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Review

Abnormal dense and dilute nuclear systems

E. E. Kolomeitsev[1,2,3,a], **D. N. Voskresensky**[1,4]

[1] Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia
[2] National Center of Physics and Mathematics, 607328 Sarov, Russia
[3] Matej Bel University, 97401 Banska Bystrica, Slovakia
[4] National Research Nuclear University (MEPhI), 115409 Moscow, Russia

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Abstract Researchers have long been interested in exotic states of matter. In the early 1970s, Migdal proposed existence of metastable or stable nuclei containing a pion condensate, and Bodmer posited existence of collapsed nuclei with a quark core. Lee and Wick proposed that compressed nuclear matter could undergo a phase transition into a scalar condensate state with effectively very light baryons. In the mid-1980s, Witten pointed out that strange quark matter may be an absolutely stable form of matter that can exist in the form of quark strangelets. These hypotheses stimulated large experimental efforts to detect these states in heavy-ion collisions and in terrestrial samples. Additionally, the cosmos is the arena for a broad search for traces of exotic matter in compact objects and other energetic phenomena. We review various approaches and arguments for a possibility of abnormal states of matter, including metastable and stable dense objects with condensates, as well as dilute nuclear objects. Among these, we discuss scalar condensate, p-wave and s-wave pion condensations in dense nuclear systems and formation of a Δ resonance matter. Then we consider a scalar mode condensation triggered by the Pomeranchuk instability in dilute nuclear matter, and a clustering. We also mention recent ideas that a relativistic rotation and a dark-matter component may stabilize abnormal nuclear systems. Finally, we list some observational anomalies which cannot be yet appropriately explained by conventional physics.

Contents

1 Introduction . . . . . .
2 The Lee–Wick model . . . . . .
3 Supercharged nuclei, nuclearites and nuclei–stars . . .
3.1 Supercharged nuclei in a nutshell . . . . . .
3.2 Dibaryons and neutral light bosons . . . . . .
3.3 Rotating nuclear systems . . . . . .
3.4 Anomalous supercharged nuclei consisting dark matter . . . . . .
4 Pion mode and condensation in dense nuclear systems
4.1 Pion-nucleon interaction amplitude and pion condensation . . . . . .
4.2 $\pi^{\pm}$ condensate nuclear systems . . . . . .
4.3 Dense π^0-condensate ferromagnetic nuclear systems
4.4 About the s-wave pion condensation . . . . . .
5 Δ resonance matter . . . . . .
6 Strangelets, strange and hybrid stars . . . . . .
6.1 Quark models . . . . . .
6.2 Abnormal π condensate and strange compact stars and the star mass-radius relation . . . . . .
6.3 Cooling of π condensate abnormal nuclei, nuclei–stars, strangelets and strange stars . . . . . .
7 Dilute stable nuclear systems . . . . . .
7.1 Again, a scalar condensate . . . . . .
7.2 Bose condensates of nuclear clusters . . . . . .
7.3 Dilute nonequilibrium “anomalous” states . . . .
7.4 Dilute quantum droplets . . . . . .
8 Observations . . . . . .
9 Conclusion . . . . . .
References . . . . . .

In 2026 there will be the 100th anniversary of Tsung-Dao Lee and the 115th anniversary of Arkady Migdal. Their ideas sparked extensive theoretical and experimental research programs investigating anomalous dense and dilute nuclear states in the Universe and in heavy-ion collisions. Our review is devoted to developments of this field.

[a] e-mail: evgeny.kolomeytsev@umb.sk (corresponding author)

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1 Introduction

A search for *abnormal* forms of matter accompanied the development of modern physics during the twentieth century. In year 1931 Lev Landau wrote a paper [1], in which he derived an upper limit of star mass consisting of an ultra-relativistic gas. He hypothesized that stars heavier than $\sim 1.5\, M_{\odot}$ may possess regions, in which the laws of quantum mechanics (and therefore of quantum statistics) are violated. In such "unheimliche Sterne" (*Germ*: eerie stars) as they were dubbed by Landau, see [2], electroneutral mixture of protons and electrons, the only particles known by then, would be stable against particle annihilation. The discovery of neutron by James Chadwick in 1932 lead Dmitri Ivanenko to the idea [3] that nuclei may consist of protons and neutrons instead of protons and electrons as thought before. In [4–6] Werner Heisenberg introduced the theory of an atomic nucleus consisting of nucleons, — strongly interacting particles with the putative isospin 1/2, which manifest themselves as protons and neutrons depending of the nucleon isospin projections. In 1934 Walter Baade and Fritz Zwicky [7] proposed that sufficiently massive stars end their life undergoing a transition (supernova event) into compact object consisting mainly of neutrons and called, therefore, neutron stars. More than three decades the neutron stars were considered as a theoretical exotica. Only in 1967 Jocelyn Bell and Anthony Hewish reported [8] the first observation of a radio pulsar. By now neutron stars are not considered as exotica, and more than 3000 pulsars are observed, presently.

What is the ground state of the world around us has been one of the 'big' questions since the beginning of physics and then at the discussion of the role of the either. If there is some underlying symmetry behind the construction of our world, would it be possible to change the ground state? In condensed matter physics this concept was proposed by Landau in 1937 [9,10] for the description of phase transitions. He pointed out that any phase transition may be characterized by a change in the value and/or the symmetry structure of some *order parameter*, which can be used as an effective degree of freedom in the thermodynamic potential. This order parameter can have a rather complicated relation to the fundamental degrees of freedom of the considered system. Generalization to the case of an order parameter interacting with the magnetic field was done by Vitaly Ginzburg and Lev Landau in 1950 [11].

Addressing the question of a possible transformation of the ground state of our physical world it is natural to suggest a simplest choice that the order parameter should have quantum number of vacuum 0^+ (spin and parity), i.e., correspond to some scalar field. There were no scalar particles known in the mid 60 s. Nevertheless, Julian Schwinger tried to formulate a theory of strong interactions using the spin 0, spin 1, and spin 1/2 fields [12]. The first attempts to build a model of interacting hadrons using a symmetry between vector and axial currents, which enter equally in weak interactions, were undertaken in works by John Polkinghorn, Murrey Gell-Mann and Maurice Lévy [13,14]. The so-called σ-model, including scalar and pseudoscalar mesons coupled to nucleons, was formulated. The Lagrangian of this model contains the part symmetric with respect to local transformations keeping parity of the states (the chiral invariant part) and the part, which is not invariant under the parity mixing transformations. Our physical ground state (0^+) is chosen as a result of *the spontaneous symmetry breaking* in the invariant part of the Lagrangian whereas the symmetry breaking part is treated as a rather weak perturbation. A dynamical mechanism of the nucleon mass generation through the mechanism of spontaneous breaking of the chiral symmetry was elaborated in independent works by Valentin Vaks and Anatoly Larkin [15] and Yoichiro Nambu [16], applying superconductivity methods to a Heisenberg type four-fermion two-component Lagrangian.

We should certainly mention a role played by the spontaneous symmetry breaking in the unified electroweak theory of Abdul Salam, Sidney Glashow and Steven Weinberg, and the role played by the Higgs scalar boson in it [17–21]. The study of the properties of superdense matter described by unified gauge theories began in 1972 with the work of David Kirzhnits [22], who showed that the classical scalar field responsible for symmetry breaking disappears at a high enough temperature. In 1974 first grand unified theory was proposed by Howard Georgi and Sidney Glashow [23].

Now let us come to the subject of this review, i.e., discussion of yet considered as exotica, possibilities of existence of various abnormal nuclear systems. Soon after prediction of quarks by Murrey Gell-Mann and George Zweig in 1964, Dmitry Ivanenko and Dmitry Kurdgelaidze suggested that a neutron star may transform in a quark star [24], see also [25]. In 1971 Arnold Bodmer proposed that also atomic nuclei may undergo a transition in a superdense quark state [26].

In 1971 Arkady Migdal put forward the idea that ground state of atomic nuclei may contain a pseudoscalar condensate (0^-) of pions. He demonstrated occurrence of the pion condensation in a sufficiently deep potential well in scalar and electric fields forming deep potential wells for pions and suggested possibility of superdense stable or metastable nuclei glued by the pion condensate [27]. Three types of abnormal π condensate nuclei: superdense, neutron and supercharged ones were discussed in [28,29], more details see in reviews [30,31].

In 1974 Tsung-Dao Lee and Gian-Carlo Wick suggested possible existence of a superdense state of nuclear matter characterized by a value of the scalar (e.g., σ) field different from its mean value in atomic nuclei [32,33]. This idea

was reviewed in [34,35]. The simplest variant of the model contains only two fields, the neutral scalar meson field and the isospinor nucleon field. As the π condensate nuclei, the σ abnormal nuclear state can be either stable or metastable. In the former case the normal nuclei after a while would undergo the phase transition to the new state. In first works the mean field approximation was used in spite of the assumed strong interaction of the scalar meson with the nucleons. To somehow justify this assumption the authors noted analogy between the abnormal state and the Bose–Einstein condensation one. Then Lee and Margulies argued [36,37] that quantum fluctuations may not change the statement about the existence of a super-dense state of nuclear matter.

At the workshop at Bear Mountain, to north from New York City, on "BeV/nucleon collisions of heavy ions" Lee presented his results on the possible vacuum reconfiguration [32] and noted [38] that "*in order to produce the abnormal nuclear state, we must consider reactions in which (i) a large number of nucleons is involved and (ii) the nucleon density can be increased by a significant factor. One is, therefore, led to considerations of high energy collisions between heavy ions, say* $\mathrm{U}+\mathrm{U}\rightarrow Ab+\ldots$, *where Ab denotes the abnormal nuclear state.*" The event at Bear Mountain was pivotal in the conception of heavy ion physics. 1974 there were the Synchrophasotron at the Joint Institute for Nuclear Research in Dubna and a high-energy heavy-ion accelerator, the Bevalac, being completed at the Lawrence Berkeley Laboratory in Berkeley. Ideas about possibility of the production of abnormal nuclei in heavy-ion collisions gave a push to active experimental investigations. One suggested also the look for collapsed quark nuclei [26] and super-dense nuclei with pion condensate in heavy-ion collisions, see Refs. [39,40]. The possibility of achieving a high degree of nuclear matter compression in shock waves, which could contribute to the formation of anomalously dense droplets in heavy ion collisions, was investigated in Refs. [41,42]. Limits on the yields of abnormal nuclei in heavy-ion collisions and on their possible terrestrial concentrations were discussed in Refs. [43–46,46,47]. The interpretation of the data collected from heavy-ion collisions so far does not require the presence of abnormal nuclei. Feasibly, one of reasons is that compressed nuclear matter is produced in heavy ion collisions at a higher temperature than the binding energy of abnormal nuclei. Nevertheless, further dedicated experimental and theoretical analysis is needed.

Cosmos was another domain to look for an anomalous state of matter. Yakov Zeldovich liked to claim that the Universe is a poor man's accelerator: experiments don't need to be funded, and all we have to do is to collect the experimental data and interpret them properly. Years later ideas of spontaneous symmetry breaking penetrated in cosmological models, cf. [48].

Probably in 1975, Migdal returned from a conference and told one of us (D.N.V.) that during his talk, in which he mentioned a possibility of the existence of anomalous supercharged nuclei with a pion condensate, Gell-Mann asked him about a role of electrons in such highly-charged systems. Migdal suggested to think about this problem. In works [49,50] the authors developed a relativistic semiclassical treatment of the Dirac equation in deep electric potentials and the many-particle approximation for electrons occupying levels of the lower continuum. The occupation of the vacuum levels by electrons and negative muons in the deep electric potential well was named "a fermion condensation". The presence of the upper bound on the electric potential was discussed by Berndt Müller and Johann Rafelski in [51]. Preparing this manuscript, we realized that Lee also discussed a role of electrons in abnormal nuclei. He wrote in Ref. [35] that "...the abnormal state can create e^+e^- pairs. The e^+ will be sent to infinity, but a fair fraction of the e^- will be kept within the abnormal nucleus. As Z increases, the number of e^- also increases. The interplay between the added Fermi energy of e^- and the Coulomb energy may eventually bring the abnormal state to the point of instability when A reaches $\approx 10^4$." References [52–54] described the charged condensate pions and vacuum electrons in strong electric fields and suggested a possibility of existence of "nuclei–stars" of arbitrary size (up to the compact star size) glued by the charged π condensate produced in the dense nuclear matter and the electrons produced from the vacuum.

In 1979 Mikhail Troitsky and Nikolai Chekunaev discussed in [55] that at densities above the nuclear saturation density, $n_0 = 0.16\,\mathrm{fm}^{-3}$, the gain in the energy due to the pion condensation can exceed the energy loss associated with the transformation of nucleons into heavier baryon species like hyperons and Δ isobars. The similar idea was applied to quarks by Edward Witten in 1984. In Ref. [56], he argued that in deconfined quark matter, as the baryon density increases, it becomes energetically favorable to occupy the s-quark Fermi sea alongside the u- and d-quark Fermi seas. Moreover, he claimed that the proposed 'strange matter', which contains comparable numbers of u, d, and s quarks may be absolutely stable, i.e. more stable than an iron atomic nucleus. The Migdal hypothesis of abnormal nuclei with a pion condensate, the Lee–Wick hypothesis of abnormal nuclei with a σ condensate and the Witten strange matter hypothesis may play an important role in cosmology, compact stars, cosmic ray physics, and relativistic heavy-ion collisions. Developing Witten's idea, Edward Farhi and Robert Jaffe examined the strange matter clusters with an intermediate baryon number $10^2 < A < 10^7$, which are in equilibrium with respect to weak interactions [57]. Later Alvaro De Rujula and Sidney Glashow suggested the possibility of the existence of quark strangelets, large-size nuclearites and strange

stars of a larger mass (when gravity comes into play), see Refs. [58,59]. The charge distribution in the (strangelets and nuclearites) and the strange stars was considered in [60] in a similar manner as was done for supercharged nuclei in Refs. [50,52,53]. The surface layer of strange objects (strangelets, quark nuclearites and strange stars) has typical thickness ~ 10 fm. Also it was suggested that there may exist stars with a hadronic shell of a macroscopic size and quark core –hybrid stars [61,62].

First investigations were done ignoring possibility of color superconductivity. As a detailed review one may use Ref. [63]. The phenomenon of the Cooper pairing was known to exist in quark matter starting from works [64,65], where the pairing gaps were estimated to be $\lesssim$ MeV, as for nucleons. Then, an interest to a possibility of quark stars revived after arguments were given in [66,67] in favor of large values of the quark-quark pairing gaps, ~ 100 MeV, see Ref. [68] and references therein.

The axion quark nuggets (AQN) model was proposed in Refs. [69–71]. The latter model incorporates axion domain walls formed during the QCD phase transition. The domain walls act as stabilizing structures, alleviating the need for a first-order phase transition as in the original Witten model. A generic feature of the quark nuggets is the electrosphere. In case of antimatter axion quark nuggets the electrosphere of the $\sim 10^{-8}$ cm depth contains positrons. The axion quark nuggets behave as conventional dark matter components in the low density environment and become strongly interacting macroscopically large objects in the relatively high-density environment. The available energy due to matter–antimatter annihilation can be as high as ~ 2 GeV per baryon charge.

If dark matter particles were superheavy (with mass $\gtrsim$ TeV) and negatively charged, they could compensate the Coulomb field being embedded inside a normal nuclear matter drop that leads to the stability of nuclear drops of a large size [72].

Although the initial studies [28,32,33] discussed the possibility of the sigma or pion condensation occurring at densities both greater than and less than n_0, it soon became evident that the latter possibility was unrealistic. On the other hand various models of equation of state of the dilute nearly isospin-symmetric nuclear matter demonstrate the presence of region of the first-order phase transition and spinodal instability at nucleon densities $0.3n_0 \lesssim n \lesssim 0.7n_0$ [73–77]. In this density region the compressibility of nuclear matter is negative and the scalar Landau–Migdal parameter of the Fermi liquid $f_0(n)$ is smaller than -1, that causes the Pomeranchuck instability. In [78] it was shown that the Pomeranchuck instability may lead to a condensation of an effective scalar field in a Fermi liquid. The presence of such condensate would remove instabilities in first and zeroth sounds. Owing to the condensate, a novel metastable state might exist at nuclear subsaturation density for $N \sim Z$, where N is the number of neutrons and Z is the number of protons. In [79] it was realized that the stability region remains also in case of significantly isospin-asymmetric matter.

It was found in Refs. [80,81] that a rapidly rotating drop of nuclear matter, being not bound in the rest frame, may become bound in the rotation frame because of the formation of a charged pion condensate giant vortex. This observation invites looking for the nuclear drops with high angular momenta.

A few words about the notion "abnormal" that we use in the title are in order. Often, a new concept seems 'abnormal' but is considered as normal after a while. Therefore, we should clarify beforehand what we mean by the term abnormal nuclear systems. Various definitions were used in the literature: the scalar-condensate abnormal nuclei; the pion-condensate abnormal nuclei and nuclei–stars; nuggets; strangelets; nuclearites; strange stars and hybrid stars; quark stars and some others. Different authors have given different meanings to these terms. We will basically follow the definitions introduced by pioneers in this field: The term *abnormal nuclei* was used by Migdal to describe hypothetical superdense (with a density larger than several n_0) stable or metastable nuclei glued together by a pion condensate as well as by Lee who applied it to hypothetical superdense nuclei glued together by a scalar condensate. Farhi and Jaffe in 1984 introduced term *strangelets* to describe hypothetical stable quark systems of a microscopic size bound more strongly than ^{57}Fe, which should contain a similar number of up, down, and strange quarks. Hypothetical quark objects of a larger (macroscopic) size De Ruhula and Glashow in 1984 named nuclearites. Below all abnormal stable hypothetical nuclear systems of such a size will be called *nuclearites*. For such objects the gravity contributes still much weaker than nuclear forces. When the baryon number becomes $A \gtrsim 10^{56}$, the gravity starts to contribute significantly to the binding, and we are dealing with *nuclei–stars*. For nuclei–stars consisting of the strange quark matter (bound even at smaller A) we will use the term *strange stars*. Compact stars with quark interiors (which are not necessarily self-bound without gravitational attraction) and hadron exteriors of a macroscopic size are usually called *hybrid stars*.

The existence of stable or metastable nuclear systems of various sizes would have numerous important consequences in physics. Therefore, it would be worthwhile to continue theoretical studies and, especially, experimental searches of such objects.

The paper has the following structure. In Sect. 2 we focus on the Lee–Wick model of abnormal nuclei with a scalar (e.g. a σ field) condensate. We discuss the relationship between the Lee–Wick model and the standard RMF models, as well as the RMF models with σ-scaled hadron masses and coupling constants. Section 3 considers various aspects of the model of supercharged nuclei held together by light charged bosons, if

the latter existed. In reality the role of the light boson could be played by an ordinary pion, which can become effectively lighter in a dense nuclear medium and/or in external fields, for example in rapidly rotating nuclear matter. In Sect. 4 we discuss density dependence of the p-wave pion–nucleon–nucleon and pion–nucleon–Δ isobar interactions which may result in charged and neutral pion condensates and a possibility of existence of superdense abnormal nuclei as proposed by Migdal. These phenomena can be modeled using the σ model. Then we turn our attention to the subtleties associated with the description of the s-wave pion-nucleon interaction and the s-wave pion condensation. In Sect. 5 we consider Δ-resonance matter. The uncertainty in the Δ optical potential allows for a wide variation in its value. For certain values, metastable or even stable resonance matter and abnormal nuclei may exist. The concept of Δ-resonance matter is somewhat similar to Witten's idea of strange quark matter. In the latter case the occupation of the strange quark Fermi sea results in a decrease of the energy. The energy of the state also decreases with a decrease of the strange quark mass and the bag constant, the values of which are not well known. In Sect. 6 we focus on the hypothesis of strangelets and strange and hybrid stars. Ordinary neutron stars and stars with exotic matter have different mass-radius relations, and their cooling histories are different. These topics will be discussed. Section 7 presents the idea of the scalar field condensation and possible existence of dilute metastable nuclear states. A scalar condensate field, determined by the value of the Landau–Migdal parameter in the scalar channel, may appear owing to the Pomeranchuk instability occurring in a dilute nuclear matter. Thereby we apply the idea of Lee and Wick of the possible existence of dense abnormal nuclei held together by a scalar field. Then, we consider the possibility of the formation of a self-bound dilute Bose–Einstein condensate state, e.g.,of α particles in the β-equilibrated system described with the help of a scalar condensate field. Next, we discuss anomalous states in nonequilibrium nuclear systems. Section 8 discusses anomalies that defy conventional physics interpretations, part of which could be related to abnormal nuclear systems. Conclusions are drawn in Sect. 9.

Throughout the text we use the system of units where $\hbar = c = 1$.

2 The Lee–Wick model

In this section we present the idea of Lee and Wick in a nutshell. We consider the Lagrangian density of nucleons interacting with a scalar field ϕ:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - U(\phi) + \bar{\psi}\left(\not{p} - m_N + g\phi\right)\psi, \tag{1}$$

where $m_N = 939\,\text{MeV}$ is the nucleon mass, ψ is the nucleon bispinor field, g is the strong interaction coupling constant, and $U(\phi)$ is the scalar field potential. The scalar field ϕ will be treated on the mean-field level. Then the nucleon spectrum is $\sqrt{m_N^{*2} + p^2}$ with the effective nucleon mass $m_N^* = m_N - g\phi$. While in the vacuum $\phi = 0$, Lee and Wick suggested that in the nucleon medium at some critical density the effective nucleon mass $m_N^*(n)$ can reach zero. It would occur for $\phi_c = m_N/g$. The energy density gain at this transition can be estimated as

$$\Delta E(n) \approx U(\phi_c) + \nu\frac{p_\text{F}^4}{4\pi^2} - m_N n - \frac{3p_\text{F}^2}{10m_N}n, \tag{2}$$

where $p_\text{F} = (3\pi^2 n/\nu)^{1/3}$ is the nucleon Fermi momentum and $\nu = 1$ for the neutron matter and $\nu = 2$ for the isospin-symmetric nuclear matter. Then the critical density can be estimated as

$$n_c = n_{c,0}\left(1 + \frac{3p_{\text{F},c}}{4m_N} + \frac{9p_{\text{F},c}^2}{20m_N^2} + O\left(p_{\text{F},c}^3/m_N^3\right)\right),$$
$$n_{c,0} = \frac{U(\phi_c)}{m_N} = \nu\frac{p_{\text{F},c}^3}{3\pi^2}. \tag{3}$$

For a simplistic estimate we can take $U(\phi) = \frac{1}{2}m_\phi^2\phi^2$, where m_ϕ is the mass of the scalar meson. Then, neglecting the correction $O(p_{\text{F},c}/m_N)$ we find

$$n_c \simeq n_{c,0} = \frac{m_\phi^2 m_N}{2g^2}. \tag{4}$$

Using typical values for the strong interaction constant $g \sim 10$ and the mass of the scalar meson, $m_\phi \sim m_\sigma \sim 700$ MeV, we obtain $n_c \sim 2n_0$. The energy gain per nucleon for $n > n_c$ and for $(n - n_c)/n_c \lesssim 1$ can be estimated as $\mathcal{E} = \Delta E/n \simeq -(1 - n_c/n)m_N$. These rough estimates show to a possibility that the ordinary nuclei may be exist in a metastable state and after a while they may undergo the first-order phase transition to the superdense ground state. The energy of each nucleon inside the abnormal nucleus is p, and outside $\sqrt{m_N^2 + p^2}$. Thus, there is a potential barrier holding the nucleons in the given abnormal state.

The main shortcoming of the above simplified argumentation is that we did not consider the equation of motion for the ϕ field following from the Lagrangian (1) or perform the minimization of the energy density with respect to ϕ. To remove this shortcoming one has to consider potential $U(\phi)$, which has the spontaneous symmetry breaking pattern. Let us take the potential in the form

$$U(\phi) = \frac{1}{2}\mu^2\phi^2 - \frac{\mu\sqrt{\lambda}}{\sqrt{2}}(1 - \delta)\phi^3 + \frac{1}{4}\lambda\phi^4, \quad \mu, \lambda > 0, \tag{5}$$

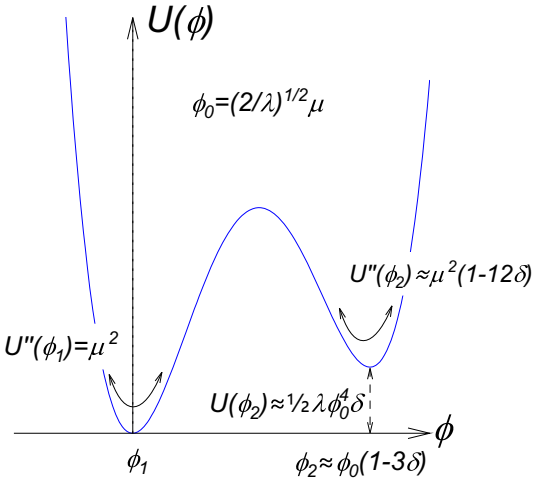


Fig. 1 Potential (5) for $\mu > 0$, $\lambda > 0$ and $0 < \delta < 1 - 2\sqrt{2}/3$

which for $0 < \delta < 1 - 2\sqrt{2}/3 \approx 0.057$ has two local minima: one at $\phi_1 = 0$ and another one at

$$\phi_2 = \frac{\phi_0}{4}\left(3(1-\delta) + \sqrt{9(1-\delta)^2 - 8}\right) = \phi_0(1 - 3\delta + O(\delta)), \quad \text{where} \quad \phi_0 = \sqrt{2}\frac{\mu}{\sqrt{\lambda}}. \tag{6}$$

The global minimum of the potential is realized at $\phi = 0$ that corresponds to the ground state. The second minimum is higher,

$$U(\phi_2) \approx \frac{1}{2}\lambda\phi_0^4\left(\delta + O(\delta^2)\right), \tag{7}$$

and the parameter δ is responsible for lifting the symmetry of two minima. In both minima the potential has the similar curvature $m_\phi^2 = U''(\phi_1) = \mu^2$ and $U''(\phi_2) = \mu^2(1 - 12\delta + O(\delta^2))$. The potential is illustrated in Fig. 1.

The coupling of the scalar field to nucleons is defined as previously so that the nucleon mass becomes very small closely to the new minimum $\phi \sim \phi_2$,

$$g = \frac{m_N}{\phi_0} = \frac{m_N}{\mu}\sqrt{\frac{\lambda}{2}} = \frac{m_N}{m_\phi}\sqrt{\frac{\lambda}{2}}. \tag{8}$$

The critical density $n_{c,0}$ in (3) can be estimated now with the help of Eq. (7) as

$$n_{c,0} = \frac{\lambda\phi_0^4}{2m_N}\delta = \frac{2m_\phi^3}{\lambda m_N}\delta = \frac{m_\phi^2 m_N}{g^2}\delta, \tag{9}$$

where to get the second and third equalities we used Eqs. (6) and (8). In contrast to the more rough estimate (4) the critical density (9) is determined by the symmetry-breaking parameter δ. Let us now consider the energy density of the system taking into account the nucleon kinetic energy

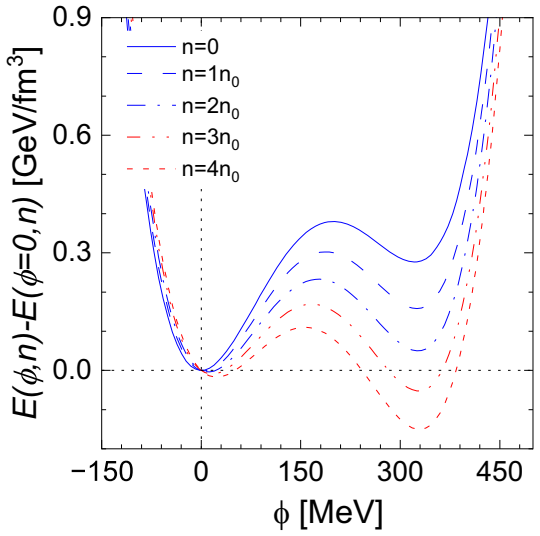


Fig. 2 Difference in the energy density (10) due to the development of the scalar field for ISM as function of ϕ for various nucleon densities calculated with the model parameters (11)

$$E(\phi, n) = U(\phi) + \nu\int_0^{p_F}\frac{\mathrm{d}pp^2}{\pi^2}\sqrt{(m_N - g\phi)^2 + p^2} \tag{10}$$

and plot $\Delta E(\phi, n) = E(\phi, n) - E(0, n)$ as a function of ϕ for various values of the nucleon density. In Fig. 2 we present the results obtained for isospin-symmetric matter (ISM), $\nu = 2$, and for the set of parameters

$$\mu = m_\phi = 800\,\mathrm{MeV}\,, \quad \lambda = 10\,, \quad \delta = 0.03\,. \tag{11}$$

We see that with a density increase the relative height of the second minimum (at $\phi \neq 0$) decreases and at some density it becomes a global minimum. At this density the nucleon mass changes abruptly as shown in Fig. 3 by the solid line. The dashed line presents the result obtained with a value $\delta = 0.02$. The transition density decreases, respectively.

In Fig. 4 we plot the binding energy per nucleon,

$$\mathcal{E}(n) = \frac{1}{n}E(\min\phi(n), n) - m_N\,. \tag{12}$$

We see, that for the parameter set (11), the vanishing nucleon mass leads to tremendous binding of the nuclear matter with a minimum realized at $\sim 12\,n_0$ with the binding energy of -340 MeV, see the solid line in Fig. 4. To reduce the binding one can take into account the nucleon–nucleon repulsion. To simulate it one can include a term quadratic in the nucleon density in expression (10),

$$E(\phi, n) \to E(\phi, n) + \frac{1}{2}k\frac{n^2}{n_0}, \tag{13}$$

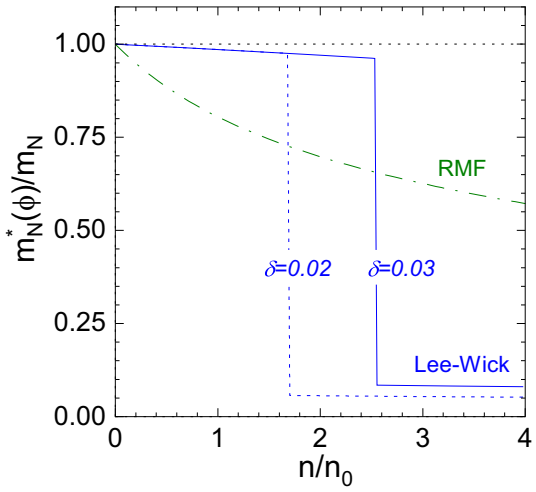


Fig. 3 Effective nucleon mass for ISM as a function of the nucleon density. Solid line is calculated for the Lee–Wick model with parameters (11). Dashed line is drawn for $\delta = 0.02$. Dash-dotted line corresponds to the RMF model (15) with parameters (16)

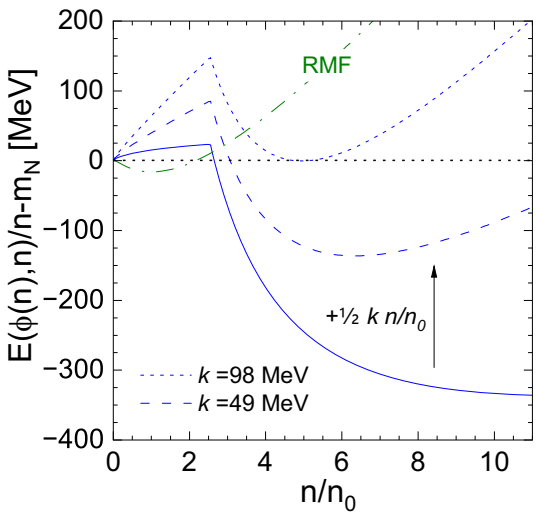


Fig. 4 Binding energy per nucleon (12) for ISM as a function of nucleon density. Difference in the energy density (10) due to the development of the scalar field is calculated for parameters (11). Solid line shows the result of the Lee–Wick model. Dashed lines include nucleon–nucleon interaction term (13). The dash-dotted curve shows the RMF calculation with parameters (16)

where k is treated as a parameter. An increase of k reduces binding, which disappears completely at the critical value, $k = 98$ MeV, see the short-dashed line in Fig. 4. Reducing the value of k, we find that the minimum in the energy appears at $n \sim 5\,n_0$, and at the half critical value, $k = 49$ MeV the minimum is located at the density $\sim 6.4\,n_0$ and has the depth of -136 MeV, see the dashed line in Fig. 4.

The same year when Lee and Wick proposed their model, Walecka formulated a relativistic mean-field model (RMF) [82,83]. Here one should also mention previous works by Johnson and Teller, and by Dürr [84,85], where the nuclear saturation arises due to the difference between scalar and vector nucleon densities. The Walecka model includes interaction of nucleons with mean fields of the scalar σ and vector ω mesons allowing the σ field dependence of the nucleon mass. Coupling constants of σ and vector ω mesons with nucleons were fitted to describe binding energy and saturation density of nuclear matter. Incompressibility at nuclear saturation proved to be unrealistically high and effective nucleon mass low. To improve the fit Boguta included possible scalar field self-interactions depending on two extra fitting parameters [86]. Also ρ meson mean field was included. These changes made the RMF models more flexible, see [87]. Doing in Eq. (10) replacements

$$\phi \to \sigma\,, \quad \mu \to m_\sigma\,, \quad g \to g_\sigma\,,$$
$$\lambda \to c g_\sigma^4 \quad \delta \to 1 + \frac{\sqrt{2}}{3\sqrt{c} m_\sigma} b g_\sigma m_N\,, \tag{14}$$

and adding the repulsive contribution due to the ω meson exchange with the mass m_ω and the coupling constant g_ω, we obtain the RMF energy density

$$E(\sigma, n) = \frac{m_\sigma^2 \sigma^2}{2} + \frac{b}{3} m_N\,(g_\sigma \sigma)^3 + \frac{c}{4}\,(g_\sigma \sigma)^4 + \nu \int_0^{p_\mathrm{F}} \frac{\mathrm{d}p p^2}{\pi^2} \sqrt{(m_N - g_\sigma \sigma)^2 + p^2} + \frac{g_\omega^2 n^2}{2 m_\omega^2}. \tag{15}$$

The magnitude of the scalar field follows from the equation $\partial E(\sigma, n)/\partial \sigma = 0$.

As example, taking the parameter of binding energy per nucleon $\mathcal{E}_0 = 16$ MeV, compressibility modulus $K = 275$ MeV and the effective nucleon mass $m_N^*(n_0) = 0.805$ at the saturation density n_0, as we did in Ref. [88], we obtain parameters

$$\frac{g_\sigma^2 m_N^2}{m_\sigma^2} = 184.356\,, \quad \frac{g_\omega^2 m_N^2}{m_\omega^2} = 87.5996,$$
$$b = 5.53871 \times 10^{-3}, \quad c = 2.29759 \times 10^{-2}. \tag{16}$$

The density dependence of the effective nucleon mass of the ISM calculated within the RMF model is shown by the dash-dotted line in Fig. 3. The mass decreases smoothly with the density increase without any jumps in difference to the Lee–Wick model. The binding energy per nucleon (12) for the nuclear matter ($\nu = 2$) as a function of nucleon density is depicted in Fig. 4, solid line for Lee–Wick model. Long and short dashed lines include the nucleon–nucleon interaction term (13). The dash-dotted line shows the RMF calculation

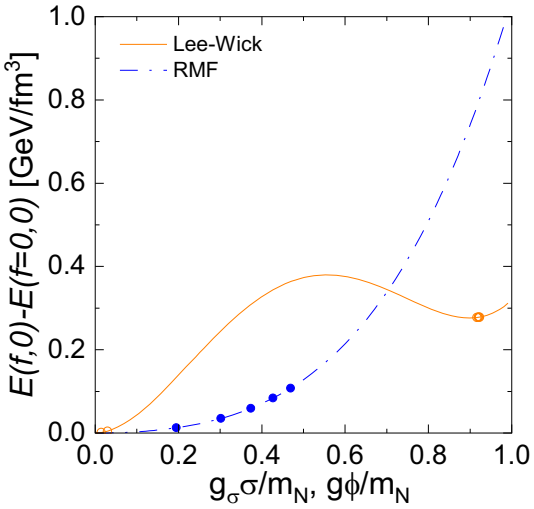


Fig. 5 Potentials of the scalar field for the Lee–Wick model (5) with parameters (11) and for the RMF model with parameters (16) as functions of the dimensionless scalar field. Dots show the values of the dimensionless magnitude of the scalar field for five values of the nucleon density from $1\,n_0$ till $5\,n_0$ with the step $1\,n_0$

with parameters (16). The Lee–Wick abnormal state does not appear in the latter model.

Various modifications of the original Walecka model including ρ mesons, a non-linear σ field dependent non-linear potential, possible $\omega - \sigma$ interaction, etc, are widely and rather successfully used to describe equation of state of the hadron matter in compact stars, heavy-ion collisions and finite nuclei, cf. [87,89] and references therein. Usually, one assumes that there is no pseudoscalar mean-field of pions in the spin balanced system. However, including pionic excitations beyond the mean-field level improves the description of the equation of state of nuclear matter at finite temperatures, e.g., in application to heavy-ion collisions [90]. At low temperatures, which are of main our interest below, the contribution of pions to the equation of state is rather small.

In spite of the success in describing various phenomena at densities $\lesssim 2\,n_0$, RMF models should be considered as merely phenomenological extrapolations at densities $\gg n_0$. In Fig. 5 we compare the potentials of the scalar field for the Lee–Wick model and the RMF model. Dots show the values of the dimensionless magnitude of the scalar field for five values of the nucleon density from $1\,n_0$ till $5\,n_0$ with the step $1\,n_0$. For parameters fitted to the properties of the nuclear system at $n = n_0$, the Lee–Wick abnormal state does not appear at larger densities in the RMF model. As seen in Fig. 5, no information about the possible restoration of the chiral symmetry is included in the standard RMF models, whereas a mechanism for a phase transition to a new state with a small nucleon mass is build in the Lee–Wick model by construction. Although the latter model cannot describe the properties of the ordinary atomic nuclei and nuclear matter at $n \lesssim n_0$, it demonstrates the principle possibility of the existence of a new stable (or metastable) abnormal nuclear state at n significantly above n_0.

The possibility of the Lee–Wick state can be studied within an RMF model if the latter is augmented with the concept of partial restoration of the chiral symmetry. Such idea was realized in a series of works [88,91–95], where the hadron masses and coupling constants were taken to be dependent on the scalar σ field, so that not only baryon masses but also masses of other hadrons (ω, ρ and the σ mesons) decrease with an increase of the σ mean field. This approach differs from the well-known approach of Refs. [96,97] where the coupling constants were assumed to be functions of the baryon density. Therefore thermodynamical consistency of the former models is fulfilled automatically. The RMF models with the σ scaling have been shown to satisfy many experimental constraints from heavy-ion collisions and from compact star observations. For example, the models fulfil constraints imposed by directed and elliptical particle flows measured in nucleus collisions [98], describe very massive neutron stars allowing admixtures of hyperons and Δs in neutron star matter, and shift the threshold of the most efficient direct Urca (DU) neutrino reaction to occur only in the sufficiently heavy neutron stars. The latter possibility is favored by the fit of the experimental data on cooling of pulsars, e.g. see [99–102]. Only for simplicity, the parameters of the models used in [88,91–94] were chosen so that the new Lee–Wick minimum was not realized. By changing parameters of the models, new solutions can be obtained, some of which would exhibit a sudden drop of the effective nucleon mass at a critical density, cf. discussion in Sect. 3 of Ref [95]. Such a possibility was realized in the model [95] at sufficiently strongly attractive Δ optical potential, see Fig. 9 below.

3 Supercharged nuclei, nuclearites and nuclei–stars

In Refs. [28,29] three types of abnormal nuclei with a π condensate were discussed. These were named the "supercharged", "neutron" and "superheavy" anomalous nuclei. Later, it became clear that this gradation is very conditional. What is important is that these hypothetical objects may differ in their baryon number, interior density, isospin content, and the type of the condensate ($\pi^{\pm}$ or π^0) that holds them together.

3.1 Supercharged nuclei in a nutshell

We consider a nuclear system with a rather large baryon number A, approximately constant density $n \sim n_0$ at the proton number $Z \approx A/2$ (in this respect the nucleus with a large $Z \gg 1/e^2 = 137$ is supercharged although for

$Z \gg 1/e^3 \sim 10^3$ the charge is compensated by electrons produced from vacuum, see below). To demonstrate the key idea, we assume for a moment that there exists a negatively charged light scalar bosons (or pseudoscalar "light pions") with a mass m_b. Below we will show that in realistic situations, pions, which mass is changed in nuclear matter and in external fields, can play the role of such a light boson. One may ask whether these light bosons can bind supercharged nuclei, see [54], and [31].

The energy of the nucleus with approximately constant density $n \simeq n_0$ at the proton number $Z \simeq A/2$ is given by

$$\mathcal{E} \approx -\mathcal{E}_0\, A + \mathcal{E}_{\rm surf} - \int n_p V \mathrm{d}^3x - \int \frac{(\nabla V)^2}{8\pi e^2}\mathrm{d}^3x + \int \left(|\nabla\phi|^2 - V^2|\phi|^2 + m_b^2 + \tfrac{1}{2}\lambda|\phi|^2)|\phi|^2\right)\mathrm{d}^3x\,,$$
$$\mathcal{E}_0 = 16\,\mathrm{MeV}. \tag{17}$$

Here the first term is the volume binding energy, the second term produces the surface energy, which is suppressed as $1/A^{1/3}$ compared to the first term and will be dropped when the nucleus atomic number A is large as we for simplicity will assume. The proton density is $n_p \simeq n_0/2$. The third and fourth terms describe Coulomb interactions of protons with the electric field. For ordinary atomic nuclei these terms produce the Coulomb energy $3Z^2e^2/5R$, where the nucleus radius is $R \simeq r_0 A^{1/3} = (r_0/2^{1/3})Z^{1/3}$ and $r_0 \simeq 1.2\,\mathrm{fm}$. The last term in Eq. (17) is the contribution of the classical charged boson field, the coupling of the pion-pion self-interaction $\lambda \sim 1$. With an increase in charge $Z \sim A/2$, the Coulomb energy repulsion always begins to exceed the volume energy attraction. So for the existence of extended systems with $Z \sim A/2 >> 1/e^3$ the charge must be essentially compensated by electrons and μ^-. Indeed when A and Z grow, the electrical potential well, V, deepens. When it becomes $V \leq -m_b$, the charged light bosons may appear in the reactions $n \to p + b^-$ and compensate the charge of the protons. As the result, in a nucleus with $Z >> 1/e^3$, the charge-neutral interior is formed [50] surrounded by a thin (with thickness $\sim$ 10 fm) surface layer, where the strength of the electrical field is $E \sim 10^2 m_e^2 \sim 10^{18}\,\mathrm{V/cm}$ with $m_e = 0.5\,\mathrm{MeV}$ being the electron mass.

The spatial structures of the electric potential $V(r)$ and the boson field $\phi(r)$ are governed by equations

$$\Delta\phi + \left(V^2 - m_b^2 + \lambda|\phi|^2\right)\phi = 0, \tag{18}$$

$$\Delta V = 4\pi e^2\left(n_p + 2V|\phi|^2\right). \tag{19}$$

Here we put $\lambda \to 0$ to simplify consideration. Hence From these equations in case of a spherical nucleus with large A we get uniform solutions $V = -m_b$ and $|\phi|^2 = n_p/(2m_b)$ valid in the interior, except for a boundary layer of the depth determined by the typical Debye screening length, being much shorter than the nucleus size. Thus, after the formation of the charged boson condensate the potential well stops growing keeping limiting depth of $V = -m_b$.

The volume part of the energy reads now

$$\mathcal{E} = -\mathcal{E}_0 A - VZ = -\mathcal{E}_0 A + \frac{1}{2}m_b A. \tag{20}$$

We conclude that if negatively charged scalar bosons with a mass of $m_b < 2\mathcal{E}_0 \simeq (30\text{–}32)\,\mathrm{MeV}$ existed in nature, then stable supercharged nuclei and nuclearites of an arbitrary size (with A up to 10^{56} and $n \sim n_0$) would also exist, being glued by strong and electromagnetic forces, see Refs. [31,54]. For larger-A systems, gravity comes into the game, and we deal with compact stars containing a nucleon–light-boson (e.g., light pion) component.

We discussed abnormal nucleonic systems glued by negatively charged bosons but certainly, on equal footing, we could consider antinucleonic systems glued by positively charged bosons. In early Universe, the appearance of the latter anti-systems could contribute to a solution of the problem of the observable charge asymmetry of the Universe.

In the literature the exotic objects have been given various names: anomalous nuclei, abnormal nuclei, nuclear drops, nuclearites, and nuclei–stars for heavy objects. Similar self-bound objects made of quarks were called — quark nuggets or strangelets, quark nuclearites, strange stars. For objects build with the Witten's strange quark matter, we will use further on the names like strangelets, quark nuclearites and strange stars.

3.2 Dibaryons and neutral light bosons

The possibility of the existence of multiquark states was predicted by QCD-inspired models [103,104]. These works initiated a lot of experimental searches for six-quark states (dibaryons). Usually, one looked for dibaryons in the NN channel. Such dibaryons would have decay widths from a few up to a hundred MeV. The obtained results were not very conclusive and the background determination was questionable. Enthusiasm has been revived in the field after experimental reports [105–107]. Reaction $pp \to p\pi^+ X$ was studied in [105] and three narrow peaks (width $\sim$ 5 MeV) in the missing mass spectrum were seen at $M_X = 1004$, 1044 and 1094 MeV with high statistical significance. The further study in [108] indicated the narrow baryonic structures at 1004, 1044, 1094, 1136, 1173, 1249, 1277, and 1384 MeV (and possibly 1339 MeV) in the missing mass M_X and in the invariant mass $M_{p\pi^+}$. The reaction $pd \to ppX_1$ was studied in [106,107] and three peaks of the 5 MeV width were clearly observed in the missing mass M_{pX_1} spectrum

at $M_{pX_1} = 1904 \pm 2$, 1926 ± 2 and 1942 ± 2 MeV. In the missing mass M_{X_1} spectrum the peaks are located at $M_{X_1} = 966 \pm 2$, 986 ± 2 and 1002 MeV. The peaks in the pd reaction can be interpreted as the signatures of either dibaryons or nucleon resonances. Theoretical and experimental bounds on the properties of nucleon resonances were discussed in Ref. [109]. The resonance states below the pion-nucleon threshold and their consequences for nuclear systems were discussed in [110].

Rather recently several very heavy compact stars were observed. The current highest precisely measured mass is 2.08(7) $M_\odot$ for PSR J0740+6620 [111,112]. On the other hand appearance of hyperons and Δ resonances in neutron star interiors results in a decrease of the maximal neutron star mass. Nevertheless these so called the hyperon and Δ isobar puzzles can be resolved, see [95] and reference therein. Reference [110] demonstrated that in presence of light dibaryons the maximal possible mass of a neutron star would decrease below the observational limit. Appearance of rather heavy dibaryons and baryon resonances although results in a decrease of the maximal neutron star mass does not contradict to observations.

An interesting alternative interpretation of the tower of resonance states was given in [113]. It was noted that drawing all new states (both dibaryonic and nucleonic ones) on one energy scale, one obtains a tower of almost equidistant states. It was hypothesized the existence of a light pseudoscalar meson ($J^P = 0^-$) with a mass of $m_{\text{light}\,\pi} \simeq 21 \pm 2.6$ MeV. The basic idea is that the tower of excited nucleonic states is formed by the nucleon in its ground state plus 1, 2, 3, ... light pions as the quantum of excitation with the energy $m_{\text{light}\pi}$. It was argued in [110] that the presence of light charged pions would allow for existence of abnormal nuclei ($A \gtrsim 10^3$) and "nuclei stars" of arbitrary size, bound by strong and electromagnetic interactions.

Recent possible discovery is also worth mentioning: a light neutral boson (X17) with a mass of 17 MeV and a very narrow width (presumably less than 0.07 eV) was found at the accelerator of ATOMKI. Three significant anomalies were observed in electron–positron pairs emitted in the ^{3}H (p, e^-e^+) ^{4}He, ^{7}Li (p, e^-e^+) ^{8}Be, and ^{11}B $(p, e^-e^+)^{12}$C reactions [114–116]. If confirmed, the existence of this new particle could be important for particle physics and cosmology, see review [117]. Reference [118] assumed that X17 is the vector gauge boson coupled more strongly to neutrons than to protons. Reference [119] suggested another interpretation of the experimental results, assuming a light, pseudoscalar particle. In both cases the coupling of X17 with nucleons was assumed to be very weak.

On the other hand, independently of the value of the strength of the interaction it is worthwhile to ask, if existence of a light neutral boson does not contradict to the neutron star properties. Reference [120] assumed that X17 can mediate low-energy nucleon–nucleon interactions and attempted to find possible constraints on the hypothetical X17 boson from the analysis of the equation of state of neutron stars. It was shown that such a neutron star could have $M - R$ curve rather similar to that for the ordinary neutron star.

Note that since $m_{\text{X17}}/m_\pi \sim 1/8$, one could expect presence of the nucleon–light boson polarization effects of the same order as for pions, if the coupling of the light boson with nucleons were $f_{XNN} \gtrsim f_{\pi NN}/8$. In the latter case a possibility would exist for the Bose condensation of such a particles in the compact star matter (similarly to π^0 condensation) and feasibly for existence of metastable or stable nuggets and light-boson enriched nuclei–stars. Nevertheless we should repeat that [119] and [118] estimated X17-nucleon polarization effects as tiny.

3.3 Rotating nuclear systems

The fastest known pulsar, PSR J1748-2446ad, makes one revolution in 1.4 ms (angular frequency $\Omega \approx 4.5 \times 10^3$ Hz). Thus typical angular velocities of so far observed neutron stars are $\Omega R \lesssim 0.15c$, where c is the speed of light and $R \sim 10$ km is the neutron star radius. The limiting rotation frequency of a compact star — the Kepler frequency — is determined by equation $m_N \Omega_{\text{K}}^2 R^2/2 = GMm_N/R$, where G is the gravitational constant, R is the radius of the object, and M is its mass. For a compact star rotating with the frequency close to the Kepler frequency Ω_{K}, the rotation affects the equation of state resulting in an increase of the mass of the static star on the value $\lesssim (0.3 - 0.4) M_\odot$, cf. [121]. When such a compact star collapses to form a black hole, its angular rotation frequency increases due to conservation of the angular momentum up to $\Omega < 1/r_G = 1/(2MG) \sim 10^5$Hz for the compact star with the mass $M \sim M_\odot$, where $M_\odot \approx 2 \times 10^{33}$ g is the solar mass. For primordial black holes with masses $M < 10^{14}$ g, this estimate would give $\Omega \lesssim 10^{24}$ Hz.

Rotational frequencies of excited nuclei usually do not exceed $\Omega \sim 3 \times 10^{21}$ Hz, cf. [122]. Estimates yield angular momenta $L \sim \sqrt{s_{NN}} Ab/2 \lesssim 10^6 \hbar$ in peripheral heavy-ion collisions of Au+Au at the center of mass NN collision energy $\sqrt{s_{NN}} = 200$ GeV, for the impact parameter $b = 10$ fm, where A is the nucleon number of the ion [123]. The global polarization of $\Lambda(1116)$ hyperon observed by the STAR Collaboration in non-central Au–Au collisions [124] indicated existence of a vorticity with rotation frequency $\Omega \simeq (9 \pm 1) \times 10^{21}$ Hz $\simeq 7$ MeV/$\hbar$.

Magnetic fields in ordinary pulsars, like the Crab pulsar, reach values $\lesssim 10^{13}$ G at their surfaces. At the surface of magnetars magnetic fields are higher, $\gtrsim 10^{15}$ G. In the interior the magnetic field might be even stronger (up to $\sim 10^{18}$ G) depending on the assumed mechanism of the formation of the magnetic field [125]. Also, magnetic fields $\gtrsim (10^{17}$–$10^{18})$ G, $h \sim h_{\text{VA}} = Ze/R^2 \sim H_\pi (Ze^6)^{1/3}$ (where typical time

of the collision process was estimated as $\Delta t \sim R/c$) may exist in non-central heavy ion collisions at collision energies $\sim$GeV/A, as it was evaluated in Ref. [125], Z is the charge of the fireball, $H_\pi = m_\pi^2/e \simeq 3.5 \times 10^{18}$ G.

As in Sect. 3.1 we consider now negatively charged scalar bosons (or pseudoscalar pions) with a mass m_b. In a presence of a magnetic field and an electrical potential well, there exists a critical angular velocity, above which, from the vacuum in the rotation frame there arises a condensate of the charged bosons [80,81,126–128]. In the frame rotating with the angular velocity $\mathbf{\Omega} \parallel z$, the solution for the charged condensate field has the form of a vortex, $e^{il\theta}\phi$, where θ is the angle in the cylindrical system and l is the quantum number of the angular momentum. To be specific let further Ω and l be positive. In the presence of a uniform constant magnetic field $\mathbf{H} \parallel z$ and a rectangular potential well $V = -V_0 = const$ for $\sqrt{x^2+y^2} < R$, the critical angular velocity is given by $l\Omega_c = -V_0 + \sqrt{m_b^2 + |eH|(1-2\alpha)}$ with $-1 \le \alpha \le 0$.

For the case of the rotating (for simplicity cylindric) nucleus equations for the boson field and the electric potential (18) and (19) are to be replaced by the following ones [80,81],

$$\Delta_r\phi + \left[(V-\Omega l)^2 - m_b^2 - (l/r - A_\theta(r))^2\right]\phi = 0\,, \tag{21}$$

$$\Delta_r V = 4\pi e^2\left(n_p + 2(V-\Omega l)|\phi|^2\right)\,, \quad \Delta_r = \partial_r^2 + \partial_r/r \tag{22}$$

where $A_\theta(r)$ is the angular component of the vector potential of the magnetic field, and a ϕ-field self-interaction is for simplicity neglected. Equations (21), (22) should be supported by equation for the vector-potential $\mathbf{A}$. However with the ansatz that $A_\theta(r) = l/r$ in the vortex exterior we have curl$\mathbf{A} = 0$ and equation for $\mathbf{A}$ is fulfilled identically.

Now in case of a nuclear system ($N \simeq Z$, $n \approx n_0$) of a large size solutions are $V = \Omega l - m_b \le 0$, $|\phi|^2 = n_p/(2m_b)$ and instead of (20) we find [80,81]

$$\mathcal{E} = -\mathcal{E}_0 A + \frac{1}{2}(m_b - \Omega l)A + \mathcal{E}_{kin}\,,\ A = N+Z, \tag{23}$$

where $\mathcal{E}_{kin}$ is the kinetic energy of the rotation of the nucleon subsystem. The term $lA/2$ is smaller or equal to the initial angular momentum of the charged uniformly rotating nucleon sub-system $L \sim Am_N\Omega R^2$ (before appearance of the pion giant vortex, $l \gg 1$), m_N is the mass of the nucleon. At the rigid rotation Ω is determined by the initial value of the total angular momentum. The critical values of l and Ω are determined by the condition $\Omega_c l_c = m_b - 2\mathcal{E}_0$. The role of m_b can be played by the effective mass of a charged pion, which decreases with an increase of the baryon density, see Sect. 4 below. Here we considered the possibility of the second-order phase transition to a state with a giant vortex of the charged pion field. In reality, the phase transition can be of the first order with a jump of the condensate field at the critical point. Then the critical angular velocity is given by the condition $\Omega_c l_c = m_b - 2\mathcal{E}_0 + 2\mathcal{E}_{\rm cond}(\Omega_c l_c)$, where $\mathcal{E}_{\rm cond} \le 0$ is a gain in the energy per baryon due to the formation of the vortex. In the rotating frame the nugget remains self-bound until the system is not decelerated by the weak radiation force below Ω_c. For a large-size system the deceleration time can be very long.

Now we mention another possibility. In the case of rectilinearly moving and rotating media, condensation of Bose excitations on a low-lying branch of the excitation spectrum $\omega(k)$, behaving similar to that in superfluid ^{4}He and in Fermi liquids, was studied in Ref. [129], see also Ref. [130] and references therein. As shown, when the speed of rectilinear motion exceeds a critical value (typically Fermi velocity for a nucleon medium) a part of the momentum can be transferred from the normal nucleon system to the superfluid condensate of Bose excitations with energy $\omega(k_c)$. Similarly when the rotation frequency exceeds some critical value a part of the angular momentum can be transferred from the normal nucleon system to the superfluid condensate of Bose excitations [129,131]. Consequently, the angular frequency of the normal component may decrease below $\Omega_{\rm K}$, whereas it would exceed $\Omega_{\rm K}$ otherwise.

When the baryon density in the interior of the star exceeds the value $n_c^{\pi^+} \lesssim n_0$, there appears a new branch of π^+ excitations with a negative frequency $\omega(k)$ and it may become energetically profitable to transport a part of the angular momentum to the π^+ condensate droplets, for a more detail see Ref. [129] and Sect. 4.2 below. This allows to propose the following scenario. When during the initial compression stage of a supernova collapse, the density exceeds the critical density $n_c^{\pi^+}$ it becomes energetically favorable to transfer a part of the initial angular momentum from the nucleon subsystem to pion condensate drops. If the star is destroyed in the end, these drops will continue to rotate (with angular velocities $\Omega R_{\rm d} < 1$, with $R_{\rm d}$ being a size of the drop) until the angular momentum has been completely radiated from the drop surface. These drops of π condensate nuclear matter may be observed as abnormal nuclei and nuclearites propagating in cosmos.

Notice also that differentially rotating stars can support a larger star mass than their uniformly rotating or non-rotating counterparts [132]. In the case of massive remnant stars of binary merger events, differential rotation is one of the mechanisms that may provide an extra centrifugal support to stabilize the star well above the Tolman-Oppenheimer-Volkoff (uniform rotation) mass limit.

3.4 Anomalous supercharged nuclei consisting dark matter

In 1981 Cahn and Glashow [133] assumed existence of superheavy charged dark matter particles, cf. also [134]. In 2005 Glashow [135] conjectured that more specifically the dark matter may consist of the U tera-quarks and O-UUU (charge -2) baryons, and in 2006 Khlopov [136] studied OHe-hidden dark matter with $m_O \sim 1$ TeV. Reference [72] suggested existence of stable O-nuclearites of arbitrary size (up to $A \sim 10^{57}$), if there exist charged dark matter stable superheavy particles. In this case the proton charge of the nucleus is compensated by the heavy O-particles. Due to the large mass of the O particles they yield a negligible contribution to the kinetic energy. Therefore, the ordinary ISM at density $n \simeq n_0$ with embedded O-particles compensating the charge would be absolutely stable. Similarly to estimates (17), (20) performed above for supercharged nuclei, in case of the nuclear–O matter one may present the energy as

$$\mathcal{E} \simeq \mathcal{E}_0 A - \int d^3x \left(n_p - 2n_O\right) V. \tag{24}$$

With $2n_O = n_p$ we get $\mathcal{E} \simeq \mathcal{E}_0 A = -16A$ MeV.

At present there exist various anomalous events, some of which may have a relation to the discussed above scalar fields, π^- condensate, quark or dark matter nuggets, see [71] and references therein and a discussion in Sect. 8 below.

4 Pion mode and condensation in dense nuclear systems

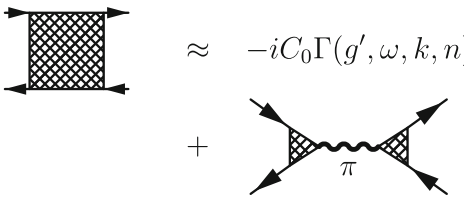
The phenomenon of a pion condensation was proposed in Refs. [27,137–139] and investigated then in numerous works, see [30,31,90] and references therein. The pion-nucleon interaction is strongly attractive in the p wave. Therefore, starting from the first works one expected that a pion condensate is formed in a state with a finite momentum $k \neq 0$ when the nucleon density exceeds some critical density $n_c^{(\mathrm{p})}$ [140–148]. In first works one anticipated that $n_c^{(\mathrm{p})}$ might be smaller than n_0. Soon, after analyses of atomic nucleus data, it became clear that $n_c^{(\mathrm{p})} > n_0$ because of a stronger repulsive effect of short-range nucleon–nucleon correlations, see [30,31] and references therein. In Ref. [28] Migdal used the name "superheavy anomalous nuclei" and then "superdense" nuclei in Ref. [30] and later in Ref. [31] simply "abnormal" pion condensate nuclei. These objects are neutron-rich to reduce the repulsive Coulomb energy. The key idea is that for $n > n_c^{(\mathrm{p})} \gtrsim n_0$ there may appear the π^+, $(\pi^+\pi^-)$, or π^0 condensates leading a decrease in the energy density by a value which can be roughly estimated as

$$E_\pi \simeq -\frac{1}{2}\beta_0(n)\left(n - n_c^{(\mathrm{p})}\right)^2 \theta\left(n - n_c\right)$$

with $\beta_0 \lesssim 1/m_\pi^2$, compare with Eq. (13) that was used above in the case of the Lee–Wick σ condensate. The quantity β_0 depends on the value of the Landau–Migdal parameter g', which is not well determined for $n \neq n_0$ and for isospin-asymmetric matter. For a smaller β_0, the pion condensate only slightly modifies the total energy density. For a larger β_0 the gain in the condensate energy may overwhelm the loss in the purely baryon repulsive contribution. In the latter case there could exist metastable or even absolutely stable abnormal π condensate nuclei and nuclei–stars.

4.1 Pion-nucleon interaction amplitude and pion condensation

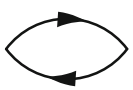
Resummation of a nucleon–nucleon interaction in the particle–hole channel in the Fermi liquid approach yields the following representation of the NN interaction amplitude in the spin-isospin channel in the isospin-symmetric matter [30,31]



(25)

Here τ and σ are Pauli matrices acting in the isospin and spin spaces, respectively, g' is the Landau–Migdal parameter of NN correlations in the spin-isospin channel, C_0^{-1} is the normalization factor — the density of states at the Fermi surface at $n = n_0$. Similar equation holds for separated $nn(pp)$ and np spin channels in isospin-asymmetric matter, where $g_{nn(pp)}$ and g_{np} Landau–Migdal parameters enter instead of the g' parameter. Note that the Δ-nucleon Landau–Migdal parameter $g'_{\Delta N} \ll g'$ and thereby we for simplicity disregarded this contribution in (25), whereas the Δ contribution in the second term of Eq. (25) is taken into account, see [31] for a more detail. $\Phi(\omega, k, n)$ stands for the famous Lindhard function corresponding to the particle-hole loop

$$i\frac{m_N^* p_\mathrm{F}}{\pi^2}\Phi(\omega,k,n) \;=\; \tag{26}$$



For nonzero temperature T it also depends on T. The first term in r.h.s. of Eq. (25) can be cast as the NN vertex correlation factor

$$\Gamma(x,\omega,k,n) = \frac{1}{1 + 2\,x\,\frac{p_\mathrm{F}(n)}{p_\mathrm{F}(n_0)}\Phi(\omega,k,n)}, \tag{27}$$

which appears not only in the spin-isopsin channel (for $x = g'$) but in other channels with other Landau–Migdal parameters $x = f, f', g$. The wavy bold line represents the full pion Green function. For $\omega \ll m_\pi$ and $k \lesssim p_F$ the main contribution to the NN amplitude is determined by the second diagram in r.h.s. of (25).

Main contributions to the pion polarization operator come from the nucleon–nucleon-hole and Δ–nucleon-hole terms corrected by the short-range correlations, for more details see [31,90,149]. The contribution of the Δ–nucleon-hole is as important as that of the nucleon–nucleon hole. Indeed, the mass of the Δ isobar is only $2m_\pi$ larger than the nucleon mass but the degeneracy factor is four times larger and the attractive $\pi N\Delta$ coupling constant is twice as large as the πNN one.

The retarded pion propagator is

$$D_\pi^R\left(\omega, \mathbf{k}, n_n, n_p\right) = \frac{1}{\omega^2 - m_\pi^2 - k^2 - \Pi^R\left(\omega, k, n_n, n_p\right)}, \tag{28}$$

where Π is the pion polarization operator, n_n and n_p are the neutron and proton densities. For $\omega = \mu_\pi$:

$$-\left[D_\pi^R(\mu_\pi, \mathbf{k})\right]^{-1} = \tilde{\omega}^2\left(\mu_\pi, k^2\right) = m_\pi^2 + k^2 + \Re\Pi^R\left(\mu_\pi, k^2\right) - \mu_\pi^2, \tag{29}$$

$\Re\Pi(\omega, k)$ is the real part of the pion polarization operator, the imaginary part $\Im\Pi(\mu_\pi, k) = 0$, $f_{\pi NN} \approx m_\pi^{-1}$ is the πNN coupling constant, and $\Gamma(n)$ is the factor taking into account the NN correlations, $\Gamma(n \to 0) \to 1$, μ_π is the pion chemical potential. In the case of the ISM, $\mu_{\pi^\pm} = 0$ in accordance with reactions $p \leftrightarrow n + \pi^+$, $n \leftrightarrow p + \pi^-$; $\mu_{\pi^0} = 0$ for the case of arbitrary isospin composition in accordance with reactions $N \leftrightarrow N + \pi^0$, $N = (n, p)$. The quantity

$$\tilde{\omega}_0^2 = \min_k \left\{\tilde{\omega}^2\left(\mu_\pi, k\right)\right\} = \tilde{\omega}^2(k_0), \tag{30}$$

where the minimum is realized at a density dependent momentum $k_0(n)$, has the meaning of the squared effective pion gap, see Ref. [31] and references therein. We notice that the in-medium π^- can play a role of the light negatively charged boson considered in Sect. 3 with the parameter m_b^2 used there replaced by the squared effective pion gap $\tilde{\omega}_0^2$.

At densities $n \sim n_c^{(\mathrm{p})}$ the pion gap behaves like $\tilde{\omega}_0^2 \sim \beta(n_c^{(\mathrm{p})} - n)$, with a constant $\beta(\mu_\pi, k_0) > 0$ (at the neglect of the quantum fluctuations) and the change in the sign of the gap indicates the onset of instability with respect to the pion condensation with the momentum k_0. The critical point $n_c^{(\mathrm{p})}$ is the same for $\pi^\pm, \pi^0$ in ISM. In the neutron star matter, $n_c^{(\mathrm{p})} = n_c^\pm$ for charged pions, and it is equal to n_c^0 for π^0. Which value $n_c^\pm$ or n_c^0 is larger depends on the model under consideration, see [30,31].

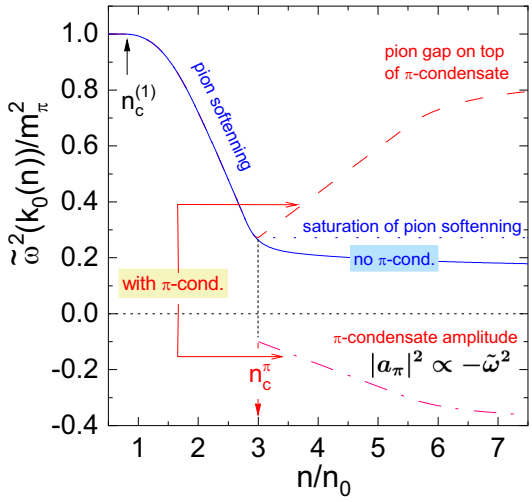


Fig. 6 Typical density dependence of the squared effective pion gap for $\pi^{\pm,0}$ in ISM and for π^0 in neutron star matter. Solid line shows $\tilde{\omega}^2(k_0(n))$ for $n_c^{(1)} < n < n_c^\pi$. For $n > n_c^\pi$ it is continued for the case when the pion condensate is not formed and the system is in a metastable state. Dotted line corresponds to the case of a saturation of the pion softenning due to artificially strong increase of the Landau–Migdal parameter g' with a density growth. Dash-dotted line characterizes the amplitude of the pion condensate appearing in the first order phase transition at n_c^π. Dashed line shows the squared effective pion gap on top of the pion condensate

In the ISM the momentum dependence of the pion gap, $\tilde{\omega}^2(0, k^2)$ develops minimum at $k = k_0 \neq 0$ at densities $n > n_c^{(1)} \simeq (0.5 - 0.7)\, n_0$, see [90] and in its vicinity one may use parameterization:

$$\tilde{\omega}_0^2(k^2) \simeq \tilde{\omega}_0^2 + \gamma \frac{\left(k^2 - k_0^2\right)^2}{4k_0^2},$$

where $\gamma \sim 1$. The squared effective pion gap $\tilde{\omega}_0^2$ changes sign at $n = n_c^{(\mathrm{p})}$. Considering the pion condensation in a finite system for $n > n_c^{(\mathrm{p})} > n_0$, one may find the spatial structure of the classical pion field solving the equation

$$\tilde{\omega}^2\left(\mu_\pi, \hat{k}\right)\phi + \lambda|\phi|^2\phi = 0\,, \quad \hat{k} = -i\nabla\,,$$

which replaces Eq. (18). Nuclear systems obeying the roton-like spectra have been extensively studied, see [31,125,149] and references therein. From a recent time, some authors became to name such a spectrum the "moat" spectrum.

A typical density dependence of the squared effective pion gap, $\tilde{\omega}^2(k_0(n))$, for $\pi^\pm, \pi^0$ in ISM and for π^0 at any N/Z is illustrated in Fig. 6, see also Ref. [31]. For $n > n_c^{(1)}$ the effective pion gap acquires a minimum at a finite momentum

k_0. With a subsequent density increase, $\tilde{\omega}^2(k_0(n))$ decreases (solid line). Various calculations done within a mean-field approximation (ignoring contributions of pion fluctuations to the pion self-energy) demonstrate that at $n > n_c^\pi$ the pion condensate with a liquid-crystal-like or a solid-like structure may appear via the second-order phase transition. Actually at $n = n_c^\pi$ the classical pion field is developed, in the first-order phase transition due to the effect of fluctuations with $k \sim k_0 \sim p_{\mathrm{F},N}$. Dashed-dotted line illustrates the sudden jump and the following growth of the pion condensate amplitude. The squared effective pion gap on top of the condensate is shown by the dashed line. It is growing with an increasing density. As we have noted, in the ISM the value n_c^π is the same for $\pi^\pm$ and π^0. In the isospin-asymmetric matter the value n_c^π depends on the sort of the pion. Actual values of the critical densities depend on poorly known density dependence of the Landau–Migdal parameters. Numerical analysis [150] performed within the "variational theory of nuclear matter", which employs the realistic two-nucleon potential in vacuum and then introduces three-nucleon interactions, gave the value $n_c^\pi \simeq 2n_0$ for the critical density of the π^+, π^-, π^0 condensation in the ISM and the value $n_c^\pi \simeq 1.3n_0$ for the π^0 condensation in the purely neutron matter. Cooling of neutron stars is appropriately described within the "nuclear medium cooling scenario" which takes into account of effects of the pion softening [31,151–153] provided $n_c^\pi \gtrsim (2-2.5)n_0$ for $\pi^\pm$ and π^0 condensates, cf. [99,100,102,154–158] and references therein. Continuation of the solid line for $n > n_c^\pi$ corresponds either to the possibility of a metastable phase in the system, when the ground state contains no liquid-crystal-like or solid-like pion condensates. Dotted line corresponds to the case of saturation of the pion softening because of an artificially strong increase of the Landau–Migdal parameter g' with the density.

Finally, we note that the resummation of the in-medium NN interaction (25) imposes that at $n > n_0$ the NN cross section has strong density dependence owing to the interplay of a moderately suppressing effect from the vertex correction (27) because of NN correlations and a strong enhancing effect of the softening of the pion mode with the increasing density and, therefore,

$$\sigma_{NN} \propto \frac{\Gamma^4\left(g', 0, k_0, n\right)}{\left(\tilde{\omega}\left(k_0(n)\right)/m_\pi\right)^4}. \tag{31}$$

4.2 $\pi^\pm$ condensate nuclear systems

Spectra of pions in the ISM and purely neutron matter were demonstrated in [143]. In the latter case for $\pi^\pm$ there are the π^+ and π^- branches, $\omega^{\pi^+}(k), \omega^{\pi^-}(k)$ (corresponding to $\omega \to m_\pi$ for $n \to 0$) and the higher lying Δ branches. It proves to be that up to large densities $\omega^{\pi^-}(k) > \mu_e$ [140,143] and the reaction $N + e \to N + \pi^- + \nu$ is forbidden. If pion were free, $\omega^2 = m_\pi^2 + k^2$, such a reaction were allowed for $\mu_e(n) > m_\pi$ resulting in efficient cooling of the neutron stars heaving such a densities in their interiors. For $n > n_c^+$ ($n_c^+ < n_0$ according to evaluations of [143]) there appears extra branch with $\omega_s^{\pi^+}(k) < 0$ corresponding to appearance of the so called π_s^+ condensate. We have mentioned this possibility in Sect. 3.3. Accordingly, for $n > n_c^+$ the reaction $p \to n + \pi^+$ becomes possible resulting in the production of π^+'s occupying this branch. For $n = n_c^\pm > n_c^+$ the branches $-\omega_s^{\pi^+}(k)$ and $\omega^{\pi^-}(k)$ coincide at $k = k_0 \neq 0$, where $\omega_s^{\pi^+}(k_0) + \omega_\pi^-(k_0) = 0$. Thereby for $n > n_c^\pm$ the reactions $N \leftrightarrow N + \pi^+ + \pi^-$ become energetically favorable and there appears $\pi^\pm$ condensate. Appearance of $\pi^\pm$ condensation for $n > n_c^\pm$ results in the efficient neutrino cooling processes, e.g. via $N + \pi_{cond}^- \to N + e + \bar{\nu}$ reaction.

At large densities the baryon subsystem is essentially rearranged. In the approach exploiting the chiral symmetry [144–146] it was shown that instead of two different Fermi seas for protons and neutrons, a common Fermi sea for baryon quasiparticles, being superpositions of proton neutron and Δ isobar states mixed due to interactions with the developed pion condensate, is formed. At densities much larger than $n_c^{(\mathrm{p})}$ the condensate becomes developed and the squared amplitude of the pion condensate reaches its maximum value $f_\pi^2/2$ for $\pi^\pm$, where $f_\pi \approx 92$ MeV is the pion weak decay constant and the electric charge density of the baryon subsystem becomes equal to $n/2$. Taking into account these effects, and electron and muon populations, the energy density acquires the form [52]

$$E_{B+\pi+e+\mu} = E_B + E_\pi + E_e + E_\mu\,, \tag{32}$$

$$E_{e,\mu} \simeq \int_0^{\sqrt{V^2 - m_{e,\mu}^2}} \frac{\mathrm{d}pp^2}{\pi^2}\sqrt{m_{e,\mu}^2 + p^2}\,\Theta\left(|V| - m_{e,\mu}\right),$$

$$E_\pi = \frac{V^2 f_\pi^2}{2} + \frac{1}{3}\left(m_\Delta^* - m_N^*\right) n - \frac{81}{50} f_{\pi NN}^2 (1 - g')n^2,$$

where E_B is the energy density of the baryonic subsystem, g' is the isospin Landau–Migdal parameter, and m_Δ^* is the effective mass of the Δ isobar (one may put for simplicity $m_\Delta^* - m_N^* \simeq m_\Delta - m_N = 293\,\mathrm{MeV}$), $\Theta(x)$ is the step function. It is taken into account that in case of the developed condensate $\mu_n \simeq \mu_p$. For more details the reader may consult Refs. [31,52]. The electric charge of protons is screened by the condensate of negative pions and by electrons and muons produced via the weak β processes and from the vacuum (at rather large values of Z) [52]. The electric potential is found from the Poisson equation for a spherical system of radius R and constant baryon density n,

$$\Delta V = 4\pi e^2\left(n_e + n_\mu + n_h\right)\,, \tag{33}$$

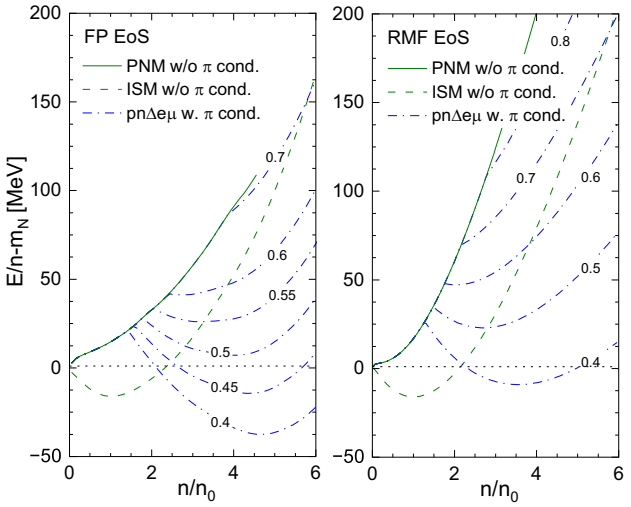


Fig. 7 Binding energies per nucleon for the PNM (solid lines), the ISM (dashed lines), and for the beta-equilibrium matter ($pn\Delta e\mu$) with the developed π^- condensate (dash-dotted lines). The latter lines are calculated for several values of the Landau–Migdal parameter g' characterising the strength of the short-range nucleon–nucleon correlations; the parameters are shown by labels on the lines. On the left panel, the nuclear part of the energy is taken from the FP equation of state [159]. On the right panel we use the RMF model [160]. Extra explanations are given in the paper body. The choice of parameters is such that the RMF equation of state supports the maximum neutron star mass of $2.09\,M_\odot$ [88]

$$n_h = \left(\tfrac{1}{2}n + f_\pi^2 V\right)\Theta(R-r),$$
$$n_{e,\mu} = -\frac{1}{3\pi^2}\left(V^2 - m_{e,\mu}^2\right)^{3/2}\Theta(|V| - m_{e,\mu}).$$

Here $f_\pi^2 V$ is the contribution of the developed π^- condensate as it follows from the consideration within the σ model and $n/2$ is the total baryon charge density in the presence of the developed condensate [146], n_e and n_μ are contributions of electrons and muons. Nullifying the r.h.s. of Eq. (33), similarly to that we did above for Eq. (19), demonstrates the charge neutrality condition in the interior of a large size system, for $R \ll l_V$ where l_V is the Debye screening length.

In Fig. 7 by solid curves we show the binding energy per nucleon for the purely neutron matter (PNM). Other curves show calculations for the ISM with and without the developed condensate done with the help of Eq. (32). Actually this expression does not hold for $n \lesssim n_c^{(p)}$ since it was derived for $n \gg n_c^{(p)}$, see a discussion in [31]. However we conjecture that for $n \gtrsim (2-3)n_0$ we already can deal with the developed pion condensate On left panel the nucleon part (dashed line) is calculated with a soft Friedman–Pandharipande (FP) equation of state [159] and on right panel with a stiffer RMF equation of state, see Eq. (15). In the RMF case parameters are tuned to describe the saturation density $n_0 = 0.160\,\mathrm{fm}^{-3}$, the binding energy $\mathcal{E}_0 = -16.0\,\mathrm{MeV}$, the compressibility modulus $K = 273.6\,\mathrm{MeV}$, the effective nucleon mass $m_N^*(n_0) = 0.755\,\mathrm{MeV}$, and the value of the coefficient of the nuclear symmetry energy at saturation, $J = 32.0\,\mathrm{MeV}$. With these parameters the maximum neutron star mass is $2.09\,M_\odot$, being larger $2\,M_\odot$ in agreement with the modern experimental data. The attraction strength of the $\pi^\pm$ condensate contribution, E_π in Eq. (32), is controlled by the value of the g' Landau–Migdal parameter. The density dependence of this parameter is poorly known, therefore following [31,52] we allow for its variation. So, the energies per nucleon of the baryon, $pn\Delta e\mu$ matter with the pion condensate (dash-dotted lines) are shown for several values of g'. Note that various fits of the atomic nuclei data produce the value of $g'(n_0) \approx 0.7$. At this instance to avoid a possible misunderstanding we should mention that different authors use different normalizations of g' and distinct expressions for the pion polarization operator. The smaller $g'(n)$ is the stronger is the pion condensate attraction for $n > n_c^{(\mathrm{p})}$. In [31] there were given some arguments for a possible decrease of $g'(n)$ with increasing density. For the FP equation of state (left panel in Fig. 7) the energy of the pion condensate matter becomes negative in some interval of density for $g'(n > n_c^{(\mathrm{p})}) < 0.48$. In this case there may exist superdense abnormal pion condensate nuclei, nuclearites and nuclei–stars of arbitrary size bounded by nuclear forces rather than by gravity (for $10^3 \lesssim A \lesssim 10^{56}$), as it was suggested in Ref. [52]. The maximal net charge of an abnormal nucleus with $A \gg 1/e^3$ observed at infinity behaves as $A^{1/3}$. In this case the extra charge is with necessity screened by electrons and muons occupying the vacuum shell. The pion condensate state becomes an absolute ground state of nuclear matter if $g' < 0.45$, then $\min_n\{E_{B+\pi}\} < -16\,\mathrm{MeV}$. For $0.48 < g' < 0.62$ there is a minimum in energy $E_{B+\pi}$ that would correspond to a positive energy metastable state and for $0.45 < g' < 0.48$ to a negative energy metastable state. For the RMF nucleon equation of state, which we used (see right panel in Fig. 7) the abnormal negative energy states may exist for $g' < 0.43$ and the absolute ground state of matter appears for $g' < 0.38$. The metastable states with positive energy are possible for $0.43 < g' < 0.63$.

4.3 Dense π^0-condensate ferromagnetic nuclear systems

Possibility of neutron-enriched abnormal π^0 condensate nuclei was discussed in [28,29]. Ferromagnetic behavior of π^0 condensate was studied within the σ model first in [147]. The $\sigma\pi^0$ running wave condensate in the neutron matter corresponds to a state, in which all neutron spins are aligned, presumably with some macroscopic domain structure. Recently there were found that extra contributions to the p-wave π^0 condensate term in the Lagrangian may arrive from the so-called Wess–Zumino–Witten axial anomaly term describing the anomalous interaction of the neutral pion field with the

external electromagnetic field and the rotation, and a related pion contribution to the baryon current, cf. [161–165]. Comparison of the ferromagnetic phase of the π^0 condensate appearing due to the axial anomaly term and the alternating layer structure of the pion condensate [148] demonstrated that the latter state is probably energetically more favorable [163]. These effects were recently studied in Ref. [166].

4.4 About the s-wave pion condensation

Pion off-mass-shell effects are of primary importance for description of the pion spectra and a possibility of the pion condensation. The most important contribution to the pion polarization operator is given by the p-wave pion-nucleon and the pion-Δ-isobar interactions. For an s-wave part of the pion polarization operator already first works employed the Weinberg-Tomozawa expression, $\Pi^s_{\pi^-} = C(n_n - n_p)\,\omega$, with $C \simeq 1/(2f_\pi^2) \simeq 1/m_\pi^2$, which does not contribute in case of the ISM. Parametrization of the optical pion-nucleus potential [30,31,90] used the fully off-mass-shell pion-nucleon amplitude, which fulfills the current algebra theorems and the canonical PCAC condition (see the MSTV model in [166]). Oppositely, Refs. [167–170] (see the KKW model in [166]) used the on-mass-shell pion-nucleon amplitude, taking incoming and outgoing pion 4-momenta such that $q^2 = q'^2 = m_\pi^2$, that does not allow to fulfill the so called Adler and Weinberg current algebra conditions. Subtleties associated with the current algebra theorems and field redefinitions were discussed in [171]. The s-wave pion-nucleon interactions in the linear sigma model, and in the Manohar–Georgi and Gasser–Sainio–Svarc models with finite number of terms in Lagrangians, as well as in a general phenomenological approach were reviewed.

With the MSTV model for the s-wave pion-nucleon interaction the s-wave pion condensation in the ISM does not occur at least up to very high densities. Oppositely, if the KKW model [167–170] were valid, the s-wave pion condensation in the ISM would be expected to occur already for $n > n_c^{(s)} \sim (1.4 - 2.5)n_0$, if the nucleon–nucleon correlations are not taken into account, resulting in the energy density gain

$$E_\pi^{\rm KKW}(n) \simeq -\frac{m_\pi^4}{2\lambda}\left(\frac{n}{n_c^{(s)}} - 1\right)^2 \Theta(n - n_c^{(s)}). \qquad (34)$$

In Fig. 8 we plot the energy per particle of the ISM with and without contribution of the s-wave pion condensation (34). We use the same RMF model as on the right panel of Fig. 7. The pion-pion self interaction parameter is chosen to be $\lambda = m_\pi^2/(2f_\pi^2)$. The value of the softening of the equation of state essentially depends on the value of the critical density. As we see, for $n_c^{(s)} \gtrsim 2.5\,n_0$ the softening is weak, whereas

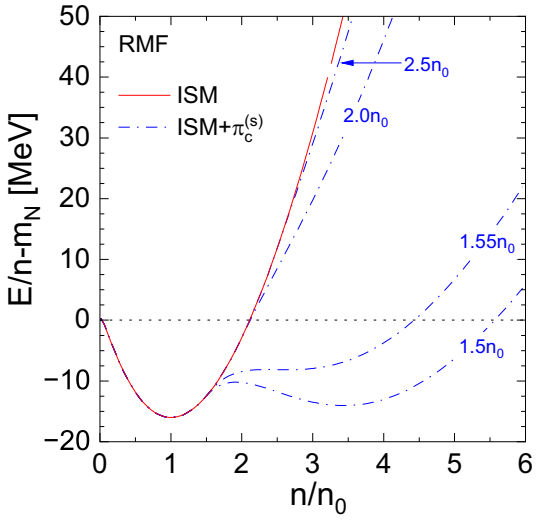


Fig. 8 Binding energy per nucleon for the ISM with and without the neutral $\pi^{\pm,0}$ condensate (solid and dash-dotted lines, respectively). The baryon part of the energy for ISM is calculated in the framework of the RMF model used on the right panel of Fig. 7. The dashed dotted curves are calculated within the KKW model for different values of the critical density $n_c^{(s)}$ indicated by the line labels. In calculations we used $\lambda = m_\pi^2/(2f_\pi^2)$

for $n_c^{(s)} \leq 1.55\,n_0$ the effect is strong and there appears possibility of a metastable state with the s-wave pion condensate. Thereby one could expect to observe experimental manifestations of the s-wave pion condensate in heavy-ion collisions. We must, however, emphasize that the estimate of the value of the critical density $n_c^{(s)}$ remains strongly model-dependent even within the on-mass shell based model [167–170]. For example, the NN correlation effects in the s-wave pion polarization operator, which cannot be constrained from the experiments at present, could shift the condensation critical point to higher values [166].

5 Δ resonance matter

Above we indicated that the pion-nucleon-Δ attraction promotes a decrease of the effective pion gap and occurrence of the pion condensation at smaller baryon density. Now we will review the idea of Δ resonance matter.

The idea of a stable or metastable baryon-resonance matter was introduced in Ref. [55]. The Fermi momentum of the nucleon is $p_{\rm F} \propto n^{1/3}/\nu^{1/3}$, the total Fermi energy of baryons grows as $E_{\rm F} \propto n^{2/3}\nu^{2/3}$ where ν is the degeneracy factor. When the nucleon Fermi energy, $E_{\rm F}$, reaches the difference between effective masses $m^*_\Delta - m^*_N$, it becomes energetically favorable to create $\Delta(1232)$ isobars. The appearance of any new degree of freedom results in a softening of the equation of state. The nucleon–nucleon-hole contribution to the pion polarization operator is $\Pi_{\pi NN} \propto -\nu p_{\rm F} k^2 \propto \nu^{2/3} n^{1/3}$,

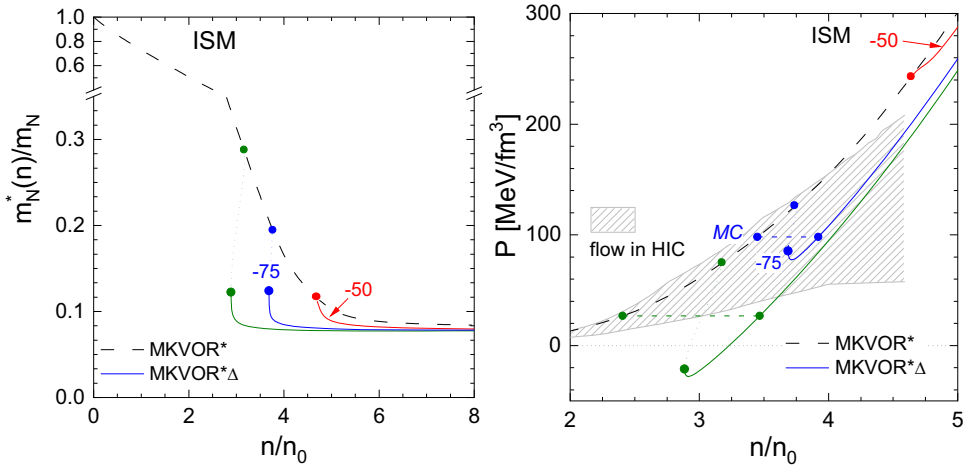


Fig. 9 Left panel: Effective nucleon mass as a function of density in the ISM without Δ isobars (dashed line) and with the allowance for Δs (solid continuations). Right panel: Pressure as a function of density without and with Δs shown by dashed line and dashed line with dotted–solid continuations, respectively. The hatched region indicates the constraint obtained from the analysis of the momentum distributions (directed flow) of particles produced in heavy-ion collisions [98]. The calculations are done within the RMF models MKVOR* and MKVOR*Δ for several values of the Δ potential at saturation density $U_\Delta = -50, -75$, and -100 MeV, see Ref. [95] for details

Therefore, the latter quantity effectively corresponds to the ν^2 times higher density compared to the case of $\nu = 1$. The Δ–nucleon–hole contribution is $\Pi_{\pi\Delta N} \propto -nk^2$, see [172]. The extra Δ–Δ-hole attractive term to the pion polarization operator leads to a further decrease of the critical density of the pion condensation. Moreover, let us recall that the spin-isospin degeneracy factor for Δ is $\nu = 16$ compared to 4 for nucleons and that $f_{\pi N\Delta} \simeq 2 f_{\pi NN}$.

Δ isobars are produced copiously in relativistic heavy-ion collisions forming a resonance matter [173]. A possibility of density isomer states was discussed in [160,174–176] within RMF models and the chiral model of the nucleon.

Reference [95] employed the RMF models of the baryonic matter, in which all involved hadron masses are assumed to be functions of the scalar σ mean field. Below we consider two specific examples of such a model: the MKVOR* model not including Δ isobars and the MKVOR*Δ model including Δ isobars. The key ingredient of the RMF model with Δ isobars is the magnitude of the Δ optical potential at the saturation density, U_Δ. It controls the balance between attraction ($\sigma\Delta$ coupling constant) and repulsion ($\omega\Delta$ coupling constant). Although, one typically expects U_Δ to be of the order of the nucleon optical potential ~ -50 MeV, uncertainties in the extraction of the Δ properties in nuclear matter allow for variations of U_Δ in a broad interval. In the left panel of Fig. 9 we show the results from [95] for the effective nucleon mass $m_N^*(n)$ in ISM as a function of the density at zero temperature for MKVOR* model and for MKVOR*Δ model for different values of the Δ optical potential. In Ref. [95] it was shown that for $U_\Delta > -67$ MeV, m_N^* decreases monotonously in the MKVOR*Δ model with an increase of the baryon density n and reaches a small value in the high-density limit. For deeper potentials the Δ appears in matter abruptly at some critical density depending on the potential U_Δ that leads to a characteristic jump in the m_N* density dependence. This behaviour resembles the sharp change of the nucleon mass in the Lee–Wick model, see Fig. 3. On the right panel of Fig. 9 we show the pressure $P(n)$ at zero temperature as a function of the density with and without Δ isobars (models MKVOR* and MKVOR*Δ, respectively). The hatched region indicates the constraint of the pressure extracted from analysis of the momentum distributions (directed flow) of particles produced in the heavy-ion collisions [98]. For $U_\Delta = -50$ MeV, the derivative of the pressure gets a discontinuity that corresponds to a second-order phase transition owing to the appearance of Δ isobars. For $U_\Delta < -67$ MeV, $P(n)$ demonstrates the behaviour typical for the first-order phase transition, except that in our case there is a specific back banding in the $P(n)$ dependence. The horizontal lines connecting points ($P(n_1^{\rm MC}) = P(n_2^{\rm MC})$, $\mu(n_1^{\rm MC}) = \mu(n_2^{\rm MC})$) show the Maxwell construction (MC). For $U_\Delta < -91.4$ MeV, $P(n)$ crosses zero at two values of the density. One of these zeros corresponds to unstable state, other (right one), to metastable or even stable state corresponding to the Δ resonance matter. The presence of a pion condensate would result in a further enhancement of the Δ abundance.

Note that the occupation of the new Δ Fermi sea occurs at nucleon densities exceeding a critical value $n > n_c \sim (2-4)n_0$, which depends on the N/Z ratio and parameters of the model under consideration. Appearance of Δ isobars makes

the nuclear matter more stable. Besides this, the occupation of the Δ Fermi sea additionally promotes a decrease of the effective pion gap and a pion condensation at smaller baryon density.

In addition to the occupation of the Δ Fermi sea, in the neutron star matter (at $N \gg Z$), Λ, Σ^- and even Ξ^- hyperon Fermi seas can be occupied at approximately the same densities, $n > (2-3)n_0$. The appearance of any new fermion degree of freedom decreases the energy and, consequently, the maximum compact star mass. Such a decrease of the maximum neutron star mass has been observed in many models [177–181]. These problems are known as the "hyperon puzzle" and the "Δ puzzle". They have been resolved in the RMF models, in which the hadron masses are assumed to be functions of the scalar σ mean field [93–95].

6 Strangelets, strange and hybrid stars

The idea of stable strange quark matter proposed by Witten in 1984 [56] is in some sense similar to the mentioned above idea of stable Δ resonance matter by Troitsky and Chekunaev from year 1979 [55]. A quark nugget (strangelet or nuclearite) is a baryonic state composed of quarks in a deconfined phase. There is a correspondence between this state and nontopological solitons with baryon constituents studied by Lee and collaborators and then by Sidney Coleman [182–186]. In Ref. [56] Witten estimated the energy per quark for a three-flavor system (strange quark matter) to be $\sim 90\%$ of the energy of a two-flavour system. This additional energy gain could be enough to compensate the energy loss due to the heavier strange quark mass leading, thereby, to a stable quark system. The strange quark matter can be stable or metastable for a wide range of strong interaction parameters. If the strange quark matter is more stable than the nucleus of Fe and the baryon number is $A \ll 10^{56} - 10^{57}$ one speaks about strangelets and quark nuclearites, and if the baryon number is large, $A \sim 10^{56} - 10^{57}$, about strange stars. In the latter case, the gravity essentially contributes to binding. The surface layer of these objects has typically a microscopic length ~ 10 fm. Provided the gravity is switched off some models of quark matter do not allow for the absolutely stable strange quark matter but allow for a metastable state. Other models use such parameters that the strange and nonstrange deconfined matter is unstable. In these both cases, the quark matter, nevertheless, may exist in interiors of compact stars, named in this case hybrid stars. All such stars have macroscopic hadron shells. Sometimes some authors conflate mentioned terms calling hybrid stars as strange stars or quark stars. From our point of view, the most important is that strangelets, quark nuclearites and strange stars could be stable already without assistance of gravity. The difference is only that in the case of strange stars the baryon number typically is $A \sim 10^{56} - 10^{57}$, and then the gravity contributes essentially to binding. For object with a smaller A the gravity is less important. However, the important difference exists between strange stars and hybrid stars, since the latter contain hadronic shells of macroscopic size. Also we should note that in all quark nuggets with $A \gg 1/e^3$ up to a strange star size, the net electric charge is screened mainly by electrons and muons similarly to the case of the pion condensate nuggets and nuclei–stars, cf. results of Refs. [60] and [49,50].

For strange matter to form, there must initially be a sufficient number of strange quarks. Since the probability of multiple weak processes occurring simultaneously is enormously suppressed, the large amount of strangeness required to satisfy conditions of Witten's hypothesis can only be obtained under very specific conditions. As far as we know, these conditions can only be met in the Early Universe and/or in the cores of massive stellar objects. Discussions of hypothetical strangelets and hybrid and strange stars began with the works [24–26,56,57,187] and remain a hot topic till these days, see, e.g., Refs. [63,121,188–190] and references therein.

6.1 Quark models

The most commonly employed model of quark matter is the MIT bag model [191]. By balancing the vacuum pressure outside the bag with the pressure of quarks inside it, one mimics phenomena of quark confinement and asymptotic freedom [57]. The quark matter is stable, metastable or unstable in these models in dependence mainly on the assumed value of the bag constant B and the strange quark mass m_s. A large value of B excludes the existence of hybrid stars. With a decrease of B, first the most massive neutron stars acquire a quark core, becoming hybrid stars. For a small B and a smaller value of m_s, strange quark matter becomes absolutely stable, i.e., more stable than ^{56}Fe.

In addition to the MIT-bag based approaches, the Nambu–Jona–Lasinio (NJL) model is frequently employed, see Ref. [192]. However the latter does not describe quark confinement, and its preferred sets of parameters generally do not allow for stable strange quark matter [193]. Competition of inhomogeneous chiral phases and superconducting quark phases in the NJL model was studied in [194].

The quark objects are anticipated to be color superconductors. The attraction comes either from the one-gluon exchange, or from a nonperturbative four-point interaction motivated by instantons, or from nonperturbative gluon propagators. Various quark-paring patterns were investigated: the color-flavor-locked (CFL) phase, the two-flavor color superconductivity (2SC) phase, the color spin-locking (CSL) phase, etc., see Refs. [68,121]. The 2SC phase allows for unpaired quarks of one color. The CFL phase deals with very large values of the baryon chemical potential. In this

case the color superconductivity is complete in the sense that the diquark condensation produces a gap for quarks of all three colors and flavors. The value of the gap for CFL and 2SC phases can be of the order of 100 MeV. Since the CFL phase is expected to occur at ultra-high-density, its presence in compact stars is more questionable than presence of the 2SC phase. Owing to strong interactions, a fluctuation region near the critical point of the color superconductivity is expected to be broad and it is not excluded that some effects of the color superconducting fluctuations can be manifested in heavy-ion collisions [195].

Depending on the value of the diquark coupling a region of the phase diagram was found where the inhomogeneous chiral and the 2SC condensates may coexist, see Ref. [196] and references therein. Let us add here that at large densities QCD may exhibit a moat regime in the scalar-pseudoscalar sector [197] similar to the occurrence of the minimum in the effective pion gap, see Sect. 4. Also, one uses the quark models with quark masses varying with the baryon number density, see Ref. [198]. The RMF approach proposed in Ref. [199] gives a simple way to model confinement of quarks via introducing the particular density dependence of quark masses divergent for low baryon densities and a density-dependent screening effect. The screening is described analogously to an excluded volume effect for a hadronic matter model with density-dependent coupling constants. Effects of various pasta phases occurring in the quark-hadron phase transition [200] were then investigated in Ref. [201] for a set of RMF equations of state for both the hadron matter and the quark one. Parameters used in these works allow for hybrid stars but exclude stable quark star configurations.

Some constraints can be gained from the lattice QCD calculations. The standard bag and NJL models cannot reproduce the lattice data without significant modifications. However, the lattice results at zero and small finite chemical potential are appropriately described within phenomenological quasiparticle models employing massive quarks and gluons with masses dependent on temperature [202–205]. These models were also used to extrapolate the lattice equation of state to the case of cold baryon matter. This has been done in Ref. [206] and then in Refs. [207–209] and in other works.

The quasiparticle model [208] for the equation of state of the quark-gluon matter at finite temperatures and baryon chemical potential, which is tuned to the lattice QCD data [210,211], was used in Ref. [212] to study the possibility of a hadron-to-quark transition in the neutron star matter. For the hadronic equation of state the RMF model [88] was used. For comparison two "light-bag" and "heavy-bag" models were also considered. The "light-bag" models with conventional current quark masses ($m_u = 5, m_d = 7, m_s = 150$ MeV) and vanishing gluon mass and values of the bag constant $B^{1/4} = 155$ MeV and $B^{1/4} = 200$ MeV were used. These values of $B^{1/4}$ MeV are typical for models applied

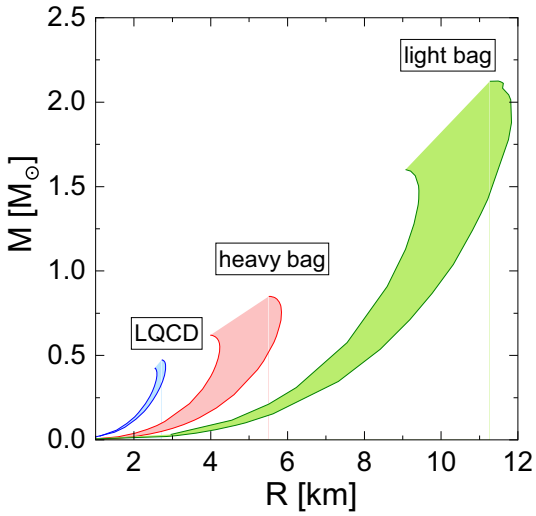


Fig. 10 Strange star mass-radius relation for the light-bag and heavy-bag models and the quasiparticle model [208] tuned to the lattice QCD data. For details see text and Ref. [212]. The bands correspond to variations of pairing gap Δ between 0 ad 100 MeV for the CFL type of quark superconductivity

to the treatment of hybrid and strange stars [213,214]. It proved to be that the "light-bag" models cannot describe the equation of state obtained in the lattice QCD calculations. Also it was considered a "heavy-bag" model with parameters $m_u = m_d = 330$ MeV, $m_s = 450$ MeV, $m_g = 600$ MeV, and $B^{1/4} = 183$ MeV. This model can reasonably reproduce the lattice results at temperatures not too close to the critical temperature. Also a more specific quark quasiparticle model [208] was introduced, which fits well the lattice QCD calculations above the critical temperature. The general result of Ref. [212] is that the families of ordinary neutron stars and pure quark stars are well separated. The hadron-quark interface can be constructed only between the light-bag model and unrealistically stiff Walecka RMF model [82]. For the quark matter model fitted to the lattice data the hybrid star cannot be constructed. The application of these models to the quark stars is illustrated in Fig. 10. The light-bag model produces neutron stars with masses up-to 1.5-−2.1 $M_\odot$ and radii $9 < R < 12$ km. For the heavy-bag model, neutron star becomes lighter and more compact, and the model tuned to the lattice QCD supports only stars with very low masses ($M \lesssim 1 M_\odot$), high densities ($n \gtrsim 10 n_0$), and small radii ($R \lesssim 6$ km). However the situation remains rather uncertain. Some studies provide arguments for stability or metastability of two-flavor quark matter, while others argue against the stability of quark matter, e.g. see [215,216] and references therein.

We also should mention the possibility of a quarkyonic phase [217], which may occur between hadron and quark phases. Thus the temperature–baryon density, $T - n$, phase diagram of the nuclear matter proves to be very rich. With

increasing density at a moderate temperature there may appear region, in which we may deal with the stable or metastable π or σ condensate matter or the strange quark matter.

Besides finite size strangelets and strange stars, also a branch of strange white dwarfs was also discussed [218]. Such dwarfs would have conventional nuclear crusts and strange quark matter cores. Possible candidates for strange dwarfs were suggested in Ref. [219]. Potential implications for gravitational wave astronomy are highlighted in Ref. [220].

Recently, other state of deconfined matter was proposed in Ref. [71] based on the model described in Ref. [69]. The compact soliton-like object consists of a core of quarks (antiquarks) in a color-superconducting state surrounded by an electrosphere of positrons (electrons), and enclosed by an axion domain wall. Large axion domain walls can form large bubbles filled with u, d and s quarks. Earlier, the idea that soliton-like configurations could serve as dark matter arose in Refs. [182–186]. Among other conditions, the possibility of existence of these objects is based on the feasible existence of a conserved charge associated with an ungauged unbroken continuous internal symmetry. In Ref. [183] such objects were called Q balls. Large Q balls may form so-called boson stars. There is a scenario that the dark matter is associated with slow light bosons with a mass $m \sim 10 - 22\,\mathrm{eV}$ and the huge de Broglie wavelength of $\sim 1\,\mathrm{kpc}$. This is called the fuzzy dark matter. A soliton is surrounded by an envelope resembling a cold dark matter hallo [221]. Recent work [222] proposes a model for the cosmological formation of superheavy Q balls as MACHOs (massive compact halo objects), the objects in the mass range $(10^{-7} - 10^{-6})\,M_\odot$. Also it shows that it is possible to produce asteroid mass Q balls. A review of these topics can be found in [222–224].

Finally, if the mass distribution of strangelets is obtained, it is possible to formulate well-focused campaigns to search for these objects, cf. [190].

The most promising signals of the scalar and pion condensate nuclei–stars and strange stars seem to be an abnormal mass-radius relation [31,225], an r mode spin-down [78,226,227], and specific stellar cooling properties [155,228–232].

6.2 Abnormal π condensate and strange compact stars and the star mass-radius relation

For quite some time there has been a consensus that most neutron stars being produced in supernova explosions have masses near $1.4M_\odot$ close to the neutron star maximum mass $M_{\max} \simeq 1.5M_\odot$ [233,234]. For $M > M_{\max}$ one deals with the black hole. Most of the existing that time hadronic equations of state could describe neutron stars with such masses. However then compact stars of different masses were observed. By now it is well established that the compact star masses vary in a broad interval [235]. At present the minimal value of the accurately measured mass is $1.174(4)\,M_\odot$ for the pulsar PSR J0453+1559 [236]. There was the claim that the object HESS CCO XMMU J173203.3-344518 has a very small mass of $0.77^{+0.20}_{-0.17}M_\odot$ [237]. The analysis of Ref. [237] was criticized in Ref. [238] with the conclusion that a small mass source is not needed for the description of observed spectra. Thereby one should consider the result [237] with a caution.

Today, the highest precisely-measured mass of a neutron star is $2.08(7)\,M_\odot$ for PSR J0740+6620 [111,112]. Recent analyses of the NICER data indicate that there is no significant variation of star radii for stars with masses varying between 1.4 and $2.0\,M_\odot$, $R \simeq (12-13)$ km. A more detailed discussion can be found in Ref. [239]. Although any strong softening of the equation of state leads to a decrease of the radius for the objects with fixed mass, e.g. see Ref. [212], the maximum mass of a pion-condensate nucleus-star, a hybrid star, or a strange star can be smaller or larger than the purely hadronic star, depending on the utilized model. In this respect we should mention that recently, the mass and radius of the Type-I X-ray burster and accretion-powered millisecond pulsar XTE J1814-138 were inferred in Ref. [240] to be $M = (1.21 \pm 0.05)\,M_\odot$ and $R = 7.0 \pm 0.4\,\mathrm{km}$ (with 68.3% credibility). If confirmed, the compact object of such a small radius could not be described by a conventional purely hadronic equation of state. Then, this object would be a candidate for a strange star, a pion condensate nucleus-star, a compact object with the Lee–Wick σ condensate, or something else.

Reference [241] suggested that a massive cold neutron star can undergo the first-order phase transition to the pion condensate state with a blowing off a part of matter or at least a giant glitch. Then a similar idea was applied to the first-order phase transition to a hybrid (quark-hadron) star configuration [242]. In majority of papers the later term "hybrid star" is associated with the star heaving a quark interior and an extended hadron exterior although consequences of any strong first-order phase transition are approximately the same, see further discussion in [31]. Then, an interesting idea appeared that a neutron star and a star, where the first-order phase transition to the state with a superdense interior (pion condensate, σ condensate, quark matter or something else) occurred, can have the same mass at very different radii. If mass-twins were observed, it would be an evidence for the existence of compact stars different from ordinary neutron stars (e.g. hybrid stars, strange stars, pion-condensate nuclei–stars, or something else), see Refs. [201,243–246] and references therein.

6.3 Cooling of π condensate abnormal nuclei, nuclei–stars, strangelets and strange stars

Below we show that nuclei–stars held together by a pion condensate and neutron stars containing a broad pion condensation region in their interiors as well as hybrid stars with an extended region of a quark 2SC phase are probably rather massive rapidly cooling stars, with $M \gtrsim 1.5\,M_\odot$.

At present time there exist data on the surface temperatures of some pulsars as function of their age, see [247,248]. Therefore, it is worthwhile to check whether existing models can describe them. The data can be separated on three groups: slow, intermediate, and rapid coolers. The data impose that the averaged neutrino emissivity for the slow and rapid coolers should differ by factor $\gtrsim 10^3$. The minimal cooling scenario, in which important in-medium effects are disregarded, cannot appropriately describe the data. Shortcomings of this scenario are discussed in Refs. [99,100,155].

The most efficient processes of the neutrino production in neutron stars are one-nucleon processes because of their large phase-space volume. These are the DU processes, e.g. $n \to p + e + \bar{\nu}_e$ and $p + e \to n + \nu_e$, and the processes going on the mesonic condensates, e.g. on the pion condensate, such as $n + \pi^-_{\rm cond} \to n + e + \bar{\nu}_e$. The emissivity of the DU process can be roughly estimated as

$$\epsilon_\nu^{\rm DU} \sim 10^{27}\,(n_e/n_0)^{1/3}\,e^{-\Delta/T} T_9^6 \Theta\left(n - n_c^{\rm DU}\right) \frac{\rm erg}{\rm cm^3 \cdot s}, \tag{35}$$

where T_9 is the temperature measured in 10^9 K, n_e is the density of electrons. The suppression factor $e^{-\Delta/T}$ appears for temperatures below the critical temperature of a nucleon pairing T_Δ, and a pairing gap $\Delta \propto (T_\Delta - T)\Theta(T_\Delta - T)$, where $\Theta(x)$ is the step-function. The DU process occurs for $n > n_c^{\rm DU}$, when the fraction of protons exceeds $\sim 10\%$. The DU process is so efficient that, if it occurred, the stars with $M > M_{\rm DU} + 0.1 M_\odot$ would cool so rapidly that they would not be manifested in the cooling data [88,249]. Therefore, the deficiency of the "minimal cooling + DU" scenario is that it would require all stars in the cooling diagram (temperature vs. time) to have a mass close to the threshold for the DU processes to occur in the star center, with an uncertainty of only $0.1 M_\odot$. However now it is known that masses of neutron stars proved to be essentially different. Moreover, there are equations of state in the literature, in which the DU process either does not appear in the whole available density interval or it occurs only for $M > 1.5 M_\odot$. For example, in case of the famous variational equation of state [150] the threshold mass for the DU process is $M_{\rm DU} \simeq 2 M_\odot$.

In works [99,100,102,154–158] the "nuclear medium cooling scenario" was developed. It relied on the results of Refs. [151–153,155] where it was assumed that neutron stars with measured surface temperatures (first data fixed only upper limits on surface temperatures) may have very different masses, and that neutrino emissivity of the star strongly depends on the nucleon density and, consequently, on the neutron star mass. This dependence appeared since the in-medium modification of the pion exchange was taken into account that significantly affected two-nucleon reaction rates. Within the "nuclear medium cooling scenario" the DU reactions were not allowed, at least for stars with masses $M < 1.5 M_\odot$.

Another process of the one-nucleon origin is a reaction going on the pion condensate, e.g. $n + \pi^-_{\rm cond} \to n + e + \bar{\nu}$. Its emissivity is roughly estimated as

$$\epsilon_\nu^\pi \sim 10^{27} \frac{\phi^2}{m_\pi^2} \left(\frac{n_e}{n_0}\right)^{1/3} e^{-\Delta/T} T_9^6 \Theta\left(n - n_c^\pi\right) \frac{\rm erg}{\rm cm^3 \cdot s}, \tag{36}$$

where $n_c^\pi \sim (1.5 - 4) n_0$ depending on the model, and ϕ is the amplitude of the pion condensate field, $\phi \sim (0.1 - 1) m_\pi$. This process would be too efficient as the DU process, if n_c^π were smaller $2 n_0$, $M \lesssim (1.3 - 1.35) M_\odot$. It would affect most of the neutron star population and lead to a contradiction with the measured Log(N) - Log(S) distribution. We should also mention the DU-like processes occurring on hyperons and Δ isobars at densities exceeding the corresponding critical densities. Values of these emissivities are similar to those for the π condensate processes, cf. [101,102]

In the presence of nucleon pairing (for $T < T_\Delta$) a new important process of the quasi-one-nucleon origin occurs. It is associated with breaking and formation of nucleon pairs, so the name — the pair-breaking-formation (PBF) process [153, 250–252], e.g. $n_{\rm pair} \to n + \nu + \bar{\nu}$. Its emissivity is estimated as

$$\epsilon_\nu^{\rm PBF} \sim 10^{28}\,(\Delta/{\rm MeV})^7\,(T/\Delta)^{1/2} e^{-2\Delta/T} \frac{\rm erg}{\rm cm^3 \cdot s}. \tag{37}$$

The in-medium renormalization of the vertices is crucially important for the PBF processes [252].

Note that processes going on π condensate and the PBF processes accounting the renormalization of the vertices are purely in-medium processes, which would not be consistent to include in the minimal cooling scheme disregarding effects of the polarization of the medium. However the PBF process was included, whereas the π^- condensate processes were disregarded, even though the latter would occur already in the stars with $M \gtrsim 1 M_\odot$, since the electron chemical potential would then reaches the bare pion mass if in-medium effects were switched off. The *pionization* of the neutron star matter can only be precluded if the effects of the medium on pions are taken into account [30].

In the absence of the DU process and PBF process in the minimal cooling scheme the main role in the neutron

star cooling is taken by the modified Urca processes, e.g. $n+n \to n+p+e+\bar{\nu}_e$. Its emissivity is roughly estimated as

$$\epsilon_\nu^{\rm MU} \sim 10^{21} \left(\frac{n_e}{n_0}\right)^{1/3} e^{-2\Delta/T} T_9^8 \frac{\rm erg}{\rm cm^3 \cdot s}. \tag{38}$$

Within the "nuclear medium cooling scenario" it was shown that the emissivity of two-nucleon reactions, e.g. $n+n \to n+p+e+\bar{\nu}_e$, — the medium-modified Urca processes (MMU) — significantly increases with an increasing of the central density (and the mass) of the object due to strong modification of the NN interaction as demonstrated in Fig. 26. Already in Refs. [151–153] it was suggested that (i) the central densities (masses) of cooling neutron stars are different, and (ii) the nucleon–nucleon scattering is significantly enhanced in denser matter, cf. Eq. (31). The appropriate formalism for calculation of the reaction rates in medium is the non-equilibrium closed diagram formalism employing the exact Green's functions. It was developed in the application to the neutrino emissivity problem in Ref. [153] using the quasiparticle approximation and then in Ref. [253] was extended by inclusion of finite nucleon widths, see also [155]. In the absence of the DU processes, the MMU processes are dominant already for $n > n_c^{(1)}$, where $n_c^{(1)} < n_0$, see Fig. 6. Its emissivity is roughly estimated as

$$\epsilon_\nu^{\rm MMU} \sim 10^{21} \frac{F(n)}{F(n_0)} \left(\frac{n_e}{n_0}\right)^{1/3} e^{-2\Delta/T} T_9^8 \frac{\rm erg}{\rm cm^3 \cdot s}, \tag{39}$$

where the factor $F(n)$ appears due to a strong density dependence of the NN interaction amplitude for $n > n_c^{(1)}$,

$$F(n) \simeq 3 \left(\frac{n}{n_0}\right)^{10/3} \frac{[\Gamma(n)/\Gamma(n_0)]^6}{\left[\tilde{\omega}(k_0(n))/m_\pi\right]^8}. \tag{40}$$

Here $\Gamma^{-1} \simeq 1 + C(n/n_0)^{1/3}\Phi(0,k_0)$ is the NN correlation factor, see (27), $\Phi(0,k_0) \simeq 1$, and $C \simeq 1.4-1.6$ is expressed via the Landau–Migdal parameter in the spin channel, $\tilde{\omega}(k_0(n))$ is the effective pion gap introduced in (30). Since $\tilde{\omega}(k_0(n))$ decreases with increasing density one easily recovers the required factor from one to $\gtrsim 10^3$ in emissivity for stars with masses from 1 $M_\odot$ to 2 $M_\odot$ required to describe the cooling data, appropriately. The pion condensation is not needed for that but, if occurred at $n \gtrsim (2.5-3)n_0$, it would not contradict the cooling data. The neutron stars with the pion condensate in their interiors and the pion condensate nuclei–stars are then rather massive rapidly cooling stars with $M \gtrsim 1.5 M_\odot$.

A modification of the reaction rates due to multiple nucleon scattering, — the Landau–Pomeranchuk–Migdal effect,— was evaluated in Ref. [253]. Accounting for it, the neutrino emissivities acquire extra pre-factors of the type

$$\frac{\omega_\nu^2}{\omega_\nu^2 + \Gamma_N^2(n,T)}, \tag{41}$$

where ω_ν is the neutrino energy and Γ_N is the nucleon width, $\Gamma_N \sim \epsilon_{\rm F}(1-\pi^2T^2/\epsilon_{\rm F}^2)$ and $\epsilon_{\rm F}$ is the nucleon Fermi energy. The pre-factor essentially deviates from unity only for the neutrino energy $\omega_\nu \lesssim \Gamma_N$, with $\omega_\nu \sim (3-5)T$ depending on the reaction type. Thus, the multiple scattering effects are important for the description of mergers and the initial stage of the cooling of the compact stars [155,253–255].

There are two stages in the cooling of compact objects: the initial stage, when neutrinos are trapped, and the stage of a long-time cooling when neutrinos are radiated from the compact object by the direct reactions, cf. [31]. At the initial stage of the compact star evolution (lasts from minutes to several hours depending on the mass of the object and the model) the neutrinos and antineutrinos are captured inside the star until their mean-free path will not reach the radius of the object. In case of hybrid stars there are two quantities playing the role of R. One is the radius of the quark or pion or sigma condensate interior core and other is the radius of the object (the core plus the hadronic exterior). The mean free paths of the neutrino and antineutrino are determined by the most efficient processes. Roughly in the process under consideration the ratios of the inverse mean free pathes are proportional to the ratio of the emissivities [256]. As function of the temperature the emissivity of the DU process is $\propto T^6$, see Eq. (35) and for the MU and MMU processes it is $\propto T^8$, see Eqs. (39) and (40). The ratio of the emissivities of the DU and MU processes is $\epsilon_{\rm DU}/\epsilon_{\rm MU} \sim 10^5/T_9^2$. The emissivity of the MU process exceeds that of the DU one for $T \gtrsim 30$ MeV. For the in-medium modified Urca processes, MMU, the emissivity increases sharply with grows of the density. Therefore the MMU processes become more efficient than the DU processes at a smaller temperature. For the compact stars with $T > T_{\rm opac}(R)$ neutrinos and antineutrinos are trapped inside the compact star. The heat flows to the surface due to the heat conductivity. The path length of neutrinos is much larger than for quarks and hadrons and the heat conductivity is therefore governed by neutrinos. In case of hybrid stars the neutrino transport changes on the boundary between the quark and hadron phases. For the hadron phases this stage was studied in Refs. [31,152,256–259]. In-medium effects proved to be very important, see Refs. [31,151–153,155,253,254]. As it was mentioned, appropriate optical theorem technique for the calculation the reaction rates within the nonequilibrium diagram formalism was developed within the quasiparticle approximation in [153] and with inclusion of the baryon width effects in Ref. [253]. Similar effects should be included in the description of a quark phase. With inclusion of in-medium effects associated with a decrease of the effec-

tive pion gap at an increasing baryon density, the value $T_{\rm opac}$ diminishes. For more massive neutron and hybrid stars one may expect that it is below 1 MeV.

The neutrino emissivity of the quark matter was first calculated by Naoki Iwamoto in Ref. [228]. In Ref. [229] the surface temperature of the strange star was found to be lower than that of a neutron star of the same age (ordinary or with mesonic condensates). The effect of the color superconductivity on the cooling of strange and hybrid stars for $T < T_{opac}$ has been first studied in Refs. [230–232] and in some subsequent works. Some models show that cooling can be so fast that strange stars may presently have temperatures smaller than the cosmic microwave background radiation [260,261].

In the case of the 2SC superconducting phase, the most efficient process is the DU process going on unpaired quarks. The cooling evolution is then very rapid that does not agree with the cooling data. Thereby, it is important to have a model, which does not allow for unpaired quarks. Since there exist other attractive quark pairing channels with a rather weak attraction (X channels), Ref. [232] introduced a hypothetical 2SC+X phase of two-flavor quark matter, where all the quarks are paired, some strongly in the 2SC channel and some weakly in the X channel. Typical values of pairing gaps Δ_X in the X channel should be in the range 10 keV—1 MeV, as for example in the case of the CSL pairing [262,263]. Nevertheless, Ref. [232] did not identify the X channel with a spin one pairing channel like the CSL one simply because this phase does not coexist with the 2SC phase, which was assumed as the dominant pairing structure. Furthermore, Ref. [232] supposed that there are no Goldstone bosons in the 2SC+X phase. As the result it was demonstrated that the cooling data can be appropriately described at such assumptions.

Since we have mentioned the axion-based model of nuggets, we should also mention a possible axion contribution to cooling of neutron stars. The emission of axions by neutron stars was, first, discussed in Ref. [264] without the inclusion of NN interactions in the hadron phase. Then in Ref. [152] it was shown that axion emission should be strongly affected by in-medium baryon-baryon interactions in the hadron phase similarly to the corresponding reaction of the neutrino production. Recent discussion of the axion contribution to the cooling of the quark matter can be found in [265].

7 Dilute stable nuclear systems

7.1 Again, a scalar condensate

The analysis of two independent nuclear emulsion experiments exposed to beams of ^{16}O and ^{56}Fe ions with 2A GeV kinetic energy allowed for the interpretation of some events as the relatively rare occurrence of anomalous nuclear fragments interacting with an unexpectedly large cross section as described in Ref. [266]. These objects were called anomalons [267]. Although these results were not reproduced by subsequent measurements, uncertainty remains.

Below we discuss a possibility of existence of very dilute stable or metastable nuclear systems. We will consider the nuclear matter at zero temperature as a normal Fermi liquid. The quasiparticle part of the nucleon Green's function close to the Fermi surface can be presented as

$$G_N(\epsilon,\mathbf{p},n)=\frac{a}{\epsilon-\xi_{\mathbf{p}}+i\,0\,{\rm sign}\epsilon},$$
$$\xi_{\mathbf{p}}=\frac{p^2-p_{\rm F}^2}{2\,m_N^*}\approx v_{\rm F}\,(p-p_{\rm F})\ ,\quad v_{\rm F}=p_{\rm F}/m_N^*. \tag{42}$$

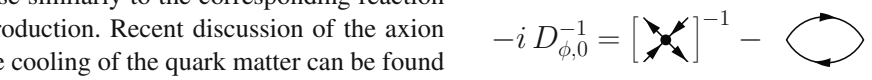
The non-pole part of the Green's function is hidden in the quasiparticle renormalization factor a, effective nucleon mass m_N^* and the quaisparticle interaction amplitude in the particle-hole channel, $\Gamma_0^\omega=f_0/(a^2N)$, which we are interested here. The interaction is expressed here through the scalar Landau–Migdal parameter f_0 and the density of states at the Fermi surface $N=\nu m_N^* p_{\rm F}/\pi^2$. For ISM the degeneracy factor $\nu=2$. The parameter f_0 is related to the compressibility of the matter as follows

$$K_N=n\frac{{\rm d}^2E_N}{{\rm d}n^2}=\frac{p_{\rm F}^2}{3m_N^*}(1+f_0), \tag{43}$$

where E_N is the energy density of the nucleon system. Parameter K_N is related to the standard compressibility modulus of the nuclear matter at saturation that we mentioned in Sect. 2 as $K=9K_N$.

The particle-hole scattering amplitude in the spin zero channel on the Fermi surface can be written through a propagator of the scalar bosonic mode [78]:

$$T_{\rm ph,0}=-{\rm sgn}(f_0)D_{\phi,0}(\omega,k)\,, \tag{44}$$



The particle-hole loop calculated with the Green's functions (42) can be expressed through the Lindhard's function Φ,

$$\int_{-\infty}^{+\infty}\frac{{\rm d}\epsilon}{2\pi i}\int\frac{2{\rm d}^3p}{(2\pi)^3}G_+G_-=-a^2N\Phi(s,x)\,,$$
$$s=\frac{\omega}{kv_{\rm F}}\,,\ x=\frac{k}{p_{\rm F}}. \tag{45}$$

In the limit $x \ll 1$ the Lindhard's function is

$$\Phi(s,x) \approx 1 + \frac{s}{2}\log\frac{s-1}{s+1} - \frac{x^2}{12\left(s^2-1\right)^2}. \tag{46}$$

It attains an imaginary part for $s < 1$, and in the limit $s \ll 1$ we have

$$\Phi(s,x) \approx 1 + i\frac{\pi}{2}s - s^2 - \frac{x^2}{12}. \tag{47}$$

The spectrum of excitations in the scalar channel, $\omega(k)$, is determined from the pole of $T_{\mathrm{ph},0}$,

$$f_0^{-1} = -\Phi(s,x). \tag{48}$$

For $-1 < f_0 < 0$, this equation has only damped solutions with $\Re s < 1$ and $\Im s < 0$. In the limit of $s, x \ll 1$, using the expansion of (47) we find for the low-lying mode

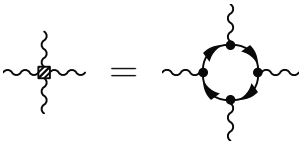
$$\omega_{\mathrm{d}}(k) \approx -i\frac{2}{\pi}kv_{\mathrm{F}}\frac{1-|f_0|}{|f_0|}, \tag{49}$$

which is valid for $1-|f_0| \ll 1$ and $k \ll p_{\mathrm{F}}$. If $f_0 < -1$, i.e. the compressibility of the system becomes negative according to Eq. (43), one has $\Im\omega_{\mathrm{d}}(k) > 0$, and the scalar excitation modes grow exponentially with time as $\propto \exp(+\Im\omega_{\mathrm{d}}t)$. This instability is called the Pomeranchuk instability of a Fermi liquid. It may result in the spinodal instability which appears in systems with a van der Waals-like equations of state. Another possibility is the formation of the Bose condensate of a scalar field, in analogy with the Lee–Wick state, but in dilute matter [78].

We assume that growing unstable bosonic modes may form the Bose condensate described by a scalar boson complex field ϕ. Such a scalar field can be introduced by means of the Hubbard–Stratonovich transformation or by a formal replacement of the contact interaction with an exchange of an artificial heavy scalar boson [78]. The effective Lagrangian density of the condensate field, taken in the simplest form $\phi = \phi_0 e^{-i\omega_c t + i\mathbf{k}_0\mathbf{r}}$, can be presented as

$$\begin{aligned}\mathcal{L}_\phi &= -\mathrm{sgn}(f_0)D^{-1}_{\phi,0}(\omega_c,k_0)|\phi_0|^2 - \tfrac{1}{2}\Lambda(\omega_c,k_0)|\phi_0|^4,\\ D^{-1}_{\phi,0}(\omega,k) &= (\Gamma^{\omega}_0)^{-1} + a^2N\Phi\left(\frac{\omega}{kv_{\mathrm{F}}},\frac{k}{p_{\mathrm{F}}}\right).\end{aligned} \tag{50}$$

The self-interaction term Λ is determined in the leading order by the loop integral of four fermion Green's functions

$i\Lambda(\omega,k) =$  $+\ldots$ (51)

It was evaluated in [268,269] and [270] in case of one-component fermion system. Energetically preferable is the state with $\omega_c = 0$ as argued in Ref. [78]. Then we can estimate for $k_0 \ll p_{\mathrm{F}}$,

$$\Lambda(0,k_0) \approx a^4\lambda\left(1 + k_0^2/2p_{\mathrm{F}}^2\right), \quad \lambda = \xi\frac{\nu m_N^{*3}}{\pi^2 p_{\mathrm{F}}^3}. \tag{52}$$

The factor ξ is introduced to demonstrate uncertainty in the value of the self-interaction parameter λ, which may arise from other diagrams beyond the leading one in (51). Below we vary it to illustrate sensitivity of our numerical results.

The equation for the condensate amplitude following from the Lagrangian (50) reads

$$\begin{aligned}&a^2N\widetilde{\omega}_s^2(k_0)\phi_0 + \Lambda(0,k_0)\,|\phi_0|^2\phi_0 = 0,\\ &\widetilde{\omega}_s^2(k_0) = |f_0(k_0)|^{-1} - \Re\Phi(0,k_0).\end{aligned} \tag{53}$$

The quantity $|\widetilde{\omega}_s(k_0)|$ plays the role of an effective gap in the excitation spectrum of the scalar quanta similarly to that we considered above in the case of the pion-nucleon interaction. The amplitude of the condensate field, $|\phi_0|$, and the Bose condensate energy-density term, E_{B}, become for $\widetilde{\omega}_s^2(k_0) < 0$,

$$\begin{aligned}|\phi_0|^2 &= -\frac{N\widetilde{\omega}_s^2(k_0)}{a^2\lambda}\left(1 + \frac{k_0^2}{2p_{\mathrm{F}}^2}\right)^{-1},\\ E_{\mathrm{B}} &= -\frac{N^2\widetilde{\omega}_s^4(k_0)}{2\lambda}\left(1 + \frac{k_0^2}{2p_{\mathrm{F}}^2}\right)^{-1}.\end{aligned} \tag{54}$$

Simplifying consideration we further put $k_0 = 0$. The case $k_0 \neq 0$ should be considered separately.

The energy of the system per nucleon including the nucleon part and the contribution from the Bose condensate is as follows

$$\mathcal{E}_N + \mathcal{E}_B = \left(E_N + E_{\mathrm{B}}[f_0]\right)/n. \tag{55}$$

Reference [78] employed a simple phenomenological parametrization for the volume part of the energy per nucleon, $\mathcal{E}_N$ as an expansion in powers of p_{F}/m_N proposed in Ref. [271],

$$\mathcal{E}_N(n) = \frac{3p_{\mathrm{F}}^2}{10\,m_N} - c_1\frac{p_{\mathrm{F}}^3}{m_N^2} + c_2\frac{p_{\mathrm{F}}^4}{m_N^3} + c_3\frac{p_{\mathrm{F}}^5}{m_N^4}, \tag{56}$$

where parameters c_1, c_2 and c_3 are expressed through values of the binding energy $\mathcal{E}_0 = -\mathcal{E}_N(n_0)$, compressibility modulus K of the nuclear matter and the nuclear saturation density n_0. Taking $m_N = 939\,\mathrm{MeV}$ for the free nucleon mass and appropriate values $n_0 = 0.17\,\mathrm{fm}^{-3}$, $\mathcal{E}_0 = -16\,\mathrm{MeV}$ and $K = 285\,\mathrm{MeV}$ we find [78], $c_1 = 3.946$, $c_2 = 3.837$, $c_3 =$

13.10. From Eq. (56) the volume part of the nucleon contribution to the scalar Landau–Migdal parameter f_0 follows as

$$f_0(n) = \frac{3m_N^*}{p_{\rm F}^2} n \frac{{\rm d}^2\left(n\mathcal{E}_{N,{\rm v}}\right)}{{\rm d}n^2} - 1 = \frac{m_N^*}{m_N} - 1 + \frac{m_N^*}{m_N}\frac{p_{\rm F}}{m_N}\left(-6c_1 + c_2\frac{28p_{\rm F}}{3m_N} + c_3\frac{40p_{\rm F}^2}{3m_N^2}\right). \quad (57)$$

The energy per particle and the scalar Landau–Migdal parameter are shown in Fig. 11 as functions of the nucleon density. The nucleon volume part of the energy per particle and the parameter f_0 given by Eq. (57) are presented on the left panel in Fig. 11 by the dotted lines. There is a broad interval of densities, $n_c^{(\rm l)} < n < n_c^{(\rm u)}$ where $f_0 < -1$ with $n_c^{(\rm l)} = 0.422 \times 10^{-2}\, n_0$ and $n_c^{(\rm u)} = 0.655\, n_0$. Similar dependence $f_0(n)$ is obtained for the DD2 equation of state, see Fig. 3 in [272]. Notice that taking into account of the clustering effects [273,274] may affect this dependence, see Fig. 9 in [272]. We will focus on the alternative taking into account effect of possible Bose condensation. The condensate contribution proves to be strongly attractive creating a minimum at $n \sim 0.4\, n_0$ for a reduced value of λ for $\xi < 1/2$. The energies per nucleon $\mathcal{E}_N + \mathcal{E}_B$ are shown on the left upper panel by thin lines labeled by values of the parameter ξ. They are calculated according to Eq. (55) with $f_0(n)$ given by Eq. (57).

In the presence of the condensate the incompressibility of the system changes as $K = K_N + K_{\rm B}$, $K_{\rm B} = n\frac{d^2E_{\rm B}}{dn^2}$, and, therefore, the scalar Landau parameter changes too

$$f_0 \to f_0^{\rm tot} = f_0 + \frac{3}{2\epsilon_{\rm F}} K_{\rm B}\left[f_0^{\rm tot}\right]. \quad (58)$$

We used the variational approach to find the solution of Eq. (58), see Ref. [78], and we fixed the values of $f_0^{\rm tot}$ at the ends of the instability interval $f_0^{\rm tot}(n_c^{(l)}) = f_0^{\rm tot}(n_c^{(u)}) = -1$. Thereby we postulated that the condensate arises by the second-order phase transition. Taking into account the possibility of a first-order phase transition, the energy could be further decreased. The energy per particle in our case is constructed using Eq. (43):

$$\mathcal{E}_{\rm tot}^{\rm (MF)}(n) = \frac{1}{n}\int_{n_c^{(\rm l)}}^{n} {\rm d}n' \int_{n_c^{(\rm l)}}^{n'} {\rm d}n'' \frac{2\epsilon_{\rm F}\left(n''\right)}{3n''}\left(f_0^{\rm tot}\left(n''\right) + 1\right) + C\,. \quad (59)$$

The constant C is fixed by requiring $\mathcal{E}_{\rm tot}^{\rm (MF)}(n_c^{(\rm u)}) = \mathcal{E}_N(n_c^{(\rm u)})$. The energy (59) corresponds to a mean-field approximation when the reconstruction of the excitation spectrum due to an interaction with the condensate field is not taken into account.

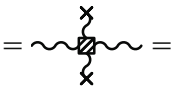
The result of calculations using (59), (58) is shown on the left panel in Fig. 11 by tick lines for different value of the ξ parameters. The self-consistency requirement changes the condensate contribution essentially. The mean-field energy $\mathcal{E}_{\rm tot}^{\rm (MF)}$ becomes now only a smoothly decreasing function of the density and no minimum appears. Nevertheless, as it was expected, the decrease of the λ parameter (smaller ξ) leads to the decrease of the energy. The density dependence of $f_0^{\rm tot}$ is flatter than that of f_0 for densities $0.2 < n < n_c^{(\rm u)}$ and $f_0^{\rm tot} > f_0$. At smaller densities $n \lesssim 0.1\, n_0$ we have oppositely $f_0^{\rm tot} < f_0$. We have to notice that our solutions are accurate at the level of 0.1% for $0.01\, n_0 \lesssim n \lesssim 0.55\, n_0$ but the accuracy is getting worse when density approaches $n_c^{(\rm u)}$. Then our solution can be viewed as a smooth extrapolation to $n_c^{(\rm u)}$.

The interaction of the condensate with over-condensate excitations leads to appearance of a new term in the excitation propagator,

$$D_{\phi,0}^{-1} \longrightarrow D_\phi^{-1}(\omega,k) = D_{\phi,0}^{-1}(\omega,k) - \delta\Sigma_\phi(\omega,k)\,,$$

where the additional term is

$$-i\delta\Sigma_\phi(\omega,k) = \quad = -i2\Lambda(\omega,k)\,|\phi_0|^2\,. \quad (60)$$



In such a way we include both mean-field and quadratic fluctuation contributions.

The new spectrum of excitations on top of the condensate is determined now by the equation $D_\phi^{-1}(\omega,k) = 0$. For $k_0 = 0$ and $-1 \ll \tilde{\omega}_s^2(0) < 0$ we find

$$\omega_{\rm d} \approx i\frac{2}{\pi}kv_{\rm F}\tilde{\omega}_s^2(0) = i\frac{2}{\pi}kv_{\rm F}\frac{1-\left|f_0^{\rm tot}\right|}{\left|f_0^{\rm tot}\right|}\,,$$

where we used Eqs. (54) and (52). We pay attention to opposite sign compared to Eq. (49) for $|f_0^{\rm tot}| > 1$. Thus, we have now $\Im\omega_{\rm d} < 0$. Hence, in the presence of the condensate, excitations are damped and the Fermi liquid becomes free from the Pomeranchuk instability.

The particle-hole interaction is also changed in the presence of the condensate. There appears a new term in equation for the particle-hole amplitude (44) after the replacement $D_{\phi,0}^{-1} \longrightarrow D_\phi^{-1}$. The new term can be included in the renormalized local interaction

$$\frac{1}{f_{0,\rm ren}} = \frac{1}{f_0^{\rm tot}} + 2\,\tilde{\omega}_s^2(0) = -\frac{f_0^{\rm tot}}{1+2f_0^{\rm tot}}\,, \quad (61)$$

where we used $\tilde{\omega}_s^2(0) = -1 - 1/f_0^{\rm tot}$. Thus, if originally $f_0 < -1$ and therefore $f_0^{\rm tot} < -1$, the renormalized interaction yields $-1 < f_{0,\rm ren} < -1/2$. Hence, the first-sound modes become stable in the Fermi liquid with the condensate. The quantity $f_{0,\rm ren}$ is plotted on the right lower panel in Fig. 11.

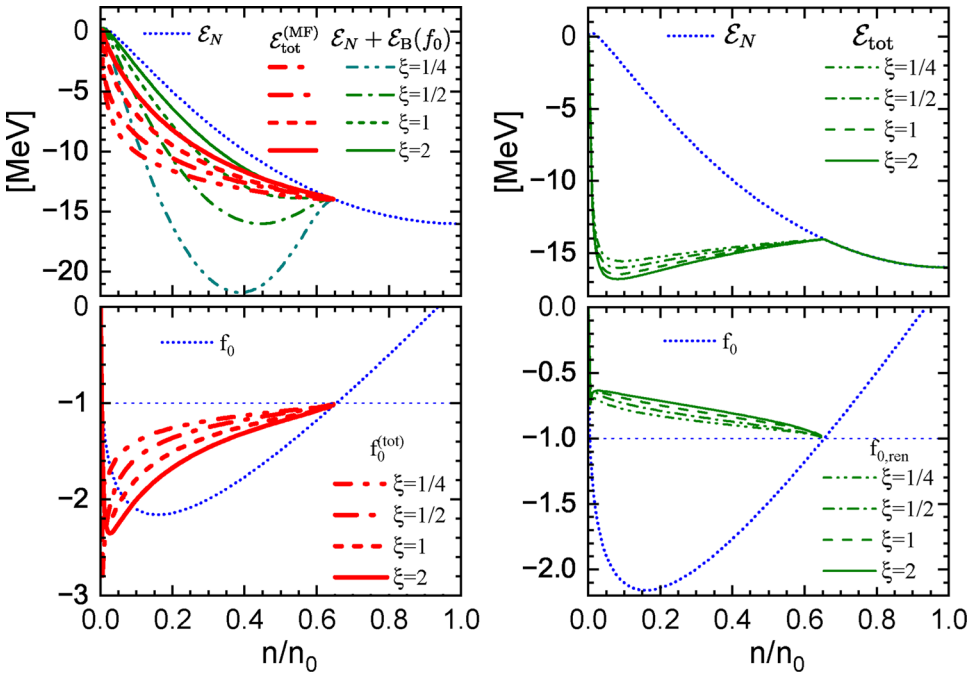


Fig. 11 Energy per particle (upper panel) and the scalar Landau–Migdal parameter (lower panel) as functions of the nucleon density

Knowing the value $f_{0,\rm ren}$ one can reconstruct the energy per particle $\mathcal{E}_{\rm tot}(n)$ of the system using Eq. (59) after the replacement $f_0^{\rm tot} \to f_{0,\rm ren}$ in the integrand. The results are shown on the right upper panel in Fig. 11. We observe the appearance of a minimum of the function $\mathcal{E}_{\rm tot}(n)$. For $\xi = 1$ the minimum is at $n_{\rm min} = 0.085\, n_0$ with $\mathcal{E}_{\rm min} = -16.01\,$MeV. Variation of the parameter ξ in interval $\xi = 1/4$–2.0 leads to variations of the values $n_{\rm min} = (0.82$–$0.93) \times 10^{-1}\, n_0$ with $\mathcal{E}_{\rm min} = -(16.8$–$15.6)\,$MeV. So the minimum in energy can correspond to a new stable or metastable state of the dilute nuclear matter compared with the usual saturation energy of $-16\,$MeV used to specify the equation of state of the ISM.

Finally, we should mention that the obtained results are valid, if on the one hand ϕ_0 is rather small and on the other hand fluctuations on the top of the condensate yield a yet smaller contribution, since the self-interaction of excitations on top of the condensate and feedback of fluctuations on the mean field were disregarded.

Considering finite system one must take into account the surface and Coulomb energies. We parameterize the surface energy as in [29,30],

$$\mathcal{E}_{\rm S} A = a_{\rm S} A^{2/3}\,, \quad a_{\rm S} \approx C_{\rm S}\Big(\frac{n}{n_0}\Big)^{1/3} \frac{\mathcal{E}(n,z)}{\mathcal{E}(n_0, z=1/2)}\,, \quad C_{\rm S} = 18\,{\rm MeV}\,, \tag{62}$$

where $z = Z/A$.

With the account for the surface energy (62) the Landau–Migdal parameter, $f_0^{\rm S}$, takes the following form

$$f_0^{\rm S}(n) = f_0(n) + \frac{C_{\rm S}}{A^{1/3}} \frac{1}{\mathcal{E}_{N,\rm v}(n_0)} \Big(\frac{n}{n_0}\Big)^{1/3} \frac{m_N^*}{m_N} \times \Big(\frac{3}{5} - c_1 \frac{28 p_{\rm F}}{9 m_N} + c_2 \frac{40 p_{\rm F}^2}{9 m_N^2} + c_3 \frac{6 p_{\rm F}^3}{m_N^3}\Big)\,. \tag{63}$$

Following [78], as an example, we consider a system of 125 nucleons at $N \approx Z$. Then the energy per particle $\mathcal{E}_N + \mathcal{E}_{\rm S}$ saturates at density $0.95\, n_0$ with the minimum $\mathcal{E}_0^{\rm S} = -12.4\,$MeV. With taking into account the surface energy and solving Eq. (58) with $f_0^{\rm S}$ instead of f_0 we find that the new minimum appears at $n_{\rm min} = 0.74 \times 10^{-1}\, n_0$ with the depth $\mathcal{E}_{\rm min} = -13.04\,$MeV. So, $\mathcal{E}_{\rm min} < \mathcal{E}_0^{\rm S}$ and, hence, a new stable state of matter is possible in the finite size object with account for the surface energy. To analyze its stability against fusion we must include the Coulomb energy

$$\mathcal{E}_{\rm C} A \simeq a_{\rm C} z^2 A^{5/3}\,, \quad a_{\rm C} = C_{\rm C}\Big(\frac{n}{n_0}\Big)^{1/3}\,, \quad C_{\rm C} = 0.7\,{\rm MeV}\,. \tag{64}$$

A spherical nucleus is stable against an ellipsoidal deformation (stretching) if

$$\frac{Z^2}{A} < 2\frac{a_{\rm S}(n_{\rm min})}{a_{\rm C}(n_{\rm min})} = 2\frac{C_{\rm S}}{C_{\rm C}}\frac{\mathcal{E}_{\rm min}}{\mathcal{E}_0} \simeq 42\,. \tag{65}$$

We see that, because of the condensate formation, a new metastable dilute nuclear state is stable against spontaneous fission for $A \lesssim 170$.

Reference [79] studied the equation of state of isospin-asymmetric matter and demonstrated that even for the asymmetry parameter $n_p/n_n \lesssim 0.2$ there remains a broad region of proton and neutron densities where $f_0 < -1$ and, therefore, the spinodal instability may develop. For a nugget of a size R much larger than the Debye screening length $l_D \simeq \sqrt{\pi}/(2|e|V_0)$, the Coulomb contribution from protons must be screened by electrons, $n_p = n_e$, cf. [50]. Here we for simplicity assume that the scalar field is electrically neutral. The electron Fermi momentum is then $p_{\mathrm{F},e} = V_0 = (3\pi^2 n_p)^{1/3}$ and the electron kinetic energy density is $E_e = p_{\mathrm{F},e}^4/(4\pi^2)$. In the isospin-asymmetric matter, the nuclear symmetry energy must be included. A simple parametrization was proposed in [275], which is a fit to microscopic calculations

$$E_{N,\mathrm{sym}}\left(n_p, n_n\right) = J\,\frac{\left(n_p+n_n\right)^\gamma}{n_0^\gamma}\,\frac{\left(n_p-n_n\right)^2}{n_p+n_n}\,,$$
$$J = 32\,\mathrm{MeV}\,,\quad \gamma = 0.6\,. \tag{66}$$

Let $n_p = \chi n_n$, then the total energy per particle of the system is

$$\mathcal{E}(\chi) = \mathcal{E}_{\min} + J\left(\frac{n_{\min}}{n_0}\right)^\gamma \frac{(1-\chi)^2}{(1+\chi)^2} + \frac{3}{4}(3\pi^2 n_{\min})^{1/3}\frac{\chi^{4/3}}{(1+\chi)^{4/3}} \approx \mathcal{E}_{\min} + 7.3\,\mathrm{MeV}\frac{(1-\chi)^2}{(1+\chi)^2} + 149\,\mathrm{MeV}\frac{\chi^{4/3}}{(1+\chi)^{4/3}}\,, \tag{67}$$

where $n_{\min} = 0.85\times 10^{-1}\,n_0$ is used. If we take $\chi \sim 0.2$ we obtain $\mathcal{E}(\chi \sim 0.2) \approx \mathcal{E}_{\min} + 16.9\,\mathrm{MeV}$. Minimizing $\mathcal{E}(\chi)$ with respect to χ we find that the minimum is realized for $\chi_{\min} \approx 3.1\times 10^{-3}$ and in the minimum $\mathcal{E}(\chi_{\min}) \approx \mathcal{E}_{\min} + 7.3$ MeV. Thus if $\mathcal{E}_{\min}$ is of the order $-(10-17)$ MeV in the case of the isospin asymmetric nuclear matter, one may rise the question about a possibility of the existence of neutron-rich dilute compact stars glued by an collective scalar field. However, the above estimates crucially depend on the value of symmetry energy of nuclear matter at small densities. The smaller $E_{N,\mathrm{sym}}$ is, the smaller will be repulsive contributions in Eq. (67).

Above we only demonstrated a possibility of the scalar field condensation in the dilute nuclear systems, which may result in occurrence of new metastable or stable states. A more detailed analysis is still required to justify this hypothesis.

7.2 Bose condensates of nuclear clusters

At densities when $f_0(n) < -1$ the incompressibility of nuclear matter is negative and, as we have mentioned, not only the scalar particle-hole mode condensation may occur but also the spinodal instability of matter. In the low energy heavy-ion collisions the latter instability manifests itself in an abrupt increase of the production of α particles and other nuclear clusters, see [73–77,79]. With a binding energy of 7.07 MeV per nucleon α particles are bound as strongly as the most tightly bound nucleus of ^{52}Fe. Under certain conditions α particles can form a stable Bose–Einstein condensate state (see Ref. [276–278] and references therein). Reference [279] argued that the spectrum of excitations in the system of interacting α particles is similar to the spectrum of He^4 and has the roton minimum at $k \neq 0$. If so, the inhomogeneous α condensate occupying the roton minimum could occur in the moving (e.g. rotating) pieces of the α particle system. Similar effects were studied in Refs. [129,149,280], see discussion in Sect. 3.3. Also, one considers the possibility of deuteron condensation in dilute nuclear systems as discussed in Ref. [281], despite the fact that deuterons are seven times weakly bound than α particles.

Consider an extended nucleon system of large size and study whether it is possible to form a Bose–Einstein condensate from bosonic clusters consisting of an even number of nucleons. We must take into account an electric field and β equilibrium conditions, as it was done in Ref. [31,54] and discussed above in Sect. 3.

Let A is the total number of nucleons in a piece of dilute nuclear matter containing the Bose–Einstein condensate of clusters consisting of A_{cl} nucleons each (e.g. $A_{\mathrm{cl}} = 4$ for the α particle). The cluster condensate is described by a complex field ϕ. The Lagrangian density of a stationary, homogeneous condensate field ϕ can be written as

$$\mathcal{L}_{\mathrm{cl}} = (\omega+V)^2|\phi|^2 - m_{\mathrm{cl}}^2|\phi|^2 + n_e V - \frac{V^4}{4\pi^2}\,, \tag{68}$$

where $\omega + V$ is the energy of a positively charged cluster quasiparticle shifted by the electric potential V. Here, considering a large system, we dropped gradient terms with $\nabla\phi$ and ∇V and neglected ϕ self-interaction terms for simplicity, similarly to that we have done finding solutions in Sects. 3.1 and 3.3. The equation of motion for the scalar field following from the Lagrangian (68) is $(\omega+V)^2\phi = m_{cl}^2\phi$. The condition of the charge neutrality of the system requires that the positive charge of clusters is compensated by the charge of electrons, therefore $-n_{\mathrm{cl}} = n_e = V^3/(3\pi^2)$, where the number density of cluster quasiparticles is

$$n_{\mathrm{cl}} = \frac{\partial \mathcal{L}_{\mathrm{cl}}}{\partial \omega} = -2(\omega+V)|\phi|^2.$$

The energy density of a cluster is $A_{\rm cl}\big(m_N - \mathcal{E}_{\rm cl}^{\rm (bind)}\big)n_{\rm cl}$, where $\mathcal{E}_{\rm cl}^{\rm (bind)} > 0$ is the cluster binding energy per nucleon, e.g., $\mathcal{E}_{\rm cl}^{\rm (bind)} \simeq 7.07\,$MeV for the α particle. Then the energy density of the condensate of the bosonic clusters can be presented as

$$\Delta E = E_{\rm tot} - A_{\rm cl} m_N n_{\rm cl} \simeq \frac{V^4}{12\pi^2} - A_{\rm cl}\mathcal{E}_{\rm cl}^{\rm (bind)} n_{\rm cl} = \left(\frac{1}{4}(3\pi^2 n_{\rm cl})^{1/3} - A_{\rm cl}\mathcal{E}_{\rm cl}^{\rm (bind)}\right) n_{\rm cl}. \quad (69)$$

Thus, in this simplified model, the system is bound ($\Delta E < 0$) at densities

$$n < n_{\rm Mott} = (4A_{\rm cl}\mathcal{E}_{\rm cl}^{\rm (bind)})^3/(3\pi^2)$$

in the large-system limit, and it is unbound for $n > n_{\rm Mott}$. For α particles we find $n_{\rm Mott} \simeq 0.04\, n_0$.

7.3 Dilute nonequilibrium "anomalous" states

The production of pion-enriched matter occurs in ultrarelativistic heavy-ion collisions at highest RHIC and LHC energies. At expansion of the system, the temperature of the pion gas decreases. At a certain temperature, T_{chem}, the particle populations stop changing, reaching a state called chemical freeze-out, though the particles still remain in the thermal equilibrium. Then, at a later stage of the expansion up to a thermal freeze-out, till $T(t) > T_{th}$, we are dealing with a system with a dynamically fixed number of pions. At $T < T_{BEC}$, provided $T_{th} < T_{BEC}$, there may appear the Bose–Einstein condensate of pions characterized by the dynamically fixed particle number [282,283].

Another interesting situation is possible: in peripheral heavy-ion collisions, the colliding nuclei can be considered as almost freely interpenetrating beams of nucleons. At the collision energy $\gtrsim 300\,$MeV per nucleon, the nucleon Fermi spheres from each beam do not overlap in momentum space. Since at low densities, the main contribution to the pion polarization operator is $\Pi_{\pi NN} \propto -\nu p_{\rm F}(n)k^2$, $\nu p_{\rm F} \propto (n\nu^2)^{1/3}$, see Sect. 5. This corresponds to a density, which is effectively four times higher than in the case of overlapping Fermi spheres. Thus, there is a possibility of observing the effects of the p-wave pion condensation in peripheral heavy-ion collisions [166,284].

7.4 Dilute quantum droplets

The phenomenon of Efimov states in Fermi systems opens another interesting possibility. For particular values of the diluteness parameter $n|a|^3$, where n is the fermion density, a is the scattering length, a three-body interaction can provide a dominant contribution in the energy-density functional [285]. if this happens, both bosonic and fermionic systems of various sizes and densities could become self-bound. In this relation, we note that the nucleon–nucleon scattering length has an anomalously large magnitude, $a_{NN} \simeq -20\,$fm. A discussion of the role of this circumstance for description of a dilute neutron matter can be found in [286]. Finally, we notice that some experiments claimed an observation of a tetraneutron (n^4) signal, which may be associated with a bound state or an unbound resonance state, see discussion in [287].

As we have mentioned, in the spectrum of π^+ in neutron matter in the interval of densities $n_c^{\pi^\pm} > n > n_c^{\pi^+}$, where $n_c^{\pi^+} < n_0$ according estimate of Ref. [143], and $n_c^{\pi^\pm} \gtrsim n_0$, there appears the branch with $\omega_s^{\pi^+}(k) < 0$ at $k \neq 0$, as we have discussed in Sect. 4.2. Appearance of such a branch may result in some important consequences, see Ref. [90]. For instance, at the formation of the neutron star in a supernova explosion a part of the initial angular momentum can be absorbed by the π^+ condensate subsystem. Metastable π^+-multi-neutron droplets with a large angular momentum and a density $n > n_c^{\pi^+}$ can be formed, as it was discussed in Sect. 3.3. They may take away part of the total angular momentum of the star. Metastable π^+ condensate neutron-rich droplets also can be formed with some probability during heavy-ion collisions. Since density in such droplets might be smaller than n_0 they may behave as anomalons mentioned above, see Ref. [90]. Also there a possibility of appearance of condensate fragments with a large angular momentum in peripheral heavy-ion collisions was discussed.

8 Observations

Search of abnormal nuclei, nuclearites and nuclei–stars composed of absolutely stable matter is an intriguing prospect in astrophysics. Detection of such objects is not a trivial problem due to numerous observational signatures, which are superimposed on those of ordinary neutron stars and white dwarfs. Distinguishing features may be found in ultra-fast rotation (well below 1 ms), star radii less than 9–10 km, specific photon emission from a star surface, etc. Possible candidates among compact objects were identified: RX J1856.5-3754, PSR J1614-2230, PSR J0348+0432, 4U 1820-30, HESS J1731-347, XTE J1814-138. However, definitive conclusions whether these objects are anomalous or not, could not be drawn. Additionally, particular measurements of the low-mass and small-radius compact star HESS J1731-347 depend heavily on the assumed atmosphere model and are sensitive to errors in distance and inferred age. Let us add here the latest results of NICER [288] for PSR J0614-3329 giving rather small equatorial radius $10.29^{+1.01}_{-0.86}\,$km for the mass $M = 1.44^{+0.06}_{-0.07}\, M_\odot$ (median values with equal-tailed 68% credible interval).

Experimental search for abnormal nuclei was carried out in natural samples among the products of interactions between high-energy particles and matter, in fission products, in heavy-ion collisions, etc., see Refs. [39,43–46,46,47,289–293]. Reference [294] accounted for the absence of traces found in the sample previously considered in Ref. [295] and excluded strangelets with a mass $M < 55$ g. More details can be found in [190].

Although no abnormal states were found, presently a number of observations demonstrate anomalies that defy conventional physics interpretations. May be at least some of them are related to the anomalous nuclear states. We mention several such anomalies. A very heavy particle was detected passing through a balloon-borne stack of Cherenkov film, emulsion, and Lexan sheets [296]. It could be a neutron anomalous nucleus. The spectra of nuclei obtained in the aerostatic JACEE experiment with an emulsion chamber show a peculiar bend at the cosmic ray energy of 3 PeV. The change in the slope of the spectrum was suggested in Ref. [260] to be explained by the appearance of an anomalous component of cosmic rays, which is much heavier than the ordinary nuclear component and could be associated, e.g., with quark nuggets or abnormal π or σ condensate nuclei. In the transition region, both nuclei and strangelets can exist, that leads to large fluctuations in the events observed in the experiments with X-ray emulsion chambers.

Recent works [297–299] argued that there are other observations, which can be hardly explained in the framework of conventional physics. These anomalies include: unexpected correlations with temperature variations in stratosphere, the total electron content of the Earth atmosphere, correlations between earthquake activity and relative positions of planets, unexpected seasonal stratospheric temperature variations, ionospheric anomalies, and their correlations with seismic activity, as well as solar phenomena like the coronal heating problem, the origin of sunspots, and the trigger mechanism of the solar flares, see also [300,301]. The statistical significance of the observed correlations is high. Reference [71] suggested that in mentioned events one may deal with hypothetical very dense and microscopically large composite objects with the mass of the order of grams and submicrometer size consisting of nuggets of strongly interacting matter. For instance, it was suggested to look for so-called solar nano-flares with the axion quark nugget annihilation events in the solar corona. Note that, if instead of quark nuggets one considered the Lee–Wick abnormal states or the Migdal pion condensate abnormal nuclei, at least a part of physical consequences would be similar to those for strangelets.

We now mention another possibilities. In a uniform electric field with the strength of $\gtrsim (10^4 - 10^5)$ V/cm the depth of the potential V_0 exceeds $2m_e \approx 10^6$ eV when the distance between plates is $\gtrsim 10$ cm. For a 300 times larger distance between plates V_0 exceeds two pion masses, $2m_\pi \approx 280$ MeV. For comparison, the breakdown voltage for the dry air at atmospheric pressure is $(2–3) \times 10^4$ V/cm and for some types of glasses it reaches 3×10^6 V/cm. Similar estimates can be found for the cylindrical and spherical capacitors. However we must stress that despite the potential inside the capacitor may readily exceed $2m_e$ and $2m_\pi$ respectively, the probability of the spontaneous pair production from the vacuum is exponentially small in both cases, $\propto e^{-m^2/E}$. Nevertheless, electrons and charged pions and, maybe, kaons can be produced with the larger probability near the walls of the empty capacitor or in reactions, if capacitor is filled by a medium, provided the electric potential well is sufficiently deep. In the broad potential well for π^- in the region, where $-V > m_\pi$, the π^- ground state energy level crosses $\omega = 0$ and formation of the static π^- condensate field becomes energetically favorable. In the region with $V > m_\pi$ the static π^+ condensate field can be formed, if the time needed for the formation of the condensate is sufficient. Note that in the presence of a charged condensate, photons become effectively massive particles via the Anderson-Higgs mechanism. Therefore, in the case of empty capacitor the condensate region should look dark. This circumstance can help in identifying the effect.

It is worth noticing that deep potential wells can be formed in thunderstorms on Earth. During tens of minutes before a discharge, the electric voltage may reach values of $\gtrsim 10^8$ V at distance $\gtrsim (1 - 10)$ km between clouds, cf. [302]. Hence the typical potential well reaches values $V_0 \gg m_e$ at such distances. An excess of neutrons during thunderstorms has been indeed detected, however it was associated with photonuclear reactions [303]. Note that pions, which could form a condensate, can also be produced in photonuclear reactions. Strong thunderstorms may also occur in atmospheres of other planets such as Jupiter and Venus. If a nugget of a new phase of the matter passes through the thunderstorm clouds at the ultra-high voltage ($\sim 10^9$ V) between them, the nugget will be accelerated and its electron shell will be destroyed with a release of the sufficient energy for the production of baryons, pions, quarks, etc. In these processes the nugget can be destroyed that could be observed via the significant energy release.

We should notice that 10 anomalous bursts correlated with lightnings have been observed in thunderstorms occurred during 5 years [304,305]. The estimated energy from individual events within the bursts is five to six orders of magnitude higher than the energy estimated by an event rate. The multiple air showers occurred within 1 ms, defying explanation by conventional high-energy cosmic rays. Similar "exotic events" were recorded by the AUGER collaboration. The ANITA experiment detected two anomalous upward-propagating events with non-inverted polarity. The list of anomalous atmospheric events also includes the

Multi-Modal Clustering Events observed by HORIZON 10T. Among various possible explanations we indicate the possibility of the energy release due to the destruction of the previously bounded nuggets arrived from cosmos.

9 Conclusion

In 1971 Migdal suggested a possibility of existence of metastable or stable abnormal pion condensate nuclei and Bodmer assumed possibility of collapsed quark nuclei. In 1974 Lee and Wick proposed scalar (sigma) condensate abnormal nuclei. In 1984 Witten suggested quark nuggets. These pioneering suggestions paved the way for numerous subsequent theoretical and experimental investigations of various anomalous nuclear objects. New more detailed models were constructed. The review is dedicated 115-year anniversary of A. B. Migdal and 100-year anniversary of T. D. Lee. Our aim was to present the development of their ideas and discuss the modern status of the subject, demonstrating the interrelation between the different phenomena under discussion.

In Sect. 2 we reviewed the original Lee–Wick model of scalar condensate abnormal nuclei. The model describes the coupling of nucleons with a scalar mean field. Typical behaviors of the effective potential and the energy of the model as function of a scalar field ϕ are shown in Fig. 1 and Fig. 2. The role of the scalar field can be played by the σ meson and the model then can be treated as a σ model permitting chiral symmetry breaking in a dense nuclear matter. The same year when Lee and Wick proposed their model, Walecka formulated a relativistic mean-field $\sigma - \omega$ model [82,83] with two coupling constants fitted to describe the nuclear binding energy and the value of the nuclear saturation density. Various modifications of the Walecka model were introduced for the better description of nuclear matter properties. The density dependencies of the effective nucleon mass within the Lee–Wick model and in the RMF model are shown in Fig. 3. In the Lee–Wick model the effective nucleon mass undergoes a jump to a tiny value at the critical density, whereas it decreases smoothly with the density increase in the standard RMF models. With parameters, which were fitted to describe the nuclear system at $n = n_0$, at larger densities in these RMF models the Lee–Wick abnormal state does not appear. However information about chiral symmetry is lost in the standard RMF based models, whereas it is present in the Lee and Wick model. Although not permitting to describe properties of the ordinary atomic nuclei and nuclear matter at $n \lesssim n_0$, the Lee–Wick model demonstrates a principle possibility of existence of the new stable (or metastable) abnormal nuclear state at n significantly above n_0. As a possibility to reconcile these approaches, we indicated the RMF models employing the σ field scaled hadron masses and coupling constants [88,91–94], which can be treated as models using the idea of the partial chiral symmetry conservation. For simplicity the parameters of the models used in [88,91–94] were chosen such that the new Lee–Wick minimum is not realized. Changing parameters of the models we could get new solutions, some of them with a jump to a low value of the effective nucleon mass at a critical density. Such a possibility is realized in the model [95] at some choice of the attractive Δ optical potential, see Fig. 9 in Sect. 5. This question should be more carefully studied in the future.

Section 3 considered various aspects of the model of supercharged nuclei, nuclearites and nuclei–stars glued together by light charged bosons, if the latter existed. In reality the role of the light charged boson can be played by the ordinary charged pion since in the nuclear medium and in external fields the pion effective mass can be essentially decreased. For example, it was demonstrated that a rapid rotation of a nugget acts in favor of the decrease of the effective pion mass and appearance of the stable state.

Section 4 analysed the behavior of the pion degree of freedom in dense nuclear matter and the possibility of the Migdal's p-wave pion condensation, cf. Ref. [31]. Equation (25) is the final result of the resummation of the nucleon–nucleon interaction amplitude in the spin-isospin channel in the dense nuclear matter done within the Fermi liquid approach of Migdal allowing for the explicit treatment of soft (quasi-Goldstone) pion degree of freedom. For $n > n_0$ the in-medium pion exchange term in (25) becomes dominant and the nucleon–nucleon scattering cross section appreciably increases with increasing density towards the pion condensation point, see Eq. (31). The Δ isobar plays very important role because the mass of the Δ isobar is only $2m_\pi$ larger than the nucleon mass and degeneracy factor is 4 times larger. Moreover the pion-Delta-nucleon attraction coupling constant is twice larger than that of the pion-nucleon–nucleon one.

Various types of the p-wave pion condensation (π^+, $\pi^\pm$, π^0) as well as a possibility of the s wave pion condensation were mentioned. The typical effective pion mass (better saying the effective pion gap) in the dense medium depends on the pion momentum and decreases with increasing baryon density. It is shown in Fig. 6. The description of the pion degree of freedom can be formulated within the linear σ-model [31,52,144–146,172], in which both the pion and sigma field condensations can be treated simultaneously. Also the $\sigma(n)$ dependence is related to the change of the quark condensate in baryon matter. The possibility of self-bound pion condensate abnormal nuclei, nuclearites and nuclei–stars found within the chiral σ-model of pion condensation used in Ref. [52] is demonstrated in Fig. 8.

Section 5 reviewed the idea of Δ resonance matter. Occupation of the new Δ Fermi sea with increasing baryon density occurs within many models for the equation of state of neu-

tron star matter already at $n \gtrsim (2-3)n_0$. Also, Δ isobars may appear in symmetric nuclear matter. Appearance of Δ isobars results in an energy decrease making the nuclear matter more stable. It may result in a metastable or even stable ground state of the Δ resonance matter if the attractive Δ optical potential is sufficiently strong. Figure 9 (left) demonstrates an abrupt decrease of the nucleon effective mass to a small value occurring in the isospin-symmetric matter at a critical density $n \gtrsim (3-4)n_0$ in a RMF model [94] employing the σ scaled hadron masses. We note that such a behavior is also typical for the Lee–Wick model.

Section 6 reviewed ideas of Bodmer, Witten and others on quark strangelets or nuclearites and strange stars as stable baryonic states composed of deconfined quarks. These states are interconnected with nontopological solitons containing baryon constituents studied by Lee and collaborators as well as by Coleman [182–186]. The idea of stable strange quark matter by Witten 1984 [56] is somewhat similar to the idea of stable Δ resonance matter by Troitsky and Chekunaev 1979 [55]. If a model predicts the strange quark matter to be more stable than an iron nucleus, one speaks about strangelets and quark nuclearites, provided the baryon number $A \ll 10^{56} - 10^{57}$, and about strange stars for $A \sim 10^{56} - 10^{57}$. In the latter case the gravity essentially contributes to the binding. The surface layer of these objects has a typical length of ~ 10 fm. Some other models do not allow for the absolutely stable strange quark matter but allow for metastable matter at switching off the gravity. Other models use parameters that render strange and not strange (u, d quark) deconfined matter unstable. Nevertheless, we may deal with the quark matter in interiors of compact stars named hybrid stars. All hybrid stars have macroscopic hadron shells.

Various models describing quark deconfined state were reviewed. Among different models we indicated the heavy quark quasiparticle model [212] capable to describe the lattice data [210,211]. At some choice of the parameters the model [212] allows for the stable quark matter but does not permit the hybrid stars.

At $T < T_\Delta$ for the quark-quark pairing the quark matter should form a color superconductor. We briefly discussed this phenomenon. We focused on a possible manifestation of the pion and σ condensate nuclei–stars and strange stars via the measurement of the $M - R$ relation and via the cooling of these stars.

Section 7 discussed the possibility of the existence of very dilute stable or metastable nuclear systems. For this the Fermi liquid Green's function approach was employed as for the description of dense nuclear systems. However, in the case of pion-condensate nuclear objects we used nucleon–nucleon interactions in the spin channel (or in spin-isospin one), whereas in Sect. 7 we focus on the scalar channel of an NN interaction. In this channel, the interaction is expressed via the scalar Landau–Migdal parameter $f_0(n)$. First we showed that in case of approximately isospin-symmetric matter at low densities we have $f_0(n) < -1$, and there appears the Pomeranchuk instability. Then we argued in [78] that this instability may result in the formation of a static scalar-field condensate (compare with the Lee–Wick scalar field condensation). We demonstrated that a decrease in the energy due to the condensation can be sufficient to allow for the existence of very dilute metastable or even stable abnormal dilute nuclei held together by the condensate of a scalar field, see Fig. 11.

Then we studied if it is possible to form a Bose–Einstein condensate from bosonic clusters consisting of an even number of nucleons, e.g. of the α clusters or deutrons. We showed that the charged Bose cluster condensates can be described by a complex field ϕ similarly to the description of supercharged nuclei performed in Sect. 3. We estimated the energy for the dilute α condensate extended systems, which charge is compensated by the electrons. For $n > n_{\rm Mott} \simeq 0.04n_0$ according to our estimate the system is destroyed.

Finally we made some speculations about formation of metastable pion Bose–Einstein condensate during the course of the heavy-ion collisions. Section 8 discussed the situation with observations of exotic phenomena considered in the review.

Concluding, in spite of more than fifty years were past, as in early days of this field, there remain essential uncertainties in theoretical predictions due to a lack of our knowledge of the behavior of the strongly interacting dense and dilute nuclear matter. In spite of abnormal nuclear states were not observed, the number of observations grows, which are hardly explained by conventional physics. Some of such events could be associated with manifestation of anomalous nuclear systems. Let us be patient. Time flows not in vain.

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References

1. L.D. Landau, On the theory of stars. Phys. Z. Sowjetunion **1**, 285 (1932). https://doi.org/10.1016/B978-0-08-010586-4.50013-4
2. D.G. Yakovlev, P. Haensel, G. Baym, C.J. Pethick, Lev Landau and the concept of neutron stars. Phys. Usp. **56**, 289 (2013). https://doi.org/10.3367/UFNe.0183.201303f.0307
3. D. Iwanenko, The neutron hypothesis. Nature **129**, 798 (1932). https://doi.org/10.1038/129798d0
4. W. Heisenberg, Über den Bau der Atomkerne. I, Z. Phys. **77**, 1 (1932). https://doi.org/10.1007/BF01342433
5. W. Heisenberg, Über den Bau der Atomkerne. II, Z. Phys. **78**, 156 (1932). https://doi.org/10.1007/BF01337585
6. W. Heisenberg, Über den Bau der Atomkerne. III, Z. Phys. **80**, 587 (1933). https://doi.org/10.1007/BF01335696
7. W. Baade, F. Zwicky, Remarks on super-novae and cosmic rays. Phys. Rev. **46**, 76 (1934). https://doi.org/10.1103/PhysRev.46.76.2
8. A. Hewish, S.J. Bell, J.D.H. Pilkington, P.F. Scott, R.A. Collins, Observation of a rapidly pulsating radio source. Nature **217**, 709 (1968). https://doi.org/10.1038/217709a0
9. L.D. Landau, On the theory of phase transitions I. Phys. Z. Sowjetunion **11**, 26 (1937)
10. L.D. Landau, On the theory of phase transitions II. Phys. Z. Sowjetunion **11**, 545 (1937)
11. V.L. Ginzburg, L.D. Landau, On the theory of superconductivity. Zh. Eksp. Teor. Fiz. **20**, 1064 (1950). https://doi.org/10.1016/b978-0-08-010586-4.50078-x
12. J.S. Schwinger, A theory of the fundamental interactions. Ann. Phys. **2**, 407 (1957). https://doi.org/10.1016/0003-4916(57)90015-5
13. S.L. Adler, W.A. Bardeen, Absence of higher-order corrections in the anomalous axial-vector divergence equation. Phys. Rev. **182**, 1517 (1969). https://doi.org/10.1103/PhysRev.182.1517
14. M. Gell-Mann, M. Levy, The axial vector current in beta decay. Nuovo Cim. **16**, 705 (1960). https://doi.org/10.1007/BF02859738
15. V.G. Vaks, A.I. Larkin, On the application of the methods of superconductivity theory to the problem of the masses of elementary particles. Sov. Phys. JETP **13**, 192 (1961)
16. Y. Nambu, A 'superconductor' model of elementary particles and its consequences, in *Proc. of the Midwest Conf. on Theoretical Physics, Purdue, 1960*, edited by F. J. Belinfante, S. G. Artenhaus, and R. W. King (Purdue Unit. Press, Purdue, 1960)
17. S.L. Glashow, The renormalizability of vector meson interactions. Nucl. Phys. **10**, 107 (1959). https://doi.org/10.1016/0029-5582(59)90196-8
18. S.L. Glashow, Partial symmetries of weak interactions. Nucl. Phys. **22**, 579 (1961). https://doi.org/10.1016/0029-5582(61)90469-2
19. A. Salam, J.C. Ward, Weak and electromagnetic interactions. Nuovo Cim. **11**, 568 (1959). https://doi.org/10.1007/BF02726525
20. A. Salam, J.C. Ward, On a gauge theory of elementary interactions. Nuovo Cim. **19**, 165 (1961). https://doi.org/10.1007/BF02812723
21. S. Weinberg, A model of leptons. Phys. Rev. Lett. **19**, 1264 (1967). https://doi.org/10.1103/PhysRevLett.19.1264
22. D.A. Kirzhnits, Weinberg model in the hot universe. JETP Lett. **15**, 529 (1972)
23. H. Georgi, S.L. Glashow, Unity of all elementary particle forces. Phys. Rev. Lett. **32**, 438 (1974). https://doi.org/10.1103/PhysRevLett.32.438
24. D.D. Ivanenko, D.F. Kurdgelaidze, Hypothesis concerning quark stars. Astrophysics **1**, 251 (1965). https://doi.org/10.1007/BF01042830
25. N. Itoh, Hydrostatic equilibrium of hypothetical quark stars. Prog. Theor. Phys. **44**, 291 (1970). https://doi.org/10.1143/PTP.44.291
26. A.R. Bodmer, Collapsed nuclei. Phys. Rev. D **4**, 1601 (1971). https://doi.org/10.1103/PhysRevD.4.1601
27. A.B. Migdal, Stability of vacuum and limiting fields. Zh. Eksp. Teor. Fiz. **61**, 2209 (1971)
28. A.B. Migdal, Meson condensation and anomalous nuclei. Phys. Lett. B **52**, 172 (1974). https://doi.org/10.1016/0370-2693(74)90081-1
29. A.B. Migdal, G.A. Sorokin, O.A. Markin, I.N. Mishustin, Pion condensation and stability of abnormal nuclei. Phys. Lett. B **65**, 423 (1976). https://doi.org/10.1016/0370-2693(76)90434-2
30. A.B. Migdal, Pion fields in nuclear matter. Rev. Mod. Phys. **50**, 107 (1978). https://doi.org/10.1103/RevModPhys.50.107
31. A.B. Migdal, E.E. Saperstein, M.A. Troitsky, D.N. Voskresensky, Pion degrees of freedom in nuclear matter. Phys. Rept. **192**, 179 (1990). https://doi.org/10.1016/0370-1573(90)90132-L
32. T.D. Lee, Abnormal nuclear states. In: 17th International Conference on High-Energy Physics pp. III.81–83, (1974)
33. T.D. Lee, G.C. Wick, Vacuum stability and vacuum excitation in a spin-0 field theory. Phys. Rev. D **9**, 2291 (1974). https://doi.org/10.1103/PhysRevD.9.2291
34. T.D. Lee, Abnormal nuclear states and vacuum excitations. Rev. Mod. Phys. **47**, 267 (1975). https://doi.org/10.1103/RevModPhys.47.267
35. T.D. Lee, A possible new form of matter at high density. AIP Conf. Proc. **28**, 65 (1976). https://doi.org/10.1142/9789814434973_0005
36. T.D. Lee, M. Margulies, Interaction of a dense fermion medium with a scalar meson field. Phys. Rev. D **11**, 1591 (1975). https://doi.org/10.1103/PhysRevD.12.4008
37. T. D. Lee, G. C. Wick, Abnormal nuclear states. In: Mesons in Nuclei, Vol. 3, edited by M. Rho and D. Wilkinson (North-Holland, 1979) Chap. chapter 22, pp. 867–887
38. L. Lederman, editor J. Weneser, eds., *Report of the workshop on BeV/nucleon collisions of heavy ions – how and why, Bear Mountain, New York, Nov. 29 – Dec. 1, 1974*, Brookhaven Nat. Lab. (BNL, 1975)
39. V.I. Aleshin, A.Y. Balysh, V.M. Galitsky, Y.V. Kozlov, V.I. Lebedev, V.P. Martemyanov, L.A. Mikaelyan, A.A. Pomansky, V.G. Tarasenkov, Spontaneous transitions of nuclei to superdense state. Pisma Zh. Eksp. Teor. Fiz. **24**, 114 (1976)
40. V.M. Galitskii, Anomalous states and collective motions of nuclear matter. Sov. Phys. Usp. **19**, 769 (1976). https://doi.org/10.1070/PU1976v019n09ABEH005296
41. W. Scheid, H. Müller, W. Greiner, Nuclear shock waves in heavy-ion collisions. Phys. Rev. Lett. **32**, 741 (1974). https://doi.org/10.1103/PhysRevLett.32.741
42. M.I. Sobel, H.A. Bethe, P.J. Siemens, J.P. Bondorf, Shock waves in colliding nuclei. Nucl. Phys. A **251**, 502 (1975). https://doi.org/10.1016/0375-9474(75)90545-X
43. R.J. Holt, J.P. Schiffer, J. Specht, L.M. Bollinger, G.E. Thomas, S.M. Fried, J.J. Hines, A.M. Friedman, Search for stable, abnor-

mal (collapsed) nuclei in nature. Phys. Rev. Lett. **36**, 183 (1976). https://doi.org/10.1103/PhysRevLett.36.183
44. V.A. Karnaukhov, S.M. Polikarpov, Superheavy nuclei and pion condensation. JETP Lett. **25**, 304 (1977)
45. S.P. Avdeyev, V.A. Karnaukhov, G.Y. Korovin, V.D. Kuznetsov, T. Nagy, L.A. Petrov, Search for superdense nuclei of rb and cs among the products of 8-GeV proton interactions with Ta. Nucl. Phys. A **381**, 419 (1982). https://doi.org/10.1016/0375-9474(82)90368-2
46. M.K. Anikina, G.L. Vardenga, V.D. Volodin, M. Gazdzicki, A.I. Golokhvastov, A.A. Kuznetsov, Y. Lukstinsh, E.O. Okonov, S.A. Khorozov, Search for unstable superdense nuclei. Sov. Phys. JETP **58**, 890 (1983) http://jetp.ras.ru/cgi-bin/dn/e_058_05_0890.pdf
47. S.P. Avdeyev, V.A. Karnaukhov, V.D. Kuznetsov, L.A. Petrov, On the search for density isomers in interaction of relativistic ^{12}C nuclei with lead. Z. Phys. A **329**, 375 (1988). https://doi.org/10.1007/BF01290244
48. A.D. Linde, Particle physics and inflationary cosmology. Contemp. Concepts Phys. **5**, 1 (1990). arXiv:hep-th/0503203
49. A.B. Migdal, D.N. Voskresensky, V.S. Popov, About vacuum charge distribution near supercharged nuclei. JETP Lett. **24**, 163 (1976)
50. A.B. Migdal, V.S. Popov, D.N. Voskresensky, Distribution of vacuum charge near supercharged nuclei. Sov. Phys. JETP **45**, 436 (1977)
51. B. Müller, J. Rafelski, Stabilization of the charged vacuum created by very strong electrical fields in nuclear matter. Phys. Rev. Lett. **34**, 349 (1975). https://doi.org/10.1103/PhysRevLett.34.349
52. D.N. Voskresensky, G.A. Sorokin, A.I. Chernoutsan, Charge distribution in anomalous nuclei. JETP Lett. **25**, 465 (1977)
53. D.N. Voskresensky, A.I. Chernoutsan, Condensation of pions in electric field of supercharged nucleus. Sov. J. Nucl. Phys. **27**, 742 (1978)
54. D. N. Voskresensky, Charge distribution in anomalous nuclei, Ph.D. thesis, MEPhI, Moscow (1977), (**in Russian**)
55. M.A. Troitsky, N.I. Chekunaev, Structure of baryon matter and pion condensation. Yad. Fiz. **29**, 220 (1979)
56. E. Witten, Cosmic separation of phases. Phys. Rev. D **30**, 272 (1984). https://doi.org/10.1103/PhysRevD.30.272
57. E. Farhi, R.L. Jaffe, Strange matter. Phys. Rev. D **30**, 2379 (1984). https://doi.org/10.1103/PhysRevD.30.2379
58. A. De Rujula, S.L. Glashow, Nuclearites: a novel form of cosmic radiation. Nature **312**, 734 (1984). https://doi.org/10.1038/312734a0
59. A. De Rujula, Aborigines of the nuclear desert. Nucl. Phys. A **434**, 605C (1985). https://doi.org/10.1016/0375-9474(85)90525-1
60. C. Alcock, E. Farhi, A. Olinto, Strange stars. Astrophys. J. **310**, 261 (1986). https://doi.org/10.1086/164679
61. S.L. Shapiro, A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (Wiley-Interscience, New York, 1983)
62. H.A. Bethe, G.E. Brown, J. Cooperstein, Stars of strange matter? Nucl. Phys. A **462**, 791 (1987). https://doi.org/10.1016/0375-9474(87)90577-X
63. J. Madsen, Physics and astrophysics of strange quark matter. Lect. Notes Phys. **516**, 162 (1999). https://doi.org/10.1007/BFb0107314
64. B.C. Barrois, Superconducting quark matter. Nucl. Phys. B **129**, 390 (1977). https://doi.org/10.1016/0550-3213(77)90123-7
65. D. Bailin, A. Love, Superfluidity and superconductivity in relativistic fermion systems. Phys. Rept. **107**, 325 (1984). https://doi.org/10.1016/0370-1573(84)90145-5
66. R. Rapp, T. Schäfer, E.V. Shuryak, M. Velkovsky, Diquark Bose condensates in high density matter and instantons. Phys. Rev. Lett. **81**, 53 (1998). https://doi.org/10.1103/PhysRevLett.81.53
67. M.G. Alford, K. Rajagopal, F. Wilczek, QCD at finite baryon density: nucleon droplets and color superconductivity. Phys. Lett. B **422**, 247 (1998). https://doi.org/10.1016/S0370-2693(98)00051-3
68. M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, Color superconductivity in dense quark matter. Rev. Mod. Phys. **80**, 1455 (2008). https://doi.org/10.1103/RevModPhys.80.1455
69. A.R. Zhitnitsky, 'Nonbaryonic' dark matter as baryonic color superconductor. JCAP **10**, 010. https://doi.org/10.1088/1475-7516/2003/10/010
70. A. Zhitnitsky, Axion quark nuggets. Dark matter and matter-antimatter asymmetry: theory, observations and future experiments. Mod. Phys. Lett. A **36**, 2130017 (2021). https://doi.org/10.1142/S0217732321300172
71. A. Zhitnitsky, M. Maroudas, Mysterious anomalies in Earth's atmosphere and strongly interacting dark matter. Symmetry **17**, 79 (2025). https://doi.org/10.3390/sym17010079
72. V.A. Gani, M.Y. Khlopov, D.N. Voskresensky, Double charged heavy constituents of dark atoms and superheavy nuclear objects. Phys. Rev. D **99**, 015024 (2019). https://doi.org/10.1103/PhysRevD.99.015024
73. G. Röpke, L. Münchow, H. Schulz, On the phase stability of hot nuclear matter and the applicability of detailed balance equations. Phys. Lett. B **110**, 21 (1982). https://doi.org/10.1016/0370-2693(82)90943-1
74. M.W. Curtin, H. Toki, D.K. Scott, Liquid-gas phase instabilities in nuclear systems. Phys. Lett. B **123**, 289 (1983). https://doi.org/10.1016/0370-2693(83)91202-9
75. G. Bertsch, P.J. Siemens, Nuclear fragmentation. Phys. Lett. B **126**, 9 (1983). https://doi.org/10.1016/0370-2693(83)90004-7
76. H. Schulz, D.N. Voskresensky, J. Bondorf, Dynamical aspects of the liquid-vapor phase transition in nuclear systems. Phys. Lett. B **133**, 141 (1983). https://doi.org/10.1016/0370-2693(83)90546-4
77. B. Borderie, S. Piantelli, J.D. Frankland, N. Le Neindre, Negative heat capacity for hot nuclei: Confirmation with formulation from the microcanonical ensemble. Nuovo Cim. C **45**, 48 (2022)
78. E.E. Kolomeitsev, D.N. Voskresensky, Scalar quanta in Fermi liquids: zero sounds, instabilities, Bose condensation, and a metastable state in dilute nuclear matter. Eur. Phys. J. A **52**, 362 (2016). https://doi.org/10.1140/epja/i2016-16362-0
79. K.A. Maslov, D.N. Voskresensky, RMF models with σ-scaled hadron masses and couplings for the description of heavy-ion collisions below 2A GeV. Eur. Phys. J. A **55**, 100 (2019). https://doi.org/10.1140/epja/i2019-12779-1
80. D.N. Voskresensky, Pion-sigma meson vortices in rotating systems. Phys. Rev. D **109**, 034030 (2024). https://doi.org/10.1103/PhysRevD.109.034030
81. D.N. Voskresensky, Pion condensation at rotation in magnetic field, electric, and scalar potential wells. Phys. Rev. D **111**, 036022 (2025). https://doi.org/10.1103/PhysRevD.111.036022
82. J.D. Walecka, A theory of highly condensed matter. Ann. Phys. **83**, 491 (1974). https://doi.org/10.1016/0003-4916(74)90208-5
83. S.A. Chin, J.D. Walecka, An equation of state for nuclear and higher-density matter based on a relativistic mean-field theory. Phys. Lett. B **52**, 24 (1974). https://doi.org/10.1016/0370-2693(74)90708-4
84. M.H. Johnson, E. Teller, Classical field theory of nuclear forces. Phys. Rev. **98**, 783 (1955). https://doi.org/10.1103/PhysRev.98.783
85. H.-P. Duerr, Relativistic effects in nuclear forces. Phys. Rev. **103**, 469 (1956). https://doi.org/10.1103/PhysRev.103.469
86. J. Boguta, A.R. Bodmer, Relativistic calculation of nuclear matter and the nuclear surface. Nucl. Phys. A **292**, 413 (1977). https://doi.org/10.1016/0375-9474(77)90626-1
87. B.D. Serot, J.D. Walecka, Recent progress in quantum hadrodynamics. Int. J. Mod. Phys. E **6**, 515 (1997). https://doi.org/10.1142/S0218301397000299

88. E.E. Kolomeitsev, D.N. Voskresensky, Relativistic mean-field models with effective hadron masses and coupling constants, and rho-condensation. Nucl. Phys. A **759**, 373 (2005). https://doi.org/10.1016/j.nuclphysa.2005.05.154
89. L.-S. Geng, H. Toki, J. Meng, Masses, deformations and charge radii: Nuclear ground-state properties in the relativistic mean field model. Prog. Theor. Phys. **113**, 785 (2005). https://doi.org/10.1143/PTP.113.785
90. D.N. Voskresensky, Many particle effects in nucleus-nucleus collisions. Nucl. Phys. A **555**, 293 (1993). https://doi.org/10.1016/0375-9474(93)90324-Q
91. A.S. Khvorostukhin, V.D. Toneev, D.N. Voskresensky, Equation of state for hot and dense matter: sigma- omega- rho model with scaled hadron masses and couplings. Nucl. Phys. A **791**, 180 (2007). https://doi.org/10.1016/j.nuclphysa.2007.03.140
92. A.S. Khvorostukhin, V.D. Toneev, D.N. Voskresensky, Relativistic mean-field model with scaled hadron masses and couplings. Nucl. Phys. A **813**, 313 (2008). https://doi.org/10.1016/j.nuclphysa.2008.09.013
93. K.A. Maslov, E.E. Kolomeitsev, D.N. Voskresensky, Making a soft relativistic mean-field equation of state stiffer at high density. Phys. Rev. C **92**, 052801 (2015). https://doi.org/10.1103/PhysRevC.92.052801
94. K.A. Maslov, E.E. Kolomeitsev, D.N. Voskresensky, Relativistic mean-field models with scaled hadron masses and couplings: Hyperons and maximum neutron star mass. Nucl. Phys. A **950**, 64 (2016). https://doi.org/10.1016/j.nuclphysa.2016.03.011
95. E.E. Kolomeitsev, K.A. Maslov, D.N. Voskresensky, Delta isobars in relativistic mean-field models with σ-scaled hadron masses and couplings. Nucl. Phys. A **961**, 106 (2017). https://doi.org/10.1016/j.nuclphysa.2017.02.004
96. S. Typel, H.H. Wolter, Relativistic mean field calculations with density dependent meson nucleon coupling. Nucl. Phys. A **656**, 331 (1999). https://doi.org/10.1016/S0375-9474(99)00310-3
97. S. Typel, Relativistic model for nuclear matter and atomic nuclei with momentum-dependent self-energies. Phys. Rev. C **71**, 064301 (2005). https://doi.org/10.1103/PhysRevC.71.064301
98. P. Danielewicz, R. Lacey, W.G. Lynch, Determination of the equation of state of dense matter. Science **298**, 1592 (2002). https://doi.org/10.1126/science.1078070
99. D. Blaschke, H. Grigorian, D.N. Voskresensky, Nuclear medium cooling scenario in the light of new Cas A cooling data and the $2m_{\odot}$ pulsar mass measurements. Ph. Rev. C **88**, 065805 (2013). https://doi.org/10.1103/PhysRevC.88.065805
100. H. Grigorian, D.N. Voskresensky, D. Blaschke, Influence of the stiffness of the equation of state and in-medium effects on the cooling of compact stars. Eur. Phys. J. A **52**, 67 (2016). https://doi.org/10.1140/epja/i2016-16067-4
101. H. Grigorian, E.E. Kolomeitsev, K.A. Maslov, D.N. Voskresensky, On cooling of neutron stars with a stiff equation of state including hyperons. Universe **4**, 29 (2018). https://doi.org/10.3390/universe4020029
102. H. Grigorian, D.N. Voskresensky, K.A. Maslov, Cooling of neutron stars in nuclear medium cooling scenario" with stiff equation of state including hyperons. Nucl. Phys. A **980**, 105 (2018). https://doi.org/10.1016/j.nuclphysa.2018.10.014
103. R.L. Jaffe, Perhaps a stable dihyperon. Phys. Rev. Lett. **38**, 195 (1977). https://doi.org/10.1103/PhysRevLett.38.195
104. V.A. Matveev, P. Sorba, Is deuteron a six quark system? Lett. Nuovo Cim. **20**, 435 (1977). https://doi.org/10.1007/BF02790723
105. B. Tatischeff, N. Willis, M.P. Comets, P. Courtat, R. Gacougnolle, Y. Le Bornec, E. Loireleux, F. Reide, J. Yonnet, M. Boivin, Evidence for narrow baryons in inelastic pp scattering. Phys. Rev. Lett. **79**, 601 (1997). https://doi.org/10.1103/PhysRevLett.79.601
106. L.V. Fil'kov, V.L. Kashevarov, E.S. Konobeevski, M.V. Mordovskoy, S.I. Potashev, V.M. Skorkin, Search for supernarrow dibaryons in pd interaction. Phys. Rev. C **61**, 044004 (2000). https://doi.org/10.1103/PhysRevC.61.044004
107. L.V. Fil'kov, V.L. Kashevarov, E.S. Konobeevski, M.V. Mordovskoy, S.I. Potashev, V.A. Simonov, V.M. Skorkin, S.V. Zuev, New evidence for supernarrow dibaryons production in pd interactions. Eur. Phys. J. A **12**, 369 (2001). https://doi.org/10.1007/s100500170016
108. B. Tatischeff, J. Yonnet, M. Boivin, M.P. Comets, P. Courtat, R. Gacougnolle, Y. Le Bornec, E. Loireleux, F. Reide, N. Willis, Experimental evidence for narrow baryons in the mass range Experimental evidence for narrow baryons in the mass range $1.0 \leq M \leq 1.46\ GeV, 0M1.46\ GeV,$. Eur. Phys. J. A **17**, 245 (2003). https://doi.org/10.1140/epja/i2002-10145-2
109. Y.I. Azimov, R.A. Arndt, I.I. Strakovsky, R.L. Workman, Light baryon resonances: restrictions and perspectives. Phys. Rev. C **68**, 045204 (2003). https://doi.org/10.1103/PhysRevC.68.045204
110. E.E. Kolomeitsev, D.N. Voskresensky, Resonance states below pion nucleon threshold and their consequences for nuclear systems. Phys. Rev. C **67**, 015805 (2003). https://doi.org/10.1103/PhysRevC.67.015805
111. H.T. Cromartie et al., (collaboration NANOGrav), Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar. Nat. Astron. **4**, 72 (2019). https://doi.org/10.1038/s41550-019-0880-2
112. E. Fonseca et al., Refined mass and geometric measurements of the high-mass PSR J0740+6620. Astrophys. J. Lett. **915**, L12 (2021). https://doi.org/10.3847/2041-8213/ac03b8
113. T. Walcher, A Simple model to explain narrow nucleon resonances below the π threshold, arXiv (2001), arXiv:hep-ph/0111279
114. A.J. Krasznahorkay et al., Observation of anomalous internal pair creation in ^{8} Be: a possible indication of a light, neutral boson. Phys. Rev. Lett. **116**, 042501 (2016). https://doi.org/10.1103/PhysRevLett.116.042501
115. A.J. Krasznahorkay, M. Csatlós, L. Csige, J. Gulyás, A. Krasznahorkay, B.M. Nyakó, I. Rajta, J. Timár, I. Vajda, N.J. Sas, New anomaly observed in ^{4}He supports the existence of the hypothetical X17 particle. Phys. Rev. C **104**, 044003 (2021). https://doi.org/10.1103/PhysRevC.104.044003
116. A.J. Krasznahorkay et al., New anomaly observed in ^{12}C supports the existence and the vector character of the hypothetical X17 boson. Phys. Rev. C **106**, L061601 (2022). https://doi.org/10.1103/PhysRevC.106.L061601
117. C. Gustavino, X17: status and perspectives. Universe **10**, 285 (2024). https://doi.org/10.3390/universe10070285
118. J.L. Feng et al., Protophobic fifth-force interpretation of the observed anomaly in ^{8}Be nuclear transitions. Phys. Rev. Lett. **117**, 071803 (2016). https://doi.org/10.1103/PhysRevLett.117.071803
119. U. Ellwanger, S. Moretti, Possible explanation of the electron positron anomaly at 17 MeV in ^{8}Be transitions through a light pseudoscalar. JHEP **11**, 039. https://doi.org/10.1007/JHEP11(2016)039
120. A. Kanakis-Pegios, V. Petousis, M. Veselsky, J. Leja, C.C. Moustakidis, Constraints for the X17 boson from compact objects observations. Phys. Rev. D **109**, 043028 (2024). https://doi.org/10.1103/PhysRevD.109.043028
121. D. Farrell, F. Weber, M. G. Orsaria, I. F. Ranea-Sandoval, M. Canullán, R. Negreiros, T. Katayama, I. Wagaman, Fast pulsars, neutron stars, and astrophysical strange quark matter objects, arXiv (2024), arXiV:2402.08835
122. A.V. Afanasjev, Rotating nuclei: from ground state to the extremes of spin and deformation. Int. Rev. Nucl. Phys. **10**, 305 (2016). https://doi.org/10.1142/9789814733267_0008
123. H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Analogy between rotation and density for Dirac fermions in a magnetic

field. Phys. Rev. D **93**, 104052 (2016). https://doi.org/10.1103/PhysRevD.93.104052
124. L. Adamczyk et al., (collaboration STAR), Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid. Nature **548**, 62 (2017). https://doi.org/10.1038/nature23004
125. D.N. Voskresensky, N.Y. Anisimov, Properties of a pion condensate in a magnetic field. Sov. Phys. JETP **51**, 13 (1980)
126. Y. Liu, I. Zahed, Pion condensation by rotation in a magnetic field. Phys. Rev. Lett. **120**, 032001 (2018). https://doi.org/10.1103/PhysRevLett.120.032001
127. T. Guo, J. Li, C. Mu, L. He, Formation of a supergiant quantum vortex in a relativistic Bose-Einstein condensate driven by rotation and a parallel magnetic field. Phys. Rev. D **106**, 094010 (2022). https://doi.org/10.1103/PhysRevD.106.094010
128. M. Bordag, D.N. Voskresensky, Generation of a scalar vortex in a rotational frame. Phys. Rev. D **112**, 085026 (2025). https://doi.org/10.1103/y2lc-27pn
129. D.N. Voskresensky, Condensate with a finite momentum in a moving medium. J. Exp. Theor. Phys. **77**, 917 (1993)
130. E. E. Kolomeitsev, D. N. Voskresensky, Condensate of excitations in moving superfluids, PTEP **2017**, 023D01 (2017). https://doi.org/10.1093/ptep/ptw182
131. E.E. Kolomeitsev, D.N. Voskresensky, Viscosity of neutron star matter and r-modes in rotating pulsars. Phys. Rev. C **91**, 025805 (2015). https://doi.org/10.1103/PhysRevC.91.025805
132. I.A. Morrison, T.W. Baumgarte, S.L. Shapiro, Effect of differential rotation on the maximum mass of neutron stars: realistic nuclear equations of state. Astrophys. J. **610**, 941 (2004). https://doi.org/10.1086/421897
133. R.N. Cahn, S.L. Glashow, Chemical signatures for superheavy elementary particles. Science **213**, 607 (1981). https://doi.org/10.1126/science.213.4508.607
134. A. De Rujula, S.L. Glashow, U. Sarid, Charged dark matter. Nucl. Phys. B **333**, 173 (1990). https://doi.org/10.1016/0550-3213(90)90227-5
135. S. L. Glashow, A Sinister extension of the standard model to SU(3) × SU(2) × SU(2) × U(1). In: 11th International Workshop on Neutrino Telescopes, pp. 539–547 (2005), arXiv:hep-ph/0504287
136. M.Y. Khlopov, Composite dark matter from 4th generation. Pisma Zh. Eksp. Teor. Fiz. **83**, 3 (2006). https://doi.org/10.1134/S0021364006010012
137. A.B. Migdal, Phase transition in nuclear matter and non-pair nuclear forces. Sov. Phys. JETP **36**, 1052 (1973)
138. D.J. Scalapino, π^- condensate in dense nuclear matter. Phys. Rev. Lett. **29**, 386 (1972). https://doi.org/10.1103/PhysRevLett.29.386
139. R.F. Sawyer, Condensed π^- phase in neutron star matter. Phys. Rev. Lett. **29**, 382 (1972). https://doi.org/10.1103/PhysRevLett.29.382
140. A.B. Migdal, The energy of the nuclear matter in the presence of the π-condensate. Phys. Lett. B **47**, 96 (1973). https://doi.org/10.1016/0370-2693(73)90578-9
141. A.B. Migdal, π condensation in nuclear matter. Phys. Rev. Lett. **31**, 257 (1973). https://doi.org/10.1103/PhysRevLett.31.257
142. G. Baym, Pion condensation in nuclear and neutron star matter. Phys. Rev. Lett. **30**, 1340 (1973). https://doi.org/10.1103/PhysRevLett.30.1340
143. A.B. Migdal, O.A. Markin, I.N. Mishustin, The pion spectrum in nuclear matter and pion condensation. Sov. Phys. JETP **39**, 212 (1974)
144. D.K. Campbell, R.F. Dashen, J.T. Manassah, Chiral symmetry and pion condensation. 1. Model dependent results. Phys. Rev. D **12**, 979 (1975). https://doi.org/10.1103/PhysRevD.12.979
145. D.K. Campbell, R.F. Dashen, J.T. Manassah, Chiral symmetry and pion condensation. 2. General formalism. Phys. Rev. D **12**, 1010 (1975). https://doi.org/10.1103/PhysRevD.12.1010
146. G. Baym, D. Campbell, R.F. Dashen, J. Manassah, A simple model calculation of pion condensation in neutron matter. Phys. Lett. B **58**, 304 (1975). https://doi.org/10.1016/0370-2693(75)90659-0
147. F. Dautry, E.M. Nyman, Pion condensation and the sigma model in liquid neutron matter. Nucl. Phys. A **319**, 323 (1979). https://doi.org/10.1016/0375-9474(79)90518-9
148. T. Takatsuka, K. Tamiya, T. Tatsumi, R. Tamagaki, Solidification and pion condensation in nuclear medium. Alternating layer spin structure with one-dimensional localization accompanying π^0 condensate. Prog. Theor. Phys. **59**, 1933 (1978). https://doi.org/10.1143/PTP.59.1933
149. D.N. Voskresensky, Structure formation during phase transitions in strongly interacting matter. Prog. Part. Nucl. Phys. **130**, 104030 (2023). https://doi.org/10.1016/j.ppnp.2023.104030
150. A. Akmal, V.R. Pandharipande, D.G. Ravenhall, The equation of state of nucleon matter and neutron star structure. Phys. Rev. C **58**, 1804 (1998). https://doi.org/10.1103/PhysRevC.58.1804
151. D.N. Voskresensky, A.V. Senatorov, Pion excitations in a nucleonic medium may be pertinent to the luminosity of neutron stars. JETP Lett. **40**, 1212 (1984)
152. D.N. Voskresensky, A.V. Senatorov, Emission of neutrinos by neutron stars. Sov. Phys. JETP **63**, 885 (1986)
153. D.N. Voskresensky, A.V. Senatorov, Description of nuclear interaction in Keldysh's diagram technique and neutrino luminosity of neutron stars. Sov. J. Nucl. Phys. **45**, 411 (1987)
154. C. Schaab, D. Voskresensky, A.D. Sedrakian, F. Weber, M.K. Weigel, Impact of medium effects on the cooling of nonsuperfluid and superfluid neutron stars. Astron. Astrophys. **321**, 591 (1997). https://doi.org/10.48550/arXiv.astro-ph/9605188
155. D. N. Voskresensky, Neutrino cooling of neutron stars: Medium effects, in *booktitle Physics of Neutron Star Interiors*, editor edited by D. Blaschke, A. Sedrakian, and N. K. Glendenning (Springer Berlin Heidelberg, Berlin, Heidelberg, 2001) pp. 467–503, arXiv:astro-ph/0101514. https://doi.org/10.1007/3-540-44578-1_17
156. D. Blaschke, H. Grigorian, D.N. Voskresensky, Cooling of neutron stars: Hadronic model. Astron. Astrophys. **424**, 979 (2004). https://doi.org/10.1051/0004-6361:20040404
157. H. Grigorian, D.N. Voskresensky, Medium effects in cooling of neutron stars and $^3P_2$ neutron gap. Astron. Astrophys. **444**, 913 (2005). https://doi.org/10.1051/0004-6361:20052799
158. D. Blaschke, H. Grigorian, D.N. Voskresensky, F. Weber, On the cooling of the neutron star in Cassiopeia a. Phys. Rev. C **85**, 022802 (2012). https://doi.org/10.1103/PhysRevC.85.022802
159. B. Friedman, V.R. Pandharipande, Hot and cold, nuclear and neutron matter. Nucl. Phys. A **361**, 502 (1981). https://doi.org/10.1016/0375-9474(81)90649-7
160. M. Cubero, M. Schönhofen, H. Feldmeier, W. Nörenberg, Delta and pion abundances in hot dense nuclear matter and the nuclear equation of state. Phys. Lett. B **201**, 11 (1988). https://doi.org/10.1016/0370-2693(88)90070-6
161. D.T. Son, M.A. Stephanov, Axial anomaly and magnetism of nuclear and quark matter. Phys. Rev. D **77**, 014021 (2008). https://doi.org/10.1103/PhysRevD.77.014021
162. M. Eto, K. Hashimoto, T. Hatsuda, Ferromagnetic neutron stars: axial anomaly, dense neutron matter, and pionic wall. Phys. Rev. D **88**, 081701(R) (2013). https://doi.org/10.1103/PhysRevD.88.081701
163. K. Hashimoto, Possibility of ferromagnetic neutron matter. Phys. Rev. D **91**, 085013 (2015). https://doi.org/10.1103/PhysRevD.91.085013
164. T. Brauner, N. Yamamoto, Chiral soliton lattice and charged pion condensation in strong magnetic fields, JHEP **04**, 132. https://doi.org/10.1007/JHEP04(2017)132

165. X.-G. Huang, K. Nishimura, N. Yamamoto, Anomalous effects of dense matter under rotation, JHEP **02**, 069. https://doi.org/10.1007/JHEP02(2018)069
166. D.N. Voskresensky, Examination of the possibility of π^0 condensation and magnetization in freely interpenetrating nuclei. Phys. Rev. C **111**, 064903 (2025). https://doi.org/10.1103/PhysRevC.111.064903
167. J. Delorme, M. Ericson, T.E.O. Ericson, To condense or not to condense? That is the question? Phys. Lett. B **291**, 379 (1992). https://doi.org/10.1016/0370-2693(92)91391-L
168. T.E.O. Ericson, Anomalous chiral effects in media. Phys. Lett. B **321**, 312 (1994). https://doi.org/10.1016/0370-2693(94)90248-8
169. E.E. Kolomeitsev, N. Kaiser, W. Weise, Chiral dynamics of deeply bound pionic atoms. Phys. Rev. Lett. **90**, 092501 (2003). https://doi.org/10.1103/PhysRevLett.90.092501
170. E.E. Kolomeitsev, N. Kaiser, W. Weise, Chiral dynamics and pionic 1s states of Pb and Sn isotopes. Nucl. Phys. A **721**, 835 (2003). https://doi.org/10.1016/S0375-9474(03)01220-X
171. D.N. Voskresensky, *S*-wave pion condensation in symmetric nuclear matter. Phys. Rev. D **105**, 116007 (2022). https://doi.org/10.1103/PhysRevD.105.116007
172. D.N. Voskresensky, I.N. Mishustin, Polarization operator of pions at finite temperatures. Sov. J. Nucl. Phys. **35**, 667 (1982)
173. V. Metag, Near threshold particle production: a probe for resonance matter formation in relativistic heavy ion collisions. Prog. Part. Nucl. Phys. **30**, 75 (1993). https://doi.org/10.1016/0146-6410(93)90007-3
174. J. Boguta, Baryonic degrees of freedom leading to density isomers. Phys. Lett. B **109**, 251 (1982). https://doi.org/10.1016/0370-2693(82)90428-2
175. B. Waldhauser, J.A. Maruhn, H. Stöcker, W. Greiner, Sensitivity of pion production to the nuclear EoS and to the Δ's coupling constant. Z. Phys. A **328**, 19 (1987). https://doi.org/10.1007/BF01295178
176. D. Diakonov, A.D. Mirlin, Is a condensed state of nuclear matter possible? Sov. J. Nucl. Phys. **47**, 21 (1988)
177. J. Schaffner-Bielich, Hypernuclear physics for neutron stars. Nucl. Phys. A **804**, 309 (2008). https://doi.org/10.1016/j.nuclphysa.2008.01.005
178. H. Djapo, B.-J. Schaefer, J. Wambach, On the appearance of hyperons in neutron stars. Phys. Rev. C **81**, 035803 (2010). https://doi.org/10.1103/PhysRevC.81.035803
179. H.J. Schulze, T. Rijken, Maximum mass of hyperon stars with the Nijmegen es c-08 model. Phys. Rev. C **84**, 035801 (2011). https://doi.org/10.1103/PhysRevC.84.035801
180. A. Drago, A. Lavagno, G. Pagliara, D. Pigato, Early appearance of δ isobars in neutron stars. Phys. Rev. C **90**, 065809 (2014). https://doi.org/10.1103/PhysRevC.90.065809
181. A. Drago, A. Lavagno, G. Pagliara, D. Pigato, The scenario of two families of compact stars: 1. Equations of state, mass-radius relations and binary systems. Eur. Phys. J. A **52**, 40 (2016). https://doi.org/10.1140/epja/i2016-16040-3
182. R. Friedberg, T.D. Lee, A. Sirlin, A class of scalar-field soliton solutions in three space dimensions. Phys. Rev. D **13**, 2739 (1976). https://doi.org/10.1103/PhysRevD.13.2739
183. S.R. Coleman, Q-balls. Addendum: Nucl. Phys. B **262**, 263 (1985). https://doi.org/10.1016/0550-3213(86)90520-1
184. T.D. Lee, Soliton stars and the critical masses of black holes. Phys. Rev. D **35**, 3637 (1987). https://doi.org/10.1103/PhysRevD.35.3637
185. T.D. Lee, Y. Pang, Stability of mini-boson stars. Nucl. Phys. B **315**, 477 (1989). https://doi.org/10.1016/0550-3213(89)90365-9
186. T.D. Lee, Y. Pang, Nontopological solitons. Phys. Rept. **221**, 251 (1992). https://doi.org/10.1016/0370-1573(92)90064-7
187. P. Haensel, J.L. Zdunik, R. Schaeffer, Strange quark stars. Astron. Astrophys. **160**, 121 (1986)
188. F. Weber, Strange quark matter and compact stars. Prog. Part. Nucl. Phys. **54**, 193 (2005). https://doi.org/10.1016/j.ppnp.2004.07.001
189. G. Lugones, A.G. Grunfeld, Scaling and universality in strange quark stars. Phys. Rev. D **111**, 123027 (2025). https://doi.org/10.1103/59d4-4x9m
190. F. Di Clemente, M. Casolino, A. Drago, M. Lattanzi, C. Ratti, Strange quark matter as dark matter: 40 yr later, a reappraisal. Mon. Not. Roy. Astron. Soc. **537**, 1056 (2025). https://doi.org/10.1093/mnras/staf087
191. A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, Baryon structure in the bag theory. Phys. Rev. D **10**, 2599 (1974). https://doi.org/10.1103/PhysRevD.10.2599
192. S.P. Klevansky, The Nambu-Jona-Lasinio model of quantum chromodynamics. Rev. Mod. Phys. **64**, 649 (1992). https://doi.org/10.1103/RevModPhys.64.649
193. M. Buballa, M. Oertel, Strange quark matter with dynamically generated quark masses. Phys. Lett. B **457**, 261 (1999). https://doi.org/10.1016/S0370-2693(99)00533-X
194. P. Lakaschus, M. Buballa, D.H. Rischke, Competition of inhomogeneous chiral phases and two-flavor color superconductivity in the njl model. Phys. Rev. D **103**, 034030 (2021). https://doi.org/10.1103/PhysRevD.103.034030
195. D.N. Voskresensky, Fluctuations of the color-superconducting gap in hot and dense quark matter. Phys. Rev. C **69**, 065209 (2004). https://doi.org/10.1103/PhysRevC.69.065209
196. M. Buballa, S. Carignano, Inhomogeneous chiral condensates. Prog. Part. Nucl. Phys. **81**, 39 (2015). https://doi.org/10.1016/j.ppnp.2014.11.001
197. R.D. Pisarski, F. Rennecke, Signatures of moat regimes in heavy-ion collisions. Phys. Rev. Lett. **127**, 152302 (2021). https://doi.org/10.1103/PhysRevLett.127.152302
198. C.J. Xia, G.X. Peng, S.W. Chen, Z.Y. Lu, J.F. Xu, Thermodynamic consistency, quark mass scaling, and properties of strange matter. Phys. Rev. D **89**, 105027 (2014). https://doi.org/10.1103/PhysRevD.89.105027
199. M.A.R. Kaltenborn, N.-U.F. Bastian, D.B. Blaschke, Quark-nuclear hybrid star equation of state with excluded volume effects. Phys. Rev. D **96**, 056024 (2017). https://doi.org/10.1103/PhysRevD.96.056024
200. D.N. Voskresensky, M. Yasuhira, T. Tatsumi, Charge screening at first order phase transitions and hadron quark mixed phase. Nucl. Phys. A **723**, 291 (2003). https://doi.org/10.1016/S0375-9474(03)01313-7
201. K. Maslov, N. Yasutake, A. Ayriyan, D. Blaschke, H. Grigorian, T. Maruyama, T. Tatsumi, D.N. Voskresensky, Hybrid equation of state with pasta phases and third family of compact stars. Phys. Rev. C **100**, 025802 (2019). https://doi.org/10.1103/PhysRevC.100.025802
202. D.H. Rischke, J. Schaffner, M.I. Gorenstein, A. Schaefer, H. Stoecker, W. Greiner, Quasiconfinement in the SU(3) gluon plasma. Z. Phys. C **56**, 325 (1992). https://doi.org/10.1007/BF01555532
203. V. Goloviznin, H. Satz, The refractive properties of the gluon plasma in SU(2) theory. Z. Phys. C **57**, 671 (1993). https://doi.org/10.1007/BF01561487
204. A. Peshier, B. Kämpfer, O.P. Pavlenko, G. Soff, An effective model of the quark - gluon plasma with thermal parton masses. Phys. Lett. B **337**, 235 (1994). https://doi.org/10.1016/0370-2693(94)90969-5
205. M.I. Gorenstein, S.-N. Yang, Gluon plasma with a medium dependent dispersion relation. Phys. Rev. D **52**, 5206 (1995). https://doi.org/10.1103/PhysRevD.52.5206
206. A. Peshier, B. Kämpfer, G. Soff, The equation of state of deconfined matter at finite chemical potential in a quasiparticle descrip-

tion. Phys. Rev. C **61**, 045203 (2000). https://doi.org/10.1103/PhysRevC.61.045203
207. A. Peshier, B. Kämpfer, G. Soff, From QCD lattice calculations to the equation of state of quark matter. Phys. Rev. D **66**, 094003 (2002). https://doi.org/10.1103/PhysRevD.66.094003
208. Y.B. Ivanov, V.V. Skokov, V.D. Toneev, Equation of state of deconfined matter within dynamical quasiparticle description. Phys. Rev. D **71**, 014005 (2005). https://doi.org/10.1103/PhysRevD.71.014005. arXiv:hep-ph/0410127
209. R. Schulze and B. Kämpfer, Cold quark stars from hot lattice QCD, arXiv (2009), arXiv:0912.2827
210. Z. Fodor, Lattice QCD results at finite temperature and density. Nucl. Phys. A **715**, 319 (2003). https://doi.org/10.1016/S0375-9474(02)01442-2
211. F. Csikor, G.I. Egri, Z. Fodor, S.D. Katz, K.K. Szabo, and A.I. Toth, Equation of state at finite temperature and chemical potential, lattice QCD results, JHEP **05**, 046. https://doi.org/10.1088/1126-6708/2004/05/046
212. Y.B. Ivanov, A.S. Khvorostukhin, E.E. Kolomeitsev, V.V. Skokov, V.D. Toneev, D.N. Voskresensky, Lattice QCD constraints on hybrid and quark stars. Phys. Rev. C **72**, 025804 (2005). https://doi.org/10.1103/PhysRevC.72.025804
213. M. Alford, M. Braby, M.W. Paris, S. Reddy, Hybrid stars that masquerade as neutron stars. Astrophys. J. **629**, 969 (2005). https://doi.org/10.1086/430902
214. H. Heiselberg, M. Hjorth-Jensen, Phases of dense matter in neutron stars. Phys. Rept. **328**, 237 (2000). https://doi.org/10.1016/S0370-1573(99)00110-6
215. B. Holdom, J. Ren, C. Zhang, Quark matter may not be strange. Phys. Rev. Lett. **120**, 222001 (2018). https://doi.org/10.1103/PhysRevLett.120.222001
216. Y. Bai, T.-K. Chen, Excluding stable quark matter: insights from the QCD vacuum energy, arXiv:2502.20241 (2025)
217. L. McLerran, S. Reddy, Quarkyonic matter and neutron stars. Phys. Rev. Lett. **122**, 122701 (2019). https://doi.org/10.1103/PhysRevLett.122.122701
218. N.K. Glendenning, C. Kettner, F. Weber, Possible new class of dense white dwarfs. Phys. Rev. Lett. **74**, 3519 (1995). https://doi.org/10.1103/PhysRevLett.74.3519
219. A. Kurban, Y.-F. Huang, J.-J. Geng, H.-S. Zong, Searching for strange quark matter objects among white dwarfs. Phys. Lett. B **832**, 137204 (2022). https://doi.org/10.1016/j.physletb.2022.137204
220. L. Perot, N. Chamel, P. Vallet, Unmasking strange dwarfs with gravitational-wave observations. Phys. Rev. D **107**, 103004 (2023). https://doi.org/10.1103/PhysRevD.107.103004
221. L. Hui, J.P. Ostriker, S. Tremaine, E. Witten, Ultralight scalars as cosmological dark matter. Phys. Rev. D **95**, 043541 (2017). https://doi.org/10.1103/PhysRevD.95.043541
222. J. McDonald, Superheavy Q-balls and cosmology. (2025), arXiv:2508.08931
223. Y.M. Shnir, *Topological and Non-Topological Solitons in Scalar Field Theories* (Cambridge University Press, 2018)
224. E.Y. Nugaev, A.V. Shkerin, Review of nontopological solitons in theories with $U(1)$-symmetry. J. Exp. Theor. Phys. **130**, 301 (2020). https://doi.org/10.1134/S1063776120020077
225. I. Bombaci, Possible signatures for deconfined strange quark matter in compact stars. Nucl. Phys. A **681**, 205 (2001). https://doi.org/10.1016/S0375-9474(00)00504-2
226. J. Madsen, Probing strange stars and color superconductivity by r-mode instabilities in millisecond pulsars. Phys. Rev. Lett. **85**, 10 (2000). https://doi.org/10.1103/PhysRevLett.85.10
227. A. Drago, A. Lavagno, G. Pagliara, Effects of color superconductivity on the structure and formation of compact stars. Phys. Rev. D **69**, 057505 (2004). https://doi.org/10.1103/PhysRevD.69.057505
228. N. Iwamoto, Neutrino emissivities and mean free paths of degenerate quark matter. Annals Phys. **141**, 1 (1982). https://doi.org/10.1016/0003-4916(82)90271-8
229. P.M. Pizzochero, Cooling of a strange star with crust. Phys. Rev. Lett. **66**, 2425 (1991). https://doi.org/10.1103/PhysRevLett.66.2425
230. D. Blaschke, T. Klahn, D.N. Voskresensky, Diquark condensates and compact star cooling. Astrophys. J. **533**, 406 (2000). https://doi.org/10.1086/308664
231. D. Blaschke, H. Grigorian, D.N. Voskresensky, Cooling of hybrid neutron stars and hypothetical selfbound objects with superconducting quark cores. Astron. Astrophys. **368**, 561 (2001). https://doi.org/10.1051/0004-6361:20010005
232. H. Grigorian, D. Blaschke, D. Voskresensky, Cooling of neutron stars with color superconducting quark cores. Phys. Rev. C **71**, 045801 (2005). https://doi.org/10.1103/PhysRevC.71.045801
233. S.E. Thorsett, Z. Arzoumanian, M.M. McKinnon, J.H. Taylor, The masses of two binary neutron star system. Astrophys. J. Lett. **405**, L29 (1993). https://doi.org/10.1086/186758
234. G.E. Brown, H. Bethe, A scenario for a large number of low mass black holes in the galaxy. Astrophys. J. **423**, 659 (1994). https://doi.org/10.1086/173844
235. J.M. Lattimer, The nuclear equation of state and neutron star masses. Ann. Rev. Nucl. Part. Sci. **62**, 485 (2012). https://doi.org/10.1146/annurev-nucl-102711-095018
236. J.G. Martinez, K. Stovall, P.C.C. Freire, J.S. Deneva, F.A. Jenet, M.A. McLaughlin, M. Bagchi, S.D. Bates, A. Ridolfi, Pulsar J0453+1559: a double neutron star system with a large mass asymmetry. Astrophys. J. **812**, 143 (2015). https://doi.org/10.1088/0004-637X/812/2/143
237. V. Doroshenko, V. Suleimanov, G. Pühlhofer, A. Santangelo, A strangely light neutron star within a supernova remnant. Nat. Astron. **6**, 1444 (2022). https://doi.org/10.1038/s41550-022-01800-1
238. J.A.J. Alford, J.P. Halpern, Do central compact objects have carbon atmospheres? Astrophys. J. **944**, 36 (2023). https://doi.org/10.3847/1538-4357/acaf55
239. E.E. Kolomeitsev, D.N. Voskresensky, NICER data and a σ-field-dependent stiffness of the hadronic equation of state. Phys. Rev. C **110**, 025801 (2024). https://doi.org/10.1103/PhysRevC.110.025801
240. Y. Kini et al., Constraining the properties of the thermonuclear burst oscillation source XTE J1814−338 through pulse profile modelling. Mon. Not. Roy. Astron. Soc. **535**, 1507 (2024). https://doi.org/10.1093/mnras/stae2398
241. A.B. Migdal, A.I. Chernoutsan, I.N. Mishustin, Pion condensation and dynamics of neutron stars. Phys. Lett. B **83**, 158 (1979). https://doi.org/10.1016/0370-2693(79)90674-9
242. B. Kämpfer, On the possibility of stable quark and pion condensed stars. J. Phys. A **14**, L471 (1981). https://doi.org/10.1088/0305-4470/14/11/009
243. U.H. Gerlach, Equation of state at supranuclear densities and the existence of a third family of superdense stars. Phys. Rev. **172**, 1325 (1968). https://doi.org/10.1103/PhysRev.172.1325
244. N.K. Glendenning, C. Kettner, Nonidentical neutron star twins. Astron. Astrophys. **353**, L9 (2000)
245. K. Schertler, C. Greiner, J. Schaffner-Bielich, M.H. Thoma, Quark phases in neutron stars and a 'third family' of compact stars as a signature for phase transitions. Nucl. Phys. A **677**, 463 (2000). https://doi.org/10.1016/S0375-9474(00)00305-5
246. S. Benic, D. Blaschke, D.E. Alvarez-Castillo, T. Fischer, S. Typel, A new quark-hadron hybrid equation of state for astrophysics - I. High-mass twin compact stars. Astron. Astrophys. **577**, A40 (2015). https://doi.org/10.1051/0004-6361/201425318

247. A. Potekhin, https://www.ioffe.ru/astro/NSG/thermal/index.html Catalog of thermal emission of neutron stars: https://www.ioffe.ru/astro/nsg/thermal/index.html (2026)
248. A.Y. Potekhin, D.A. Zyuzin, D.G. Yakovlev, M.V. Beznogov, Y.A. Shibanov, Thermal luminosities of cooling neutron stars. Mon. Not. R. Astron. Soc. **496**, 5052 (2020). https://doi.org/10.1093/mnras/staa1871
249. T. Klähn, D. Blaschke, S. Typel, E.N.E. van Dalen, A. Faessler, C. Fuchs, T. Gaitanos, H. Grigorian, A. Ho, E.E. Kolomeitsev, M.C. Miller, G. Röpke, J. Trümper, D.N. Voskresensky, F. Weber, H.H. Wolter, Constraints on the high-density nuclear equation of state from the phenomenology of compact stars and heavy-ion collisions. Phys. Rev. C **74**, 035802 (2006). https://doi.org/10.1103/PhysRevC.74.035802
250. E. Flowers, M. Ruderman, P. Sutherland, Neutrino pair emission from finite-temperature neutron superfluid and the cooling of young neutron stars. Astrophys. J. **205**, 541 (1976). https://doi.org/10.1086/154308
251. L.B. Leinson, A. Perez, Vector current conservation and neutrino emission from singlet-paired baryons in neutron stars. Phys. Lett. B **638**, 114 (2006). https://doi.org/10.1016/j.physletb.2006.05.036
252. E.E. Kolomeitsev, D.N. Voskresensky, Neutrino emission due to Cooper-pair recombination in neutron stars revisited. Phys. Rev. C **77**, 065808 (2008). https://doi.org/10.1103/PhysRevC.77.065808
253. J. Knoll, D.N. Voskresensky, Classical and quantum many-body description of bremsstrahlung in dense matter. Ann. Phys. **249**, 532 (1996). https://doi.org/10.1006/aphy.1996.0082
254. E.E. Kolomeitsev, D.N. Voskresensky, Superfluid nucleon matter in and out of equilibrium and weak interactions. Phys. At. Nucl. **74**, 1316 (2011). https://doi.org/10.1134/S1063778811090080
255. M.G. Alford, A. Haber, Z. Zhang, Beyond modified Urca: The nucleon width approximation for flavor-changing processes in dense matter. Phys. Rev. C **110**, L052801 (2024). https://doi.org/10.1103/PhysRevC.110.L052801
256. R.F. Sawyer, A. Soni, Neutrino transport in pion condensed neutron stars. Astrophys. J. **216**, 73 (1977). https://doi.org/10.1086/155446
257. R.F. Sawyer, A. Soni, Transport of neutrinos in hot neutron star matter. Astrophys. J. **230**, 859 (1979). https://doi.org/10.1086/157146
258. B.L. Friman, O.V. Maxwell, Neutron star neutrino emissivities. Astrophys. J. **232**, 541 (1979). https://doi.org/10.1086/157313
259. R.F. Sawyer, Damping of neutron star pulsations by weak interaction processes. Astrophys. J. **237**, 187 (1980). https://doi.org/10.1086/157858
260. S.B. Shaulov, V.A. Ryabov, A.L. Shepetov, S.E. Pyatovsky, V.V. Zhukov, E.A. Kupriyanova, E.N. Gudkova, Strange quark matter and the astrophysical nature of anomalous effects in 1–100 PeV cosmic rays. JETP Lett. **116**, 1 (2022). https://doi.org/10.1134/S002136402260104X
261. E.A. Kupriyanova, S.B. Shaulov, On the cooling time of strange quark stars and possibility of their fast cooling. Ann. Phys. **465**, 169681 (2024). https://doi.org/10.1016/j.aop.2024.169681
262. T. Schäfer, Quark hadron continuity in QCD with one flavor. Phys. Rev. D **62**, 094007 (2000). https://doi.org/10.1103/PhysRevD.62.094007
263. A. Schmitt, Q. Wang, D.H. Rischke, When the transition temperature in color superconductors is not like in BCS theory. Phys. Rev. D **66**, 114010 (2002). https://doi.org/10.1103/PhysRevD.66.114010
264. N. Iwamoto, Axion emission from neutron stars. Phys. Rev. Lett. **53**, 1198 (1984). https://doi.org/10.1103/PhysRevLett.53.1198
265. A. Sedrakian, Axion cooling of neutron stars. II. Beyond hadronic axions. Phys. Rev. D **99**, 043011 (2019). https://doi.org/10.1103/PhysRevD.99.043011
266. E.M. Friedlander, R.W. Gimpel, H.H. Heckman, Y.J. Karant, B. Judek, E. Ganssauge, Anomalous reaction mean free paths of nuclear projectile fragments from heavy ion collisions at 2-AGev. Phys. Rev. C **27**, 1489 (1983). https://doi.org/10.1103/PhysRevC.27.1489
267. B. Bayman, Y. Tang, Anomalons in relativistic heavy-ion collisions. Phys. Rept. **147**, 155 (1987). https://doi.org/10.1016/0370-1573(87)90058-5
268. E.G. Brovman, Y. Kagan, Singularities of multitail ring diagrams for fermi systems. Sov. Phys. JETP **36**, 1025 (1972)
269. E.G. Brovman, A. Kholas, A general method for integration of many-point ring diagrams for fermi systems. Sov. Phys. JETP **39**, 924 (1974)
270. A. Dyugaev, Effects occurring near the critical points of phase transitions in a fermi liquid as illustrated by pion condensation. Sov. Phys. JETP **56**, 567 (1982)
271. N. Kaiser, S. Fritsch, W. Weise, Chiral dynamics and nuclear matter. Nucl. Phys. A **697**, 255 (2002). https://doi.org/10.1016/S0375-9474(01)01231-3
272. G. Röpke, D.N. Voskresensky, I.A. Kryukov, D. Blaschke, Fermi liquid, clustering, and structure factor in dilute warm nuclear matter. Nucl. Phys. A **970**, 224 (2018). https://doi.org/10.1016/j.nuclphysa.2017.11.013
273. G. Röpke, Light nuclei quasiparticle energy shift in hot and dense nuclear matter. Phys. Rev. C **79**, 014002 (2009). https://doi.org/10.1103/PhysRevC.79.014002
274. G. Röpke, Nuclear matter equation of state including two-, three-, and four-nucleon correlations. Phys. Rev. C **92**, 054001 (2015). https://doi.org/10.1103/PhysRevC.92.054001
275. H. Heiselberg, M. Hjorth-Jensen, Phase transitions in neutron stars and maximum masses. Astrophys. J. Lett. **525**, L45 (1999). https://doi.org/10.1086/312321
276. S. Typel, G. Ropke, T. Klahn, D. Blaschke, H.H. Wolter, Composition and thermodynamics of nuclear matter with light clusters. Phys. Rev. C **81**, 015803 (2010). https://doi.org/10.1103/PhysRevC.81.015803
277. G. Röpke, A. Schnell, P. Schuck, P. Nozieres, Four particle condensate in strongly coupled fermion systems. Phys. Rev. Lett. **80**, 3177 (1998). https://doi.org/10.1103/PhysRevLett.80.3177
278. P. Schuck, Y. Funaki, H. Horiuchi, G. Röpke, A. Tohsaki, T. Yamada, Alpha particle clusters and their condensation in nuclear systems. Phys. Scr. **91**, 123001 (2016). https://doi.org/10.1088/0031-8949/91/12/123001
279. J.W. Clark, E. Krotscheck, α-cluster matter reexamined. Phys. Rev. C **109**, 034315 (2024). https://doi.org/10.1103/PhysRevC.109.034315
280. E. E. Kolomeitsev, D. N. Voskresensky, Condensate of excitations in moving superfluids, PTEP **2017**, 023D01 (2017). https://doi.org/10.1093/ptep/ptw182
281. U. Lombardo, P. Nozieres, P. Schuck, H.J. Schulze, A. Sedrakian, Transition from BCS pairing to Bose-Einstein condensation in low density asymmetric nuclear matter. Phys. Rev. C **64**, 064314 (2001). https://doi.org/10.1103/PhysRevC.64.064314
282. D.N. Voskresensky, On the possibility of Bose-condensation of pions in ultrarelativistic collisions of nuclei. J. Exp. Theor. Phys. **78**, 793 (1994)
283. E.E. Kolomeitsev, D.N. Voskresensky, M.E. Borisov, Charge and isospin fluctuations in a non-ideal pion gas with dynamically fixed particle number. Eur. Phys. J. A **57**, 145 (2021). https://doi.org/10.1140/epja/s10050-021-00457-0
284. H.J. Pirner, D.N. Voskresensky, Where to look for pion condensation in heavy ion collisions. Phys. Lett. B **343**, 25 (1995). https://doi.org/10.1016/0370-2693(94)01437-H

285. A. Bulgac, Dilute quantum droplets. Phys. Rev. Lett. **89**, 050402 (2002). https://doi.org/10.1103/PhysRevLett.89.050402
286. E.E. Kolomeitsev, P.D. Lukianov, D.N. Voskresensky, Conserving approximations to dilute equilibrium systems. Pair interaction potential. Phys. Part. Nucl. **55**, 1222 (2024). https://doi.org/10.1134/S1063779624700928
287. T. Faestermann, R. Gernhäuser, Recent results on the tetraneutron, arXiv (2025), arXiv:2506.11623
288. L. Mauviard et al., A NICER view of the 1.4 solar-mass edge-on pulsar PSR J0614–3329. Astrophys. J. **995**, 60 (2025). https://doi.org/10.3847/1538-4357/ae145d
289. S. Frankel, W. Frati, V. Highland, O. Van Dyck, R. Werbeck, Experimental search for condensed nuclear states. Phys. Rev. C **13**, 737 (1976). https://doi.org/10.1103/PhysRevC.13.737
290. A.V. Kulikov, B.M. Pontecorvo, Search for radioactive superdense nuclei. Phys. Lett. B **66**, 136 (1977). https://doi.org/10.1016/0370-2693(77)90159-9
291. P.B. Price, J. Stevenson, Search for ultradense nuclei in relativistic collisions of ar on pb. Phys. Rev. Lett. **34**, 409 (1975). https://doi.org/10.1103/PhysRevLett.34.409
292. K. Frankel, J. Stevenson, Search for ultradense nuclei in relativistic collisions of ar on ca, u, and pb. Phys. Rev. C **14**, 1455 (1976). https://doi.org/10.1103/PhysRevC.14.1455
293. G. Buchwald, L.P. Csernai, G. Gräbner, J.A. Maruhn, W. Greiner, H. Stöcker, Detection of possible density isomers by means of high energetic reaction products. Z. Phys. A **303**, 111 (1981). https://doi.org/10.1007/BF01420267
294. D.M. Jacobs, G.D. Starkman, B.W. Lynn, Macro dark matter. Mon. Not. R. Astron. Soc. **450**, 3418 (2015). https://doi.org/10.1093/mnras/stv774
295. P.B. Price, M.H. Salamon, Search for supermassive magnetic monopoles using mica crystals. Phys. Rev. Lett. **56**, 1226 (1986). https://doi.org/10.1103/PhysRevLett.56.1226
296. P.B. Price, E.K. Shirk, W.Z. Osborne, L.S. Pinsky, Evidence for detection of a moving magnetic monopole. Phys. Rev. Lett. **35**, 487 (1975). https://doi.org/10.1103/PhysRevLett.35.487
297. S. Bertolucci, K. Zioutas, S. Hofmann, M. Maroudas, The Sun and its Planets as detectors for invisible matter. Phys. Dark Univ. **17**, 13 (2017). https://doi.org/10.1016/j.dark.2017.06.001
298. K. Zioutas, A. Argiriou, H. Fischer, S. Hofmann, M. Maroudas, A. Pappa, Y.K. Semertzidis, Stratospheric temperature anomalies as imprints from the dark Universe. Phys. Dark Univ. **28**, 100497 (2020). https://doi.org/10.1016/j.dark.2020.100497
299. K. Zioutas *et al.*, Atmospheric Temperature anomalies as manifestation of the dark Universe (2023) arXiv:2309.10779
300. M. Maroudas, *Signals for invisible matter from solar–terrestrial observations*, Ph.D. thesis (2022),. https://doi.org/10.12681/eadd/51922 arXiv:2404.02290
301. K. Zioutas, M. Maroudas, A. Kosovichev, On the origin of the rhythmic sun's radius variation. Symmetry **14**, 325 (2022). https://doi.org/10.3390/sym14020325
302. T.C. Marshall, M. Stolzenburg, Voltages inside and just above thunderstorms. J. Geophys. Res. Atmos. **106**, 4757 (2001). https://doi.org/10.1029/2000JD900640
303. L.P. Babich, Thunderous neutrons. Phys. Usp. **62**, 976 (2019). https://doi.org/10.3367/UFNe.2018.12.038501
304. R.U. Abbasi et al., The bursts of high energy events observed by the telescope array surface detector. Phys. Lett. A **381**, 2565 (2017). https://doi.org/10.1016/j.physleta.2017.06.022
305. T. Okuda, Telescope array observatory for the high energy radiation induced by lightning. J. Phys. Conference Series **1181**, 012067 (2019). https://doi.org/10.1088/1742-6596/1181/1/012067



E. E. Kolomeitsev is a theoretical physicist. He graduated from the Moscow Engineering Physics Institute (MEPhI), Department of Theoretical Nuclear Physics, in 1993. Defended PhD thesis "Kaon properties in nuclear matter" at the TU Dresden under the supervision of Professor G. Soff. He worked at GSI Darmstadt, ECT* Trento, NBI Copenhagen, University of Minnesota Minneapolis on hadron-hadron interactions, the particle production in heavy-ion collisions, nuclear equation of state and neutron star physics. Between 2008 and 2025 he was lecturing at the Matej Bel University in Slovakia. 2022 he defended habilitation on "Many-body aspects of neutron star physics" at the Comenius University Bratislava. Since 2018 he is also a Leading Researcher at JINR Dubna.



D. N. Voskresensky is a theoretical physicist. He graduated from the Moscow Engineering and Physics Institute (MEPhI), Department of Theoretical Nuclear Physics, in 1974. After postgraduate study, in 1977 he defended dissertation "Charge distribution in abnormal nuclei" under supervision of Professor A. B. Migdal. In 1991 he defended habilitation "Pion degrees of freedom in hot and dense nuclear matter" and got title of Professor in 1992. From 1978 he is lecturing in MEPhI. He worked on problems of quantum field theory of strong fields, theory of phase transitions in equilibrium and nonequilibrium condensed matter, nuclear physics with applications for heavy ion collisions and compact stars. He was long-term Guest Scientist at NBI, Copenhagen, GSI Darmstadt, University of Rostock, Yukawa Institute Kyoto. Since 2020 he is a Principal Researcher at JINR Dubna.