio) *ln_STR*, log (capital stock per capita) *ln_Capital_Stock_pc* and openness to trade *Openness* versus an innovation of a standard deviation in log (student–teacher ratio).
Source Author's calculations

ture suggests that the quality of human capital increases returns on investments in physical capital (Hanushek and Woessmann 2015).

Moreover, the net rate of depreciation (Net-dep) shows a smaller, albeit sustained, positive response, which could be interpreted as a signal of increasing investment in human capital that does not translate into greater depreciation of physical capital. As Hanushek and Woessmann (2015) showed, human capital increases the productivity of physical capital and mitigates its depreciation, reducing its dependence on capital-intensive sectors.

The variable openness to trade, which measures trade openness, shows a temporary negative response to the shock, returning to its initial level in the long term. This could suggest that, in the short term, an improvement in the quality of education is also related to greater trade openness (as indicated by the model), which is consistent with the idea that a more educated workforce facilitates foreign trade. The results agree that education must be accompanied by an open economy and efficient institutions to maximize growth, as noted by Aslam (2020).

Finally, the driven variable ln_STR responds to its own shock immediately with high initial volatility that progressively decreases, stabilizing with a positive effect in the long run, which is consistent with the persistent nature of integrated first-order variables.

Overall, the results support the existence of a positive and permanent effect of improvements in the student–teacher ratio on economic growth and capital accumulation, whereas the impacts on other structural variables, such as trade openness, are transitory. Impulse–response function analysis reveals a positive and persistent relationship between human capital and economic growth, which is mediated by variables such as capital stock and trade openness. This dynamic is consistent with the modern view of the human capital-based growth model, which posits that human capital accumulation requires institutional improve-

ments to fully deploy its impact on development (Aslam et al. 2024). This perspective reinforces the interpretation that, in the Colombian case, the positive responses of economic growth to educational shocks observed in the model could be amplified or sustained over time only if accompanied by institutional transformations that foster an environment conducive to learning, investment, and innovation.

7 Conclusions

The endogenization of knowledge to explain the growth rates of a country allows the accumulation of human capital to be considered as an enhancer of economic growth, and important increases in national income are the consequence of additions to capital stock of this type (Chani et al. 2014). The model used throughout this paper includes educational variables within a traditional neoclassical growth model, which is novelty to develop a calibration technique for the model proposed to determine the relationship between human capital and economic growth in an emerging economy. By calibrating the parameters with figures from the Colombian economy, the theoretical model explains the dynamics of some macroeconomic variables of the country in the face of economic policies, particularly the effect of a policy focused on improving the productivity of education.

Using as a starting point an increase in the productivity of the education factor, the positive effect on long-term economic growth has been demonstrated because of the increase in the stock of human capital and higher productivity in the labour force.

The impact in the short term, on the other hand, reflects the optimizing behaviour of the agents, such that the transition to a new stationary state with respect to the time they spend on training activities generates a negative deviation in consumption, the accumulation of physical capital, the investment of said capital and the time devoted to work. Such responses to the shock also cause a negative deviation in economic growth.

With respect to the effects on the variables after the shock on the productivity of the educational factor, none of them return to its initial stationary state, and the investment in human capital and the time allocated to training are immediately shifted to a new stationary state. The cause of this is the high correlation that they present with the factor B_t, as reflected in the correlation matrix.

The strongest effect occurs for human capital stock, which reaches a deviation above 20%, since economic growth is positively correlated with this variable, both in the correlation matrix of the historical series and that of the model. Higher rates of economic growth depend on the development of policies that improve the conditions and quality of education, encouraging the economy to concentrate higher levels of the stock of human capital (Rahim et al. 2021). In particular, favouring tertiary education will translate into a more accelerated convergence of the dynamics of the business cycle.

The effect on wages reflects the virtuous cycle of education consumption, and higher wages associated with improved productivity of the labour force have an impact on increased purchasing power. Once a basic income is covered, demand increases formal training activities (Tran et al. 2019). The results of this dynamic are presented in the impulse–response functions, where the increase in wages reaches a positive deviation above 2%.

The relationship between both forms of capital considers their complementarity: high levels of human capital stock encourage investment in physical capital, as it compensates

for the drop in the productivity of human capital (Chen 2020). The results show how physical capital deviates negatively during the first periods to subsequently increase once the investment recovers and reaches a new steady state.

Although empirical evidence has shown a positive relationship between education and economic growth, this research proposes the use of the student–teacher ratio as a proxy for the quality of education. The results indicate that the quality of education has a positive long-term relationship with a country's economic performance. Increases or decreases in this variable have the same effect on GDP per capita.

This long-term relationship reflects a nonsignificant adjustment coefficient, which indicates the absence of statistically significant adjustments towards a long-term equilibrium versus a situation of imbalance. However, there is a causal relationship between the student–teacher ratio and GDP per capita. These results are consistent with what is expected according to economic theory, since the quality of education has an impact on the quality of labour productivity, which in turn generates greater remuneration and ultimately affects an increase in the income of the economy.

The effects of the variables after the shock on the productivity of the educational factor in the VECM do not converge again to zero, as previously discussed, which is a consequence of the nonstationary nature of the series implemented for the analysis. The results of the impact of education on growth are lower than expected, which may be a consequence of the simplicity of the model under which the selection of the variables is based, since economic growth does not depend only on the factors of physical capital and human capital. In the same way, it will be necessary for future research to identify other variables that capture the quality of education and role of institutions and can better capture the relationship between human capital and economic growth.

The main theoretical and practical implications of this study are the effects of economic policies associated with the education on Colombian economic growth, which implies promoting policies related to human capital in the country to achieve sustainable development, social mobility, and overall societal well-being and to generate models that support and strength development theory in the context of emerging economy such as Colombia, which can be an input to decision makers and policy makers in the development of programs and instruments that include education as fundamental factor to achieve development and economic growth.

One of the main criticisms of RBC models is their simplicity when describing the economy in general, since this two-sector model considers only households and firms in the maximization process. The analysis can be strengthened by including the external sector, the government, monopolistic competition, and non-Ricardo agents, among others. Similarly, this work represents a starting point in understanding the effects of economic policies associated with the quality of education on Colombian economic growth, which implies promoting policies related to human capital in the country to achieve sustainable development, social mobility, and overall societal well-being that could include the following aspects (Dillon 2024; Criscuolo et al. 2024; Carstensen and Emmenegger 2023): i. strategies and programs must be aligned with market demands, technological advancements, and the global economic environment; ii. Fostering innovation and integrating economic competitiveness into the educational curricula through real-world problem-solving, critical thinking, and entrepreneurship, iii. creating frameworks that support teachers in gaining new skills, remaining updated with pedagogical innovations, and addressing diverse classrooms

needs to contribute to the development of a competitive economy; iv. Prioritizing investments in infrastructure, technology, and teacher salaries to create an equitable learning environment that supports long-term economic growth, v. promoting workforce development through retraining and upskilling programs that can help bridge the gap between education and labour market needs, vi. Addressing the need for global competencies, such as multilingualism, cross-cultural understanding, and digital literacy to attract foreign investment, foster innovation, and participate in international trade, v. encouraging the use of digital tools, online learning platforms and artificial intelligence (AI) in classrooms to enhance the learning experience and provide students with the skills necessary for the digital economy, and vi. Promoting institutional quality to generate economic growth by reducing uncertainty and promoting efficiency. These elements could guarantee economic growth, social advancement and development in Colombia.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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