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Positivity and partial wave unitarity bounds on ALP theories via amplitude methods

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Abstract: We derive the complete set of partial wave unitarity bounds on the most general Axion-Like Particle (ALP) effective interactions up to dimension 8 in the limit of large center-of-mass energy. Exploiting a recently developed formalism based on spinor-helicity techniques, we discuss the unitarity bounds for $N \to M$ (with $N, M \geq 2$) scattering amplitudes that can be relevant for ALP searches at colliders or in a variety of rare processes. Moreover, we compute positivity bounds on ALP interactions, emphasizing their complementarity with partial wave unitarity bounds. As a byproduct, we show that our results can be used to infer new positivity constraints in the Standard Model Effective Field Theory.

Keywords: Axions and ALPs, Effective Field Theories, New Light Particles, Scattering Amplitudes

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JHEP07(2026)172

Contents

1 Introduction **1**

2 Partial wave unitarity bounds in the on-shell approach **3**

3 The ALP effective field theory **5**

4 Unitarity bounds **5**

4.1 Dimension-5 and -6 operators 5

4.2 Dimension-7 operators 10

4.3 Dimension-8 operators 12

4.4 Weak-violating ALP-lepton interactions 15

5 Positivity bounds **16**

6 Phenomenological applications **22**

6.1 Dimension-5 couplings to gauge bosons 22

6.2 Dimension-7 couplings in $l^+l^- \to \phi h$ 23

6.3 Weak-violating ALPs and rare decays 23

7 Conclusions **24**

A Conventions **26**

B Amplitudes **27**

B.1 Dimension-5 and -6 interactions 27

B.2 Dimension-7 interactions 28

B.3 Dimension-8 interactions 29

C Positivity bounds in a UV extension **30**

JHEP07(2026)172

1 Introduction

Axion-like particles (ALPs) are light spin-0 bosons emerging in several extensions of the Standard Model (SM) after the spontaneous breaking of an underlying global symmetry at energies much larger than the electroweak scale [1–4]. Therefore, their lightness can be naturally explained by their pseudo-Nambu-Goldstone boson (pNGB) nature. ALPs are a generalization of the QCD axion, as their mass and symmetry breaking scale are independent parameters to be probed by experiment. Remarkably, ALPs can answer several open questions in particle physics, such as the strong CP problem [5–8], the flavor puzzle [9–13], and the electroweak scale naturalness [14]. Furthermore, ALPs can be natural dark matter candidates [15–18]. ALPs with masses below the MeV scale can leave their imprints in several cosmological and astrophysical experimental searches [19–31]. Instead, larger ALP masses can be explored at colliders and through a variety of rare processes [32–39]. ALP interactions with SM fields are typically treated model-independently by means of dimension-5 effective operators [40]. This approach has the advantage of capturing general features of large classes of ultraviolet models without sticking to any particular scenario. Quite recently,

a systematic classification of higher-dimensional ALP operators — up to dimension 8 — appeared in ref. [41] exploiting on-shell techniques [42, 43].

The pNGB nature of ALPs forbids non-derivative interactions with SM fields. As a result, ALP interactions show an inherent growth with energy, making them particularly prone to experience strong unitarity constraints. In particular, partial wave unitarity bounds — stemming from general properties of the scattering S-matrix — provide a theoretically robust way to infer upper limits on the strength of such interactions, thus complementing experimental bounds. Historically, the emergence of unitarity violation for the WW scattering process led to an upper bound on the Higgs boson mass below the TeV scale [44, 45], motivating the construction of the LHC. More recently, unitarity bounds have also been extensively discussed in the context of Effective Field Theories (EFTs) [46–57]. These bounds are inherently perturbative in nature and signal the scale at which new dynamics is expected to emerge, either via additional degrees of freedom or the onset of a strongly coupled phase.

The standard approach to calculate them exploits $2 \to 2$ scattering processes of particles with helicities h_i and proceeds through the following steps: i) expansion of the helicity amplitudes into partial waves $a^J_{h_i}$ with total angular momentum J using the Wigner rotation matrix [58] and ii) diagonalization of the partial wave scattering matrix. As a result, the tightest limit arises from the partial wave unitarity bound $|a^J_{h_i}|_{\max} \lesssim 1$ on the largest eigenvalue. However, the above method is not suited to treat either $2 \to N$ (with $N > 2$) amplitudes, which are of great relevance for future high-energy colliders, or spin-2 or higher-spin theories, relevant for EFTs of gravity, given the difficulty in calculating amplitudes via Feynman rules. By contrast, on-shell methods — which have been shown to be very efficient in capturing EFT ultraviolet effects via renormalization group equations [59–71] — turned out to be the ideal tool also to account for unitarity bounds of EFTs at high energies [72]. In particular, building on a vectorial formalism [73] based on spinor-helicity techniques [74], in ref. [72] we provided the general angular momentum basis for $2 \to 3$ amplitudes thanks to the amplitude-operator correspondence [43, 75–77]. Moreover, we discussed unitarity bounds for some dimension-6 and -8 operators in the Standard Model Effective Field Theory (SMEFT) as well as for spin-2 or higher-spin theories.

The aim of this work is to extensively apply the method of ref. [72] to ALP EFTs up to dimension 8, improving and generalizing previous results in the literature. In particular, we revisit the bounds of ref. [57] relative to dimension-5 operators and include those for weak-violating ALP interactions, which exhibit energy enhancements in various processes such as charged meson decays and W boson decays [78]. Moreover, we address the unitarity bounds for higher-dimensional operators up to dimension 8 recently classified in [41, 79, 80].

Finally, we evaluate the complete set of positivity bounds for ALP EFTs, discussing their complementarity and interplay with partial wave unitarity constraints [72]. Positivity bounds provide a powerful handle to infer the parameter space of consistent EFTs as they follow from requiring the EFT to be the low-energy limit of a unitary, local, and causal quantum field theory [81]. They have been applied across a broad range of frameworks, including the SMEFT [82–104], Higgs EFT [105, 106], and gravitational theories [107–119].

In the following, we outline the structure of the paper. In section 2 we review the method for computing partial wave unitarity bounds in the on-shell approach. In section 3

JHEP07(2026)172

we present the ALP EFT, including effective operators up to dimension 8. In section 4 we systematically present the partial wave unitarity bounds in our scenario, emphasizing the important role of a coupled-channel analysis. In section 5 we discuss positivity bounds for dimension-8 operators and their complementarity with partial wave unitarity bounds. Section 6 is dedicated to some phenomenological applications of our results. Our conclusions are discussed in section 7. In appendix A we list the conventions adopted throughout the paper, in appendix B we list the relevant amplitudes to obtain the most stringent partial wave unitarity bounds reported in the main text, while in appendix C we test the obtained positivity bounds against a specific UV extension.

2 Partial wave unitarity bounds in the on-shell approach

In this section, we review the method for computing partial wave unitarity bounds in the on-shell formalism [72]. The partial wave analysis enables one to project a generic amplitude $|\mathcal{A}_{i\to f}\rangle$ onto a kinematic basis $|\mathcal{B}^J_{i\to f}\rangle$ with definite angular momentum J as follows

$$|\mathcal{A}_{i\to f}\rangle = \sum_J a^J_{i\to f}\,|\mathcal{B}^J_{i\to f}\rangle\ , \tag{2.1}$$

where $a^J_{i\to f}$ are the partial wave coefficients. The building blocks of this decomposition are the Poincaré Clebsch-Gordan coefficients $\mathcal{C}^{J,h}_{\mathcal{I}\to *}$ defined as [73]

$$\langle P,J,h|\mathcal{I}\rangle = \mathcal{C}^{J,h}_{\mathcal{I}\to *}\,\delta^{(4)}\left(P-\sum_{i\in\mathcal{I}}p_i\right)\ , \tag{2.2}$$

that is, the overlap between the multiparticle state $|\mathcal{I}\rangle$ and the Poincaré irreducible multiparticle state $|P,J,h\rangle$, where h refers to its helicity. The coefficients $\mathcal{C}^{J,h}_{\mathcal{I}\to *}$ can be interpreted as elements of the complex vector space $V_{\mathcal{I}\to *}$, i.e., $|\mathcal{C}^{J,h}_{\mathcal{I}\to *}\rangle \in V_{\mathcal{I}\to *}$, and, correspondingly, the angular momentum basis elements read

$$|\mathcal{B}^J_{i\to f}\rangle = \sum_h |\mathcal{C}^{J,h}_{i\to *}\rangle\otimes|\mathcal{C}^{J,h}_{*\to f}\rangle \in V_{i\to f}\,, \tag{2.3}$$

where $|\mathcal{C}^{J,h}_{*\to\mathcal{I}}\rangle = |(\mathcal{C}^{J,h}_{\mathcal{I}\to *})^*\rangle \in V_{*\to\mathcal{I}}$ and $V_{i\to f} = V_{i\to *}\otimes V_{*\to f}$. The partial wave coefficients $a^J_{i\to f}$ can then conveniently be determined by extending the inner product in $V_{\mathcal{I}\to *}$ [120] to the vector space $V_{i\to f}$ as follows

$$a^J_{i\to f} = \frac{1}{2J+1}\langle\mathcal{B}^J_{i\to f}|\mathcal{A}_{i\to f}\rangle = \frac{1}{2J+1}\int \mathrm{d}\Phi_i\,\mathrm{d}\Phi_f\,\left(\mathcal{B}^J_{i\to f}\right)^*\mathcal{A}_{i\to f}\,, \tag{2.4}$$

where $\mathrm{d}\Phi_{\mathcal{I}} = \frac{1}{S_{\mathcal{I}}}\prod_{k\in\mathcal{I}}\frac{\mathrm{d}^3p_k}{(2\pi)^3 2E_k}$ is the Lorentz-invariant phase space measure related to $|\mathcal{I}\rangle$ and $S_{\mathcal{I}}$ is the symmetry factor accounting for identical particles in $|\mathcal{I}\rangle$.[1] Notice that the adopted basis normalization $\langle\mathcal{B}^J_{i\to f}|\mathcal{B}^{J'}_{i\to f}\rangle = (2J+1)\delta^{JJ'}$ has been chosen to obtain the partial wave unitarity bounds in the standard form. Indeed, this choice implies

$$\int \mathrm{d}\Phi_X\ |\mathcal{B}^J_{i\to X}\rangle\otimes|\mathcal{B}^{J'}_{X\to f}\rangle = |\mathcal{B}^J_{i\to f}\rangle\,\delta^{JJ'}\,, \tag{2.5}$$

[1] Explicit parameterizations for the n-body phase space in terms of spinor-helicity variables can be found in [60, 121–124].

JHEP07(2026)172

which, when applied to the generalized optical theorem

$$|\mathcal{A}_{i\to f}\rangle - |\mathcal{A}^*_{f\to i}\rangle = i\sum_X \int \mathrm{d}\Phi_X \; |\mathcal{A}_{i\to X}\rangle \otimes |\mathcal{A}^*_{f\to X}\rangle \;, \tag{2.6}$$

leads to

$$a^J_{i\to f} - \left(a^J_{f\to i}\right)^* = i\sum_X a^J_{i\to X}\left(a^J_{f\to X}\right)^* \tag{2.7}$$

and therefore to the partial wave unitarity bounds

$$|\mathrm{Re}\, a^J_{i\to i}| \leq 1\,, \qquad 0 \leq \mathrm{Im}\, a^J_{i\to i} \leq 2\,, \qquad |a^J_{i\to f}| \leq 1\,, \tag{2.8}$$

where $i \neq f$. The angular momentum basis elements $|\mathcal{B}^J_{i\to f}\rangle$, required by the partial wave decomposition, can be determined without passing through the Poincaré Clebsch-Gordan coefficients $|\mathcal{C}^{J,h}_{\mathcal{I}\to *}\rangle$, via the following steps:

1. Determine a set of kinematic monomials in terms of spinor-helicity variables that are consistent with particle helicities, span $V_{i\to f}$, and are mutually independent after accounting for momentum conservation and Schouten identities.[2]

2. Apply the Pauli-Lubanski operator squared [125]

$$\mathbb{W}^2_{\mathcal{I}} = \frac{1}{8}\mathbb{P}^2_{\mathcal{I}}\left(\epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mathbb{M}_{\mathcal{I},\alpha\beta}\mathbb{M}_{\mathcal{I},\gamma\delta} + \epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\alpha\dot\beta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\gamma\dot\delta}\right) + \frac{1}{4}\mathbb{P}^{\alpha\dot\alpha}_{\mathcal{I}}\mathbb{P}^{\beta\dot\beta}_{\mathcal{I}}\mathbb{M}_{\mathcal{I},\alpha\beta}\widetilde{\mathbb{M}}_{\mathcal{I},\dot\alpha\dot\beta}\,, \tag{2.9}$$

   to these monomials and construct the corresponding matrix. In the previous expression, $\mathcal{I} = i, f$ and

$$\mathbb{P}^{\alpha\dot\alpha}_{\mathcal{I}} = \sum_{i\in\mathcal{I}} \lambda^\alpha_i \widetilde{\lambda}^{\dot\alpha}_i\,, \tag{2.10}$$

$$\mathbb{M}^{\alpha\beta}_{\mathcal{I}} = \sum_{i\in\mathcal{I}}\left(\lambda^\alpha_i \frac{\partial}{\partial\lambda_{i,\beta}} + \lambda^\beta_i \frac{\partial}{\partial\lambda_{i,\alpha}}\right)\,, \tag{2.11}$$

$$\widetilde{\mathbb{M}}^{\dot\alpha\dot\beta}_{\mathcal{I}} = \sum_{i\in\mathcal{I}}\left(\widetilde{\lambda}^{\dot\alpha}_i \frac{\partial}{\partial\widetilde{\lambda}_{i,\dot\beta}} + \widetilde{\lambda}^{\dot\beta}_i \frac{\partial}{\partial\widetilde{\lambda}_{i,\dot\alpha}}\right)\,. \tag{2.12}$$

3. Find the eigenvectors of the above matrix and the associated angular momentum values $J_{\mathcal{I}}$ from the Casimir eigenvalues $-P^2_{\mathcal{I}}J_{\mathcal{I}}(J_{\mathcal{I}}+1)$.

4. Normalize the elements of the determined orthogonal basis so that their norm is $\sqrt{2J_{\mathcal{I}}+1}$.

As a consistency check, for $2\to 2$ scattering, the basis $|\mathcal{B}^J_{i\to f}\rangle$ turns out to be proportional to the Wigner d-matrix [73]. Moreover, the partial wave coefficients for $N\to M$ scattering (with $N, M \geq 2$) reproduce the elements of the reduced scattering matrix [53, 126–128] for the s-wave contributions [72].

[2] Public Mathematica packages allowing one to achieve this goal can be found in [42, 43].

JHEP07(2026)172

$\phi\psi^2 H + \text{h.c.}$		ϕX^2		$\phi^2 H^2 D^2$	
${}^5Q^{pr}_{\phi\ell eH}$	$\phi\,\bar{\ell}_p e_r H$	${}^5Q_{\phi B^2}$	$\phi\,B_{\mu\nu}\widetilde{B}^{\mu\nu}$	${}^6Q_{\phi^2 H^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)(H^\dagger H)$
${}^5Q^{pr}_{\phi quH}$	$\phi\,\bar{q}_p u_r \widetilde{H}$	${}^5Q_{\phi W^2}$	$\phi\,W^I_{\mu\nu}\widetilde{W}^{I\mu\nu}$		
${}^5Q^{pr}_{\phi qdH}$	$\phi\,\bar{q}_p d_r H$	${}^5Q_{\phi G^2}$	$\phi\,G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$		

Table 1. Dimension-5 and -6 ALP operators. Operators with +h.c. have Hermitian conjugates. The indices p, r denote weak eigenstates. The labels of the operator classes contain a cross-reference to the corresponding unitarity bounds.

3 The ALP effective field theory

ALP interactions with SM fermions and gauge bosons are generated starting from dimension-5 effective operators [40]. The ALP EFT Lagrangian takes the form

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \sum_{d,k} {}^dC_k\,{}^dQ_k \tag{3.1}$$

with $d = [{}^dQ_k] = 4 - [{}^dC_k] \in \{5, 6, 7, 8\}$. The complete set of operators invariant under a shift symmetry has been systematically classified in [41, 79, 80] up to mass dimension 8 and is reported in tables 1, 2, and 3.

4 Unitarity bounds

An interesting framework for performing a detailed analysis of unitarity constraints is that of EFTs for ALPs. Indeed, the pNGB nature of the ALP forces its interactions with SM fields to be of derivative type and, therefore, to grow with the energy. As a result, strong unitarity bounds are typically expected [57]. Clearly, as the dimensionality of an operator increases, the energy growth of the interaction also increases, leading to more stringent unitarity constraints. However, the larger the dimensionality of an operator, the smaller its expected phenomenological impact on physical observables. One could then question the approach itself of putting unitarity bounds on higher-dimensional operators if their effects are somehow expected to be subdominant in any case. However, this is not necessarily the case, as was shown for instance in [41] for the production of a Higgs-ALP pair at a lepton collider: the impact of the dimension-7 operators $\partial_\mu\phi\,D^\mu\bar{\ell}He$ and $\partial_\mu\phi\,\bar{\ell}HD_\mu e$ on the total cross section was there shown to be significantly larger than the one due to the dimension-5 operator $\phi\,\bar{\ell}He$ in a wide region of the parameter space. Motivated by these considerations, we stress the possible relevance of placing unitarity constraints on higher-dimensional ALP interactions, which constitutes the subject of this section.

4.1 Dimension-5 and -6 operators

The dimension-5 and -6 ALP operators are reported in table 1.

JHEP07(2026)172

$\phi\psi^2 XD$	
${}^7Q^{(1)pr}_{\phi\ell^2 B}$	$(\partial^\mu\phi)(\bar{\ell}_p\gamma^\nu\ell_r)B_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi\ell^2 B}$	$(\partial^\mu\phi)(\bar{\ell}_p\gamma^\nu\ell_r)\widetilde{B}_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi e^2 B}$	$(\partial^\mu\phi)(\bar{e}_p\gamma^\nu e_r)B_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi e^2 B}$	$(\partial^\mu\phi)(\bar{e}_p\gamma^\nu e_r)\widetilde{B}_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi q^2 B}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu q_r)B_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi q^2 B}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu q_r)\widetilde{B}_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi u^2 B}$	$(\partial^\mu\phi)(\bar{u}_p\gamma^\nu u_r)B_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi u^2 B}$	$(\partial^\mu\phi)(\bar{u}_p\gamma^\nu u_r)\widetilde{B}_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi d^2 B}$	$(\partial^\mu\phi)(\bar{d}_p\gamma^\nu d_r)B_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi d^2 B}$	$(\partial^\mu\phi)(\bar{d}_p\gamma^\nu d_r)\widetilde{B}_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi\ell^2 W}$	$(\partial^\mu\phi)(\bar{\ell}_p\gamma^\nu\sigma^I\ell_r)W^I_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi\ell^2 W}$	$(\partial^\mu\phi)(\bar{\ell}_p\gamma^\nu\sigma^I\ell_r)\widetilde{W}^I_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi q^2 W}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu\sigma^I q_r)W^I_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi q^2 W}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu\sigma^I q_r)\widetilde{W}^I_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi q^2 G}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu\lambda^A q_r)G^A_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi q^2 G}$	$(\partial^\mu\phi)(\bar{q}_p\gamma^\nu\lambda^A q_r)\widetilde{G}^A_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi u^2 G}$	$(\partial^\mu\phi)(\bar{u}_p\gamma^\nu\lambda^A u_r)G^A_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi u^2 G}$	$(\partial^\mu\phi)(\bar{u}_p\gamma^\nu\lambda^A u_r)\widetilde{G}^A_{\mu\nu}$
${}^7Q^{(1)pr}_{\phi d^2 G}$	$(\partial^\mu\phi)(\bar{d}_p\gamma^\nu\lambda^A d_r)G^A_{\mu\nu}$
${}^7Q^{(2)pr}_{\phi d^2 G}$	$(\partial^\mu\phi)(\bar{d}_p\gamma^\nu\lambda^A d_r)\widetilde{G}^A_{\mu\nu}$

$\phi X H^2 D^2$	
${}^7Q^{(1)}_{\phi BH^2}$	$i(\partial_\mu\phi)(H^\dagger\overleftrightarrow{D}_\nu H)B^{\mu\nu}$
${}^7Q^{(2)}_{\phi BH^2}$	$i(\partial_\mu\phi)(H^\dagger\overleftrightarrow{D}_\nu H)\widetilde{B}^{\mu\nu}$
${}^7Q^{(1)}_{\phi WH^2}$	$i(\partial_\mu\phi)(H^\dagger\overleftrightarrow{D}^I_\nu H)W^{I\mu\nu}$
${}^7Q^{(2)}_{\phi WH^2}$	$i(\partial_\mu\phi)(H^\dagger\overleftrightarrow{D}^I_\nu H)\widetilde{W}^{I\mu\nu}$
$\phi\psi^2 H^2 D$	
${}^7Q^{(1)pr}_{\phi\ell^2 H^2}$	$(\partial_\mu\phi)(\bar{\ell}_p\gamma^\mu\ell_r)(H^\dagger H)$
${}^7Q^{(2)pr}_{\phi\ell^2 H^2}$	$(\partial_\mu\phi)(\bar{\ell}_p\gamma^\mu\sigma^I\ell_r)(H^\dagger\sigma^I H)$
${}^7Q^{(1)pr}_{\phi q^2 H^2}$	$(\partial_\mu\phi)(\bar{q}_p\gamma^\mu q_r)(H^\dagger H)$
${}^7Q^{(2)pr}_{\phi q^2 H^2}$	$(\partial_\mu\phi)(\bar{q}_p\gamma^\mu\sigma^I q_r)(H^\dagger\sigma^I H)$
${}^7Q^{pr}_{\phi e^2 H^2}$	$(\partial_\mu\phi)(\bar{e}_p\gamma^\mu e_r)(H^\dagger H)$
${}^7Q^{pr}_{\phi u^2 H^2}$	$(\partial_\mu\phi)(\bar{u}_p\gamma^\mu u_r)(H^\dagger H)$
${}^7Q^{pr}_{\phi d^2 H^2}$	$(\partial_\mu\phi)(\bar{d}_p\gamma^\mu d_r)(H^\dagger H)$
$\phi\psi^2 HD^2 + \text{h.c.}$	
${}^7Q^{(1)pr}_{\phi\ell eH}$	$(\partial_\mu\phi)(\bar{\ell}_p D^\mu e_r H)$
${}^7Q^{(2)pr}_{\phi\ell eH}$	$(\partial_\mu\phi)(D^\mu\bar{\ell}_p e_r H)$
${}^7Q^{(1)pr}_{\phi quH}$	$(\partial_\mu\phi)(\bar{q}_p D^\mu u_r\widetilde{H})$
${}^7Q^{(2)pr}_{\phi quH}$	$(\partial_\mu\phi)(D^\mu\bar{q}_p u_r\widetilde{H})$
${}^7Q^{(1)pr}_{\phi qdH}$	$(\partial_\mu\phi)(\bar{q}_p D^\mu d_r H)$
${}^7Q^{(2)pr}_{\phi qdH}$	$(\partial_\mu\phi)(D^\mu\bar{q}_p d_r H)$
$\phi H^4 D^2$	
${}^7Q_{\phi H^4}$	$i(\partial^\mu\phi)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$

Table 2. Dimension-7 ALP operators. Operators with +h.c. have Hermitian conjugates. The indices p, r denote weak eigenstates. σ^I are the Pauli matrices and λ^A are the Gell-Mann matrices. The labels of the operator classes contain a cross-reference to the corresponding unitarity bounds.

JHEP07(2026)172

$\phi^2 X^2 D^2$		$\phi^2\psi^2 D^3$	
${}^8Q^{(1)}_{\phi^2 B^2}$	$(\partial^\mu\phi\,\partial_\nu\phi)B_{\mu\rho}B^{\nu\rho}$	${}^8Q^{pr}_{\phi^2\ell^2}$	$i(\partial_\mu\phi\,\partial_\nu\phi)(\bar{\ell}_p\gamma^\mu D^\nu\ell_r)$
${}^8Q^{(2)}_{\phi^2 B^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)B_{\nu\rho}B^{\nu\rho}$	${}^8Q^{pr}_{\phi^2 e^2}$	$i(\partial_\mu\phi\,\partial_\nu\phi)(\bar{e}_p\gamma^\mu D^\nu e_r)$
${}^8Q^{(3)}_{\phi^2 B^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)B_{\nu\rho}\widetilde{B}^{\nu\rho}$	${}^8Q^{pr}_{\phi^2 q^2}$	$i(\partial_\mu\phi\,\partial_\nu\phi)(\bar{q}_p\gamma^\mu D^\nu q_r)$
${}^8Q^{(1)}_{\phi^2 W^2}$	$(\partial^\mu\phi\,\partial_\nu\phi)W^I_{\mu\rho}W^{I\nu\rho}$	${}^8Q^{pr}_{\phi^2 u^2}$	$i(\partial_\mu\phi\,\partial_\nu\phi)(\bar{u}_p\gamma^\mu D^\nu u_r)$
${}^8Q^{(2)}_{\phi^2 W^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)W^I_{\nu\rho}W^{I\nu\rho}$	${}^8Q^{pr}_{\phi^2 d^2}$	$i(\partial_\mu\phi\,\partial_\nu\phi)(\bar{d}_p\gamma^\mu D^\nu d_r)$
${}^8Q^{(3)}_{\phi^2 W^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)W^I_{\nu\rho}\widetilde{W}^{I\nu\rho}$	$\phi^2\psi^2 HD^2 + \text{h.c.}$	
${}^8Q^{(1)}_{\phi^2 G^2}$	$(\partial^\mu\phi\,\partial_\nu\phi)G^A_{\mu\rho}G^{A\nu\rho}$	${}^8Q^{pr}_{\phi^2 \ell e H}$	$(\partial_\mu\phi\,\partial^\mu\phi)(\bar{\ell}_p e_r H)$
${}^8Q^{(2)}_{\phi^2 G^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)G^A_{\nu\rho}G^{A\nu\rho}$	${}^8Q^{pr}_{\phi^2 q u H}$	$(\partial_\mu\phi\,\partial^\mu\phi)(\bar{q}_p u_r \widetilde{H})$
${}^8Q^{(3)}_{\phi^2 G^2}$	$(\partial_\mu\phi\,\partial^\mu\phi)G^A_{\nu\rho}\widetilde{G}^{A\nu\rho}$	${}^8Q^{pr}_{\phi^2 q d H}$	$(\partial_\mu\phi\,\partial^\mu\phi)(\bar{q}_p d_r H)$
$\phi^4 D^4$		$\phi^2 H^2 D^4$	
${}^8Q_{\phi^4}$	$(\partial_\mu\phi\,\partial^\mu\phi)^2$	${}^8Q^{(1)}_{\phi^2 H^2}$	$(\partial_\mu\partial_\nu\phi\,\partial^\mu\partial^\nu\phi)(H^\dagger H)$
$\phi^2 H^4 D^2$		${}^8Q^{(2)}_{\phi^2 H^2}$	$(\partial^\mu\phi\,\partial^\nu\phi)(D_\mu H^\dagger D_\nu H)$
${}^8Q_{\phi^2 H^4}$	$(\partial_\mu\phi\,\partial^\mu\phi)(H^\dagger H)^2$		

Table 3. Dimension-8 ALP operators that conserve lepton and baryon numbers. Operators with +h.c. have Hermitian conjugates. The indices p, r denote weak eigenstates. The labels of the operator classes contain a cross-reference to the corresponding unitarity bounds.

ϕX^2 class. The strongest partial wave unitarity bound associated with the interaction ${}^5C_{\phi X^2}\,\phi\, X^{\mathscr{A}\mu\nu}\widetilde{X}^{\mathscr{A}}_{\mu\nu}$, where $X^{\mathscr{A}}_{\mu\nu}$ is the field strength of a generic gauge group G, $\widetilde{X}^{\mathscr{A}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\mathscr{A}\rho\sigma}$ (with $\epsilon^{0123} = 1$), and $\mathscr{A} \in \{1, \ldots, d(G)\}$ is an adjoint index, is

$$\sqrt{s}\left|{}^5C_{\phi X^2}\right| \leq \min\left\{\sqrt{\frac{4\pi}{2\,d(G)-1}}, \sqrt{\frac{8\pi}{1+\sqrt{32\,d(G)+1}}}\right\} \tag{4.1}$$

where $d(G)$ is the dimension of G. This bound is derived from the $J = 0$ partial wave coefficients of the amplitudes for $XX \to XX$ and $XX \to \phi\phi$, constructed with a double insertion of the operator ${}^5Q_{\phi X^2} = \phi\, X^{\mathscr{A}\mu\nu}\widetilde{X}^{\mathscr{A}}_{\mu\nu}$; see appendix B.1 for their expressions.

This implies that the Wilson coefficients associated with the dimension-5 bosonic operators in table 1 are subject to the following individual constraints:[3]

$$\sqrt{s}\left|{}^5C_{\phi B^2}\right| \leq \sqrt{\frac{8\pi}{1+\sqrt{33}}} \approx 1.9\,, \tag{4.2}$$

[3]In ref. [57] it is claimed that, for $\sqrt{s} \geq 260\,\text{GeV}$, the strongest bound on ${}^5C_{\phi W^2}$ arises from the $W^+W^- \to Z(\gamma)\phi$ processes as they are linearly dependent on ${}^5C_{\phi W^2}$ and scale with energy as $s^{3/2}$. By an explicit calculation of the associated scattering amplitudes, both in the broken and unbroken phases of the SM, we find instead a $s^{1/2}$ energy dependence, as it is also manifest by adopting the *equivalent gauge* of [129]. As a result, the processes $XX \to XX$ and $XX \to \phi\phi$ set the most stringent bound on ${}^5C_{\phi W^2}$ at any energy scale.

JHEP07(2026)172

$$\sqrt{s}\left|{}^5C_{\phi W^2}\right| \leq \sqrt{\frac{8\pi}{1+\sqrt{97}}} \approx 1.5\,, \tag{4.3}$$

$$\sqrt{s}\left|{}^5C_{\phi G^2}\right| \leq \sqrt{\frac{4\pi}{15}} \approx 0.92\,. \tag{4.4}$$

For example, the bound for ${}^5C_{\phi B^2}$ is obtained considering the scattering matrix

$$\mathcal{A}_{i\to f} = \begin{pmatrix} -4\,{}^5C^2_{\phi B^2}\,\frac{\langle 34\rangle^2[21]}{\langle 12\rangle} & 0 & 8\,{}^5C^2_{\phi B^2}\,[21]^2 \\ 0 & -4\,{}^5C^2_{\phi B^2}\,\frac{\langle 12\rangle^2[43]}{\langle 34\rangle} & 8\,{}^5C^2_{\phi B^2}\,\langle 12\rangle^2 \\ 8\,{}^5C^2_{\phi B^2}\,\langle 34\rangle^2 & 8\,{}^5C^2_{\phi B^2}\,[43]^2 & 0 \end{pmatrix} \tag{4.5}$$

where $i, f \in \{B^-B^-, B^+B^+, \phi\phi\}$. The associated $J = 0$ partial wave scattering matrix, after including the $1/\sqrt{2}$ factor for identical particle pairs in the initial and final states, is

$$a^0_{i\to f} = \frac{s}{4\pi}\,{}^5C^2_{\phi B^2} \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 2 & 2 & 0 \end{pmatrix}, \tag{4.6}$$

whose eigenvalues are $\frac{-s}{8\pi}(1 \pm \sqrt{33})\,{}^5C^2_{\phi B^2}$ and $\frac{-s}{4\pi}\,{}^5C^2_{\phi B^2}$. In this case, since the gauge bosons are Abelian, we have $\mathcal{A}(B^\pm B^\pm \to B^\mp B^\mp) = 0$. On the other hand, if the gauge bosons are non-Abelian, the associated amplitude for the same helicity configuration no longer vanishes and contributes to having $\frac{-s}{4\pi}(2\,d(G) - 1)\,{}^5C^2_{\phi X^2}$ as an eigenvalue, which corresponds to the first term on the right-hand side of eq. (4.1).

Stronger bounds can be obtained by considering also scattering processes where these three Wilson coefficients are simultaneously non-vanishing. In fact, in that case, the amplitudes for $BB \to WW$, $WW \to GG$, and $BB \to GG$ are each proportional to the product of the two different Wilson coefficients. As a result, if we include them, the bounds on ${}^5C_{\phi B^2}$, ${}^5C_{\phi W^2}$, and ${}^5C_{\phi G^2}$ become correlated and more stringent than when considered independently. The largest eigenvalues of the entire partial wave scattering matrix with $J = 0$ are given by the roots x_i of the cubic and quartic polynomials

$$p(x) = a_0 + a_1 x + a_2 x^2 + x^3\,, \qquad q(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + x^4\,, \tag{4.7}$$

with

$$a_0 = 23\,{}^5C^2_{\phi B^2}\,{}^5C^2_{\phi W^2}\,{}^5C^2_{\phi G^2}\,R^6\,, \tag{4.8}$$

$$a_1 = -\left(17\,{}^5C^2_{\phi B^2}\,{}^5C^2_{\phi G^2} + 7\,{}^5C^2_{\phi B^2}\,{}^5C^2_{\phi W^2} + 21\,{}^5C^2_{\phi W^2}\,{}^5C^2_{\phi G^2}\right) R^4\,, \tag{4.9}$$

$$a_2 = \left({}^5C^2_{\phi B^2} + 5\,{}^5C^2_{\phi W^2} + 15\,{}^5C^2_{\phi G^2}\right) R^2\,, \tag{4.10}$$

and

$$b_0 = -8\,{}^5C^2_{\phi B^2}\,{}^5C^2_{\phi W^2}\,{}^5C^2_{\phi G^2}\left({}^5C^2_{\phi B^2} + 3\,{}^5C^2_{\phi W^2} + 8\,{}^5C^2_{\phi G^2}\right) R^8\,, \tag{4.11}$$

$$\begin{aligned} b_1 = \Big({}^5C^2_{\phi B^2}\,{}^5C^2_{\phi W^2}\,{}^5C^2_{\phi G^2} &- 8\,{}^5C^4_{\phi B^2}\,{}^5C^2_{\phi W^2} - 24\,{}^5C^2_{\phi B^2}\,{}^5C^4_{\phi W^2} - 8\,{}^5C^4_{\phi B^2}\,{}^5C^2_{\phi G^2} \\ &- 24\,{}^5C^4_{\phi W^2}\,{}^5C^2_{\phi G^2} - 64\,{}^5C^2_{\phi B^2}\,{}^5C^4_{\phi G^2} - 64\,{}^5C^2_{\phi W^2}\,{}^5C^4_{\phi G^2}\Big) R^6\,, \end{aligned} \tag{4.12}$$

JHEP07(2026)172

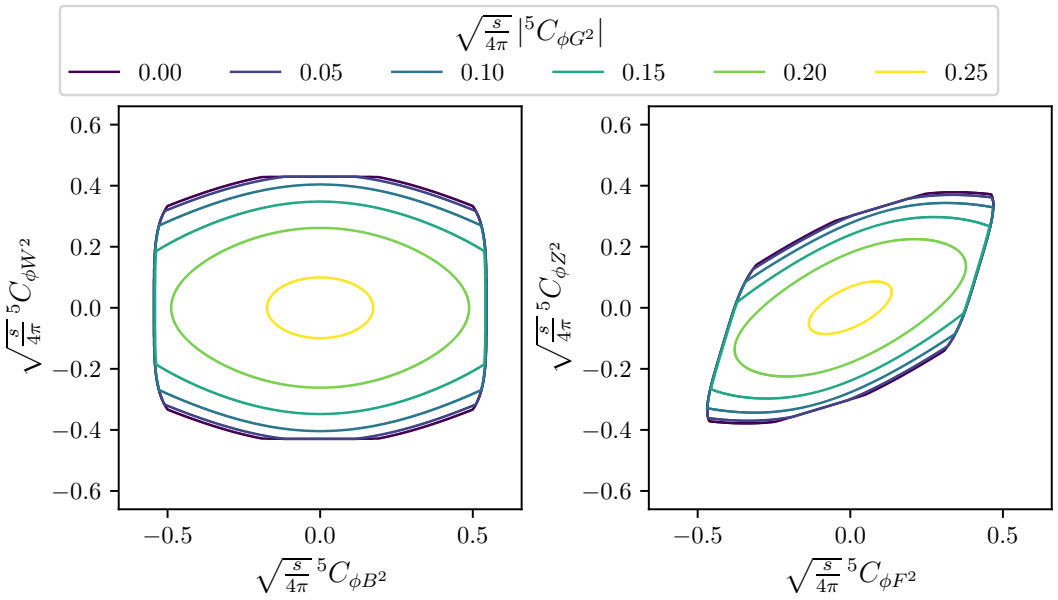


Figure 1. Plots of the boundaries of the parameter space allowed by the partial wave unitarity bounds in eq. (4.15) in the ${}^5C_{\phi B^2}$–${}^5C_{\phi W^2}$ plane (*left panel*) and in the ${}^5C_{\phi F^2}$–${}^5C_{\phi Z^2}$ plane (*right panel*) for different values of $|{}^5C_{\phi G^2}|$. The Wilson coefficients ${}^5C_{\phi F^2}$ and ${}^5C_{\phi Z^2}$ are defined as ${}^5C_{\phi F^2} = c_W^2\,{}^5C_{\phi B^2} + s_W^2\,{}^5C_{\phi W^2}$ and ${}^5C_{\phi Z^2} = s_W^2\,{}^5C_{\phi B^2} + c_W^2\,{}^5C_{\phi W^2}$ (where s_W and c_W are the sine and cosine of the weak mixing angle) and mediate the interactions of the ALP with photons and Z bosons, respectively: $\mathcal{L} \supset {}^5C_{\phi F^2}\,\phi\,F_{\mu\nu}\widetilde{F}^{\mu\nu} + {}^5C_{\phi Z^2}\,\phi\,Z_{\mu\nu}\widetilde{Z}^{\mu\nu}$.

$$b_2 = \left[{}^5C^2_{\phi B^2}\,{}^5C^2_{\phi W^2} + {}^5C^2_{\phi B^2}\,{}^5C^2_{\phi G^2} + {}^5C^2_{\phi W^2}\,{}^5C^2_{\phi G^2} - 8\left({}^5C^4_{\phi B^2} + 3\,{}^5C^4_{\phi W^2} + 8\,{}^5C^4_{\phi G^2}\right)\right] R^4\,, \tag{4.13}$$

$$b_3 = \left({}^5C^2_{\phi B^2} + {}^5C^2_{\phi W^2} + {}^5C^2_{\phi G^2}\right) R^2\,, \tag{4.14}$$

where $R = \sqrt{s/(4\pi)}$. The corresponding unitarity bounds are then found by imposing $|x_i| \leq 1$ on each root. Using the Jury stability criterion [130] (see also [131], Theorems 1.4 and 1.5), we can avoid the explicit calculation of the roots of $p(x)$ and $q(x)$. In fact, $|x_i| \leq 1$ holds for each root of $p(x)$ and $q(x)$ if and only if all the following inequalities are satisfied, respectively:

$$\begin{cases} |a_2 + a_0| \leq 1 + a_1\,, \\ |a_2 - 3a_0| \leq 3 - a_1\,, \\ a_0^2 + a_1 - a_0 a_2 \leq 1\,, \end{cases} \qquad \begin{cases} |b_1 + b_3| \leq 1 + b_0 + b_2\,, \\ |b_1 - b_3| \leq 2(1 - b_0)\,, \\ b_2 - 3b_0 \leq 3\,, \\ b_1^2 + b_2 + b_0(1 + b_0 + b_0 b_2 + b_3^2) \leq 1 + b_0^3 + 2b_0 b_2 + (1 + b_0) b_1 b_3\,. \end{cases} \tag{4.15}$$

The corresponding plots, where we take the intersection of the two regions (which is equivalent to taking the most stringent bound), are shown in figure 1.

***$\phi\psi^2 H$* class.** The individual bounds for this class of Wilson coefficients are

$$s\sum_{p,r}\left|{}^5C^{pr}_{\phi\ell eH}\right|^2 \leq 64\pi^2\,, \qquad s\sum_{p,r}\left|{}^5C^{pr}_{\phi quH}\right|^2 \leq \frac{64}{3}\pi^2\,, \qquad s\sum_{p,r}\left|{}^5C^{pr}_{\phi qdH}\right|^2 \leq \frac{64}{3}\pi^2\,, \tag{4.16}$$

JHEP07(2026)172

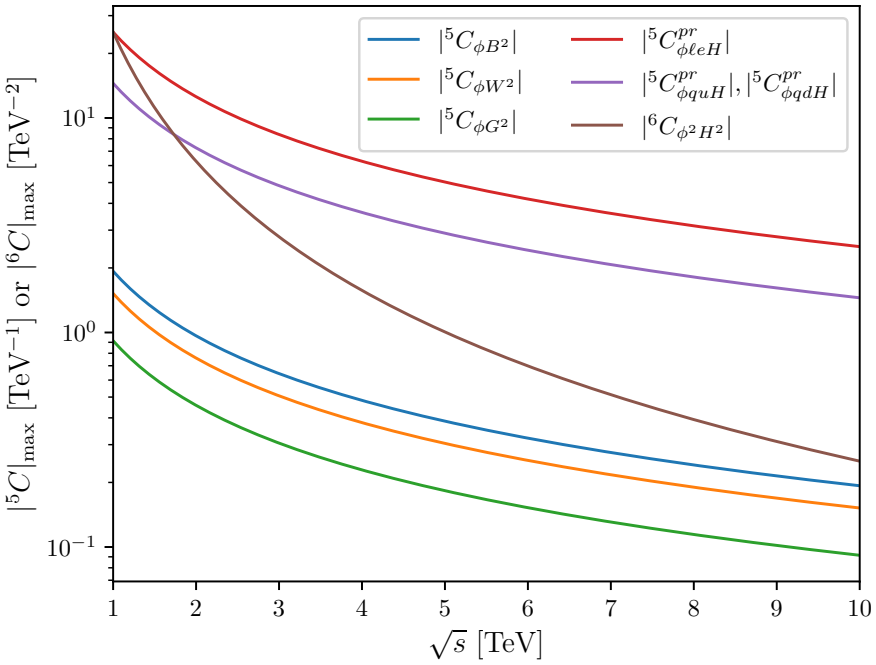


Figure 2. Summary of the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-5 and -6 ALP operators.

while the combined bound is

$$s \sum_{p,r} \left(\left| {}^5C^{pr}_{\phi \ell e H} \right|^2 + 3 \left| {}^5C^{pr}_{\phi q u H} \right|^2 + 3 \left| {}^5C^{pr}_{\phi q d H} \right|^2 \right) \leq 64\pi^2 \,. \tag{4.17}$$

These are derived from the $J = 0$ partial wave coefficients of the amplitudes for $\phi H \to \psi_p \overline{\psi}_r$.

$\phi^2 H^2 D^2$ class. The bound for this Wilson coefficient is

$$s \left| {}^6C_{\phi^2 H^2} \right| \leq 8\pi \,, \tag{4.18}$$

which is derived from the $J = 0$ partial wave coefficients of the amplitudes for $\phi\phi \to H\overline{H}$.

In figure 2, we summarize the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-5 and -6 ALP operators.

4.2 Dimension-7 operators

The dimension-7 ALP operators are reported in table 2.

$\phi \psi^2 H D^2$ class. The individual bounds for this class of Wilson coefficients are

$$s^3 \sum_{p,r} \left| {}^7C^{(1)pr}_{\phi \ell e H} + {}^7C^{(2)pr}_{\phi \ell e H} \right|^2 \leq 1024\pi^2 \,, \tag{4.19}$$

$$s^{3/2} \left| 3\, {}^7C^{(1)pr}_{\phi \ell e H} - {}^7C^{(2)pr}_{\phi \ell e H} \right| \leq 96\pi \,, \qquad s^{3/2} \left| {}^7C^{(1)pr}_{\phi \ell e H} - 3\, {}^7C^{(2)pr}_{\phi \ell e H} \right| \leq 96\pi \,, \tag{4.20}$$

JHEP07(2026)172

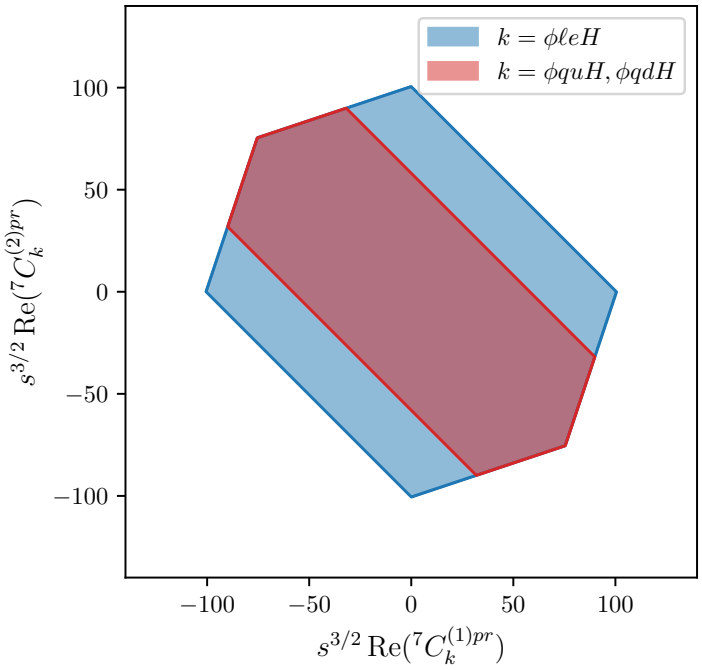


Figure 3. Parameter space allowed by partial wave unitarity bounds in eqs. (4.19)–(4.24) associated with the dimension-7 operators ${}^7Q_k^{(1)pr}$ and ${}^7Q_k^{(2)pr}$, with $k = \phi\ell eH, \phi quH, \phi qdH$ and $p, r = 1, 2, 3$. The imaginary parts of the Wilson coefficients are taken to be zero.

$$s^3 \sum_{p,r} \left| {}^7C_{\phi quH}^{(1)pr} + {}^7C_{\phi quH}^{(2)pr} \right|^2 \leq \frac{1024}{3}\pi^2 \,, \tag{4.21}$$

$$s^{3/2} \left| 3\,{}^7C_{\phi quH}^{(1)pr} - {}^7C_{\phi quH}^{(2)pr} \right| \leq 96\pi \,, \qquad s^{3/2} \left| {}^7C_{\phi quH}^{(1)pr} - 3\,{}^7C_{\phi quH}^{(2)pr} \right| \leq 96\pi \,, \tag{4.22}$$

$$s^3 \sum_{p,r} \left| {}^7C_{\phi qdH}^{(1)pr} + {}^7C_{\phi qdH}^{(2)pr} \right|^2 \leq \frac{1024}{3}\pi^2 \,, \tag{4.23}$$

$$s^{3/2} \left| 3\,{}^7C_{\phi qdH}^{(1)pr} - {}^7C_{\phi qdH}^{(2)pr} \right| \leq 96\pi \,, \qquad s^{3/2} \left| {}^7C_{\phi qdH}^{(1)pr} - 3\,{}^7C_{\phi qdH}^{(2)pr} \right| \leq 96\pi \,, \tag{4.24}$$

while the combined bound is

$$s^3 \sum_{p,r} \left(\left| {}^7C_{\phi\ell eH}^{(1)pr} + {}^7C_{\phi\ell eH}^{(2)pr} \right|^2 + 3 \left| {}^7C_{\phi quH}^{(1)pr} + {}^7C_{\phi quH}^{(2)pr} \right|^2 + 3 \left| {}^7C_{\phi qdH}^{(1)pr} + {}^7C_{\phi qdH}^{(2)pr} \right|^2 \right) \leq 1024\pi^2 \,. \tag{4.25}$$

These are derived from the $J = 0$ partial wave coefficients of the amplitudes for $\phi H \to \psi_p \overline{\psi}_r$ and the $J = 1/2$ partial wave coefficients of the amplitudes for $\phi\psi_p \to H\psi_r$. The bounds are reported in figure 3.

***$\phi\psi^2 XD$* class.** The bounds for the Wilson coefficients belonging to this class are

$$s^3 \sum_{p,r} \left| {}^7C_{\phi\ell^2 B}^{(1)pr} + i\,{}^7C_{\phi\ell^2 B}^{(2)pr} \right|^2 \leq 288\pi^2 \,, \qquad s^3 \sum_{p,r} \left| {}^7C_{\phi e^2 B}^{(1)pr} + i\,{}^7C_{\phi e^2 B}^{(2)pr} \right|^2 \leq 576\pi^2 \,, \tag{4.26}$$

$$s^3 \sum_{p,r} \left| {}^7C_{\phi q^2 B}^{(1)pr} + i\,{}^7C_{\phi q^2 B}^{(2)pr} \right|^2 \leq 96\pi^2 \,, \qquad s^3 \sum_{p,r} \left| {}^7C_{\phi u^2 B}^{(1)pr} + i\,{}^7C_{\phi u^2 B}^{(2)pr} \right|^2 \leq 192\pi^2 \,, \tag{4.27}$$

JHEP07(2026)172

$$s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi d^2B}+i\,{}^7C^{(2)pr}_{\phi d^2B}\right|^2\leq 192\pi^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi \ell^2W}+i\,{}^7C^{(2)pr}_{\phi \ell^2W}\right|^2\leq 288\pi^2\,, \tag{4.28}$$

$$s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi q^2W}+i\,{}^7C^{(2)pr}_{\phi q^2W}\right|^2\leq 96\pi^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi q^2G}+i\,{}^7C^{(2)pr}_{\phi q^2G}\right|^2\leq 144\pi^2\,, \tag{4.29}$$

$$s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi u^2G}+i\,{}^7C^{(2)pr}_{\phi u^2G}\right|^2\leq 288\pi^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi d^2G}+i\,{}^7C^{(2)pr}_{\phi d^2G}\right|^2\leq 288\pi^2\,. \tag{4.30}$$

These are derived from the $J=1$ partial wave coefficients of the amplitudes for $\phi X\to\psi_p\overline{\psi}_r$.

$\phi X H^2 D^2$ class. The bounds for the Wilson coefficients belonging to this class are

$$s^{3/2}\left|{}^7C^{(1)}_{\phi BH^2}+i\,{}^7C^{(2)}_{\phi BH^2}\right|\leq 24\pi\,,\qquad s^{3/2}\left|{}^7C^{(1)}_{\phi WH^2}+i\,{}^7C^{(2)}_{\phi WH^2}\right|\leq 24\pi\,. \tag{4.31}$$

These are derived from the $J=1$ partial wave coefficients of the amplitudes for $\phi X\to H\overline{H}$.

$\phi H^4 D^2$ class. From the $J=0$ partial wave coefficients of the amplitudes for $\phi H\to HH\overline{H}$ we obtain

$$s^{3/2}\left|{}^7C_{\phi H^4}\right|\leq 16\sqrt{6}\pi^2\,. \tag{4.32}$$

$\phi\psi^2 H^2 D$ class. The individual bounds for the Wilson coefficients belonging to this class are

$$s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi \ell^2H^2}\right|^2\leq\left(32\sqrt{6}\pi^2\right)^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{(2)pr}_{\phi \ell^2H^2}\right|^2\leq\left(32\sqrt{6}\pi^2\right)^2\,, \tag{4.33}$$

$$s^3\sum_{p,r}\left|{}^7C^{(1)pr}_{\phi q^2H^2}\right|^2\leq\left(32\sqrt{2}\pi^2\right)^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{(2)pr}_{\phi q^2H^2}\right|^2\leq\left(32\sqrt{2}\pi^2\right)^2\,, \tag{4.34}$$

$$s^3\sum_{p,r}\left|{}^7C^{pr}_{\phi e^2H^2}\right|^2\leq\left(64\sqrt{3}\pi^2\right)^2\,,\qquad s^3\sum_{p,r}\left|{}^7C^{pr}_{\phi u^2H^2}\right|^2\leq(64\pi^2)^2\,,\ s^3\sum_{p,r}\left|{}^7C^{pr}_{\phi d^2H^2}\right|^2\leq(64\pi^2)^2\,. \tag{4.35}$$

They are derived from the $J=0$ partial wave coefficients of the amplitudes for $H\overline{H}\to\phi\psi_p\overline{\psi}_r$. The combined bounds are

$$s^3\sum_{p,r}\left(\left|{}^7C^{(2)pr}_{\phi \ell^2H^2}\right|^2+3\left|{}^7C^{(2)pr}_{\phi q^2H^2}\right|^2\right)\leq\left(32\sqrt{6}\pi^2\right)^2\,, \tag{4.36}$$

$$s^3\sum_{p,r}\left(\left|{}^7C^{pr}_{\phi e^2H^2}\right|^2+3\left|{}^7C^{pr}_{\phi u^2H^2}\right|^2+3\left|{}^7C^{pr}_{\phi d^2H^2}\right|^2+2\left|{}^7C^{(1)pr}_{\phi \ell^2H^2}\right|^2+6\left|{}^7C^{(1)pr}_{\phi q^2H^2}\right|^2\right)\leq\left(64\sqrt{3}\pi^2\right)^2\,. \tag{4.37}$$

In figure 4, we summarize the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-7 ALP operators.

4.3 Dimension-8 operators

The dimension-8 ALP operators that conserve lepton and baryon numbers are reported in table 3.

JHEP07(2026)172

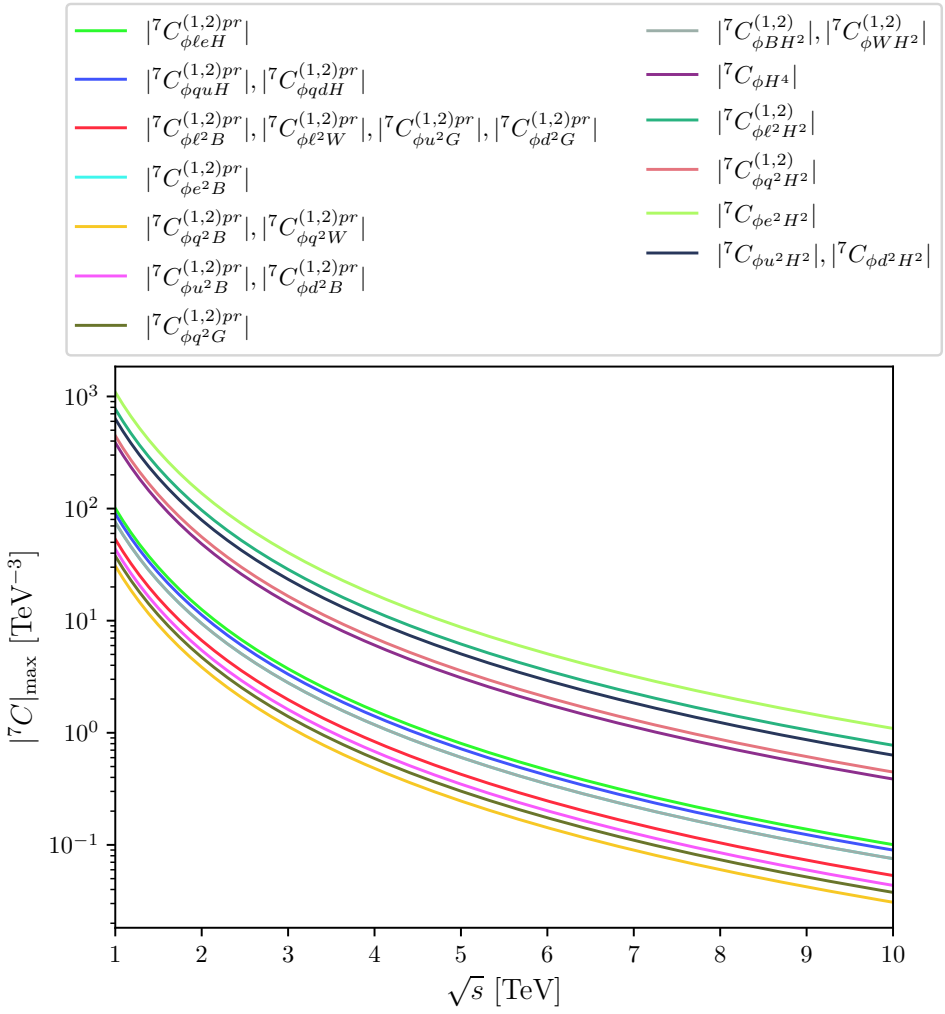


Figure 4. Summary of the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-7 ALP operators.

$\phi^4 D^4$ class. The strongest partial wave unitarity bound for this Wilson coefficient is

$$s^2 \left| {}^8C_{\phi^4} \right| \leq \frac{24}{5}\pi \tag{4.38}$$

and is derived from the $J = 0$ partial wave coefficient of the amplitude for $\phi\phi \to \phi\phi$.

$\phi^2 H^4 D^2$ class. The strongest partial wave unitarity bound for this Wilson coefficient is

$$s^2 \left| {}^8C_{\phi^2 H^4} \right| \leq 128\sqrt{2}\pi^3 \tag{4.39}$$

and is derived from the $J = 0$ partial wave coefficients of the amplitudes for $\phi\phi \to H\overline{H}H\overline{H}$.

$\phi^2 H^2 D^4$ class. The partial wave unitarity bounds for this class of Wilson coefficients are

$$s^2 \left| 3\, {}^8C^{(1)}_{\phi^2 H^2} + {}^8C^{(2)}_{\phi^2 H^2} \right| \leq 48\pi\,, \qquad s^2 \left| {}^8C^{(1)}_{\phi^2 H^2} + 2\, {}^8C^{(2)}_{\phi^2 H^2} \right| \leq 48\pi\,, \tag{4.40}$$

which are derived from the $J = 0$ partial wave coefficients of the amplitudes for $\phi\phi \to H\overline{H}$ and $\phi H \to \phi H$. They are shown in figure 7.

JHEP07(2026)172

$\phi^2 X^2 D^2$ class. The individual bounds for this class of Wilson coefficients are

$$s^2\sqrt{\left({}^8C^{(1)}_{\phi^2B^2}+4\,{}^8C^{(2)}_{\phi^2B^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2B^2}\right)^2}\leq 16\sqrt{2}\pi\,, \tag{4.41}$$

$$s^2\left[8\left|{}^8C^{(1)}_{\phi^2B^2}\right|+3\sqrt{\left({}^8C^{(1)}_{\phi^2B^2}+4\,{}^8C^{(2)}_{\phi^2B^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2B^2}\right)^2}\right]\leq 192\pi\,, \tag{4.42}$$

$$s^2\sqrt{\left({}^8C^{(1)}_{\phi^2W^2}+4\,{}^8C^{(2)}_{\phi^2W^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2W^2}\right)^2}\leq 16\sqrt{\frac{2}{3}}\pi\,, \tag{4.43}$$

$$s^2\left[8\left|{}^8C^{(1)}_{\phi^2W^2}\right|+3\sqrt{\left({}^8C^{(1)}_{\phi^2W^2}+4\,{}^8C^{(2)}_{\phi^2W^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2W^2}\right)^2}\right]\leq 192\pi\,, \tag{4.44}$$

$$s^2\sqrt{\left({}^8C^{(1)}_{\phi^2G^2}+4\,{}^8C^{(2)}_{\phi^2G^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2G^2}\right)^2}\leq 8\pi\,, \tag{4.45}$$

$$s^2\left[8\left|{}^8C^{(1)}_{\phi^2G^2}\right|+3\sqrt{\left({}^8C^{(1)}_{\phi^2G^2}+4\,{}^8C^{(2)}_{\phi^2G^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2G^2}\right)^2}\right]\leq 192\pi\,, \tag{4.46}$$

while the combined one is

$$\begin{aligned}s^4\Big[&\left({}^8C^{(1)}_{\phi^2B^2}+4\,{}^8C^{(2)}_{\phi^2B^2}\right)^2+\left(4\,{}^8C^{(3)}_{\phi^2B^2}\right)^2+3\left({}^8C^{(1)}_{\phi^2W^2}+4\,{}^8C^{(2)}_{\phi^2W^2}\right)^2+3\left(4\,{}^8C^{(3)}_{\phi^2W^2}\right)^2\\&+8\left({}^8C^{(1)}_{\phi^2G^2}+4\,{}^8C^{(2)}_{\phi^2G^2}\right)^2+8\left(4\,{}^8C^{(3)}_{\phi^2G^2}\right)^2\Big]\leq\left(16\sqrt{2}\pi\right)^2\,.\end{aligned} \tag{4.47}$$

These bounds are derived from the $J=0$ partial wave coefficients of the amplitudes for $\phi\phi\to XX$ and the $J=1$ partial wave coefficients of the amplitudes for $\phi X\to\phi X$. They are shown in figure 8.

$\phi^2\psi^2 D^3$ class. The strongest individual bounds are

$$s^2\left|{}^8C^{pr}_{\phi^2\psi^2}\right|\leq\frac{96}{5}\pi\qquad\text{with }\psi=\ell,e,q,u,d \tag{4.48}$$

and are derived from the $J=1/2$ partial wave coefficients of the amplitudes for $\phi\psi_p\to\phi\psi_r$. The collective bound is

$$s^4\sum_{p,r}\left(\left|{}^8C^{pr}_{\phi^2e^2}\right|^2+2\left|{}^8C^{pr}_{\phi^2\ell^2}\right|^2+3\left|{}^8C^{pr}_{\phi^2u^2}\right|^2+3\left|{}^8C^{pr}_{\phi^2d^2}\right|^2+6\left|{}^8C^{pr}_{\phi^2q^2}\right|^2\right)\leq\left(160\sqrt{3}\pi\right)^2 \tag{4.49}$$

and is derived from the $J=2$ partial wave coefficients of the amplitudes for $\phi\phi\to\psi_p\overline{\psi}_r$.

$\phi^2\psi^2 H D^2$ class. The individual bounds for this class of Wilson coefficients are

$$s^4\sum_{p,r}\left|{}^8C^{pr}_{\phi^2\ell eH}\right|^2\leq\left(32\sqrt{3}\pi^2\right)^2\,, \tag{4.50}$$

$$s^4\sum_{p,r}\left|{}^8C^{pr}_{\phi^2quH}\right|^2\leq(32\pi^2)^2\,,\qquad s^4\sum_{p,r}\left|{}^8C^{pr}_{\phi^2qdH}\right|^2\leq(32\pi^2)^2\,. \tag{4.51}$$

The combined one is

$$s^4\sum_{p,r}\left(\left|{}^8C^{pr}_{\phi^2\ell eH}\right|^2+3\left|{}^8C^{pr}_{\phi^2quH}\right|^2+3\left|{}^8C^{pr}_{\phi^2qdH}\right|^2\right)\leq\left(32\sqrt{3}\pi^2\right)^2\,. \tag{4.52}$$

These are derived from the $J=0$ partial wave coefficients of the amplitudes for $\phi\phi\to\psi_p\overline{\psi}_rH$.

In figure 5, we summarize the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-8 ALP operators.

JHEP07(2026)172

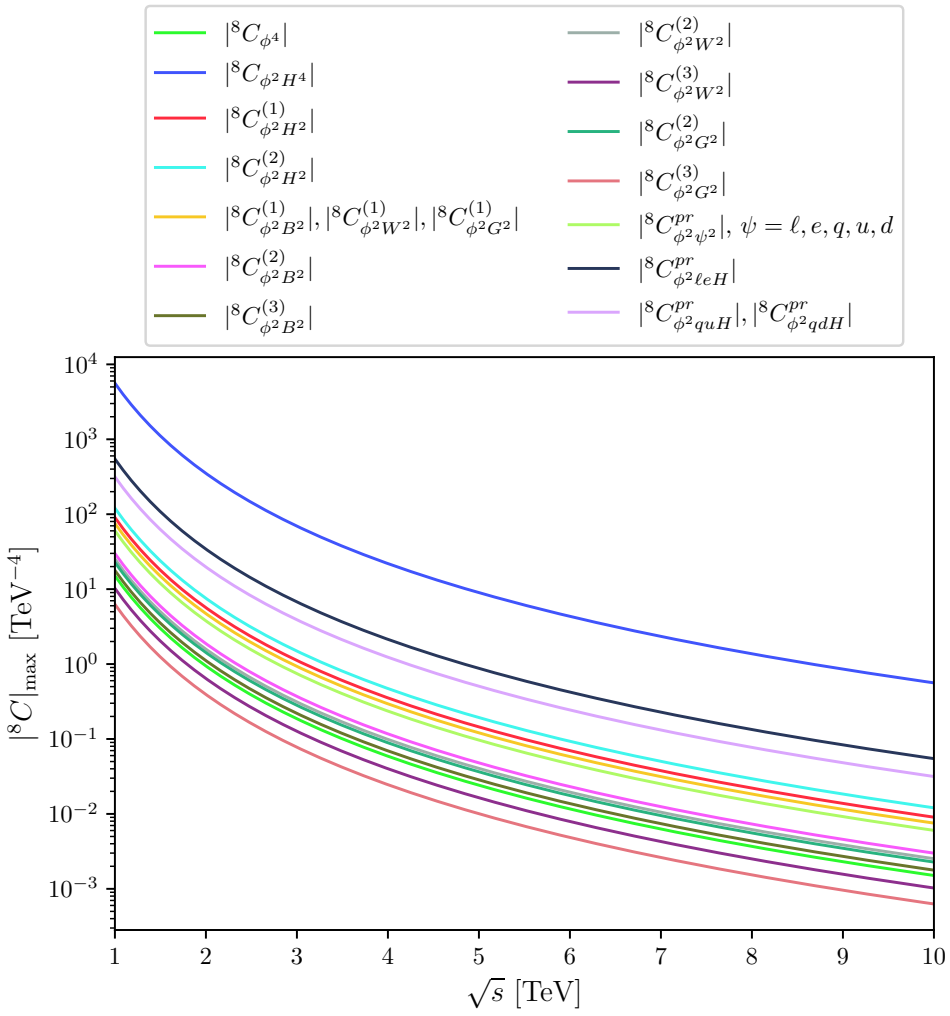


Figure 5. Summary of the marginalized partial wave unitarity bounds on the Wilson coefficients associated with the dimension-8 ALP operators.

4.4 Weak-violating ALP-lepton interactions

The EFT assumed so far was invariant under the electroweak gauge group. If we give up this assumption, ALP-lepton interactions are described by the dimension-5 Lagrangian [78]:

$$\mathcal{L} = \partial_\mu\phi\frac{\overline{g}_{ll}}{2m_l}\bar{l}\gamma^\mu l + \partial_\mu\phi\frac{g_{ll}}{2m_l}\bar{l}\gamma^\mu\gamma_5 l + \partial_\mu\phi\frac{g_{\nu_l}}{2m_l}\overline{\nu}_l\gamma^\mu P_L\nu_l\,, \qquad l = e,\,\mu,\,\tau, \tag{4.53}$$

where the $\mathrm{SU}(2)_L$ invariance is recovered for $g_{\nu_l} = \overline{g}_{ll} - g_{ll}$. The implications of the above interactions are better understood by integrating the Lagrangian by parts [78]:

$$\begin{aligned}-\mathcal{L} = {} & g_{ll}(\bar{l}i\gamma_5 l)\,\phi + \frac{e^2}{16\pi^2 m_l}\left[\frac{\overline{g}_{ll} - g_{ll} + g_{\nu_l}}{4s_W^2}W^+_{\mu\nu}\widetilde{W}^{-,\mu\nu} + \frac{\overline{g}_{ll} - g_{ll}(1-4s_W^2)}{2c_W s_W}F_{\mu\nu}\widetilde{Z}^{\mu\nu}\right.\\ & \left. - g_{ll}F_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{\overline{g}_{ll}(1-4s_W^2) - g_{ll}(1-4s_W^2+8s_W^4) + g_{\nu_l}}{8s_W^2c_W^2}Z_{\mu\nu}\widetilde{Z}^{\mu\nu}\right]\phi\\ & + \frac{ig}{2\sqrt{2}m_l}(g_{ll} - \overline{g}_{ll} + g_{\nu_l})(\bar{l}\gamma^\mu P_L\nu_l)W^-_\mu\,\phi \;+\; \text{h.c.}\,,\end{aligned} \tag{4.54}$$

JHEP07(2026)172

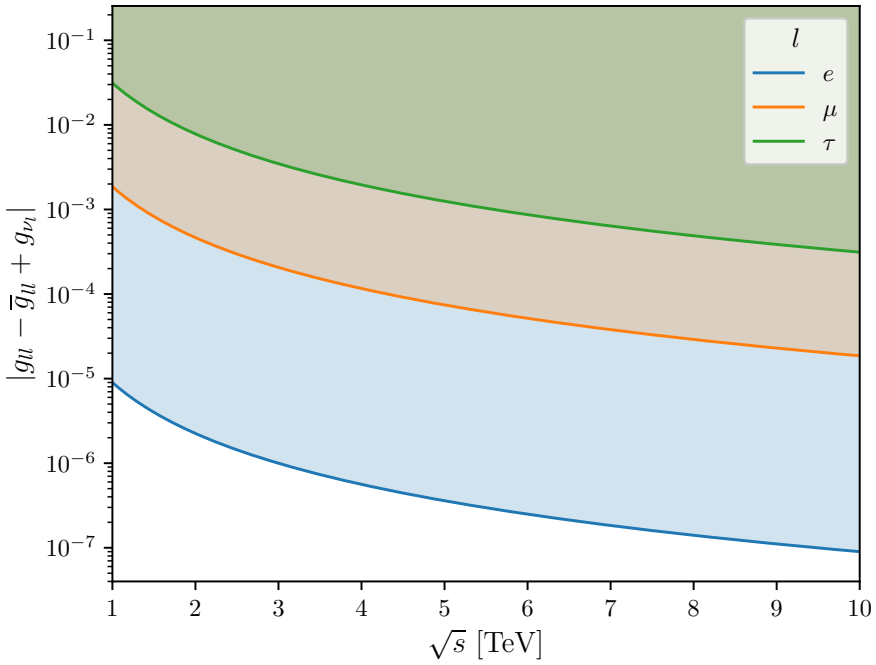


Figure 6. Excluded values of the weak-violating combinations $|g_{ll} - \overline{g}_{ll} + g_{\nu_l}|$ as function of $\sqrt{s}$ by imposing the partial wave unitarity bounds of eq. (4.55).

where s_W is the sine of the weak mixing angle. All terms in eq. (4.54) are generated in the $\mathrm{SU}(2)_L$ symmetric limit, except for the last term, which is a genuine electroweak-violating interaction. As shown in ref. [78], this term generates an (energy/m_l) enhancement in various processes such as charged meson decays and W boson decays.

The most stringent unitarity bound for the weak-violating combination of coefficients $g_{ll} - \overline{g}_{ll} + g_{\nu_l}$ is

$$\frac{s}{m_W m_l} g |g_{ll} - \overline{g}_{ll} + g_{\nu_l}| \leq 32\sqrt{2}\pi \,, \tag{4.55}$$

which is derived from the $J = 1/2$ partial wave coefficient of the amplitude for $\phi l \to W \nu_l$. In particular, we have the following bounds

$$|g_{ll} - \overline{g}_{ll} + g_{\nu_l}| \lesssim 10^{-5} \left(\frac{m_l}{m_e}\right) \left(\frac{\mathrm{TeV}}{\sqrt{s}}\right)^2 , \tag{4.56}$$

which are reported in figure 6.

5 Positivity bounds

Positivity bounds, stemming from the requirement that an EFT is the low-energy limit of a unitary, local, and causal quantum field theory [81], provide another set of conditions to infer the viable parameter space of consistent EFTs. Hereafter, we evaluate the complete set of positivity bounds for ALP EFTs, associated with dimension-8 operators, discussing their complementarity and interplay with partial wave unitarity constraints [72].

JHEP07(2026)172

Positivity bounds are derived from elastic $2 \to 2$ scattering processes whose amplitudes grow with even powers n of s, with $n \geq 2$. By considering the scattering of a superposition of states $|u\rangle = \sum_i u_i |i\rangle$ and $|v\rangle = \sum_j v_j |j\rangle$

$$\mathcal{A}(s,t) = \mathcal{A}(u,v \to u,v) = \sum_{i,j,k,l} u_i^* v_j^* u_k v_l \mathcal{A}(i,j \to k,l)\,, \tag{5.1}$$

we can set in the forward limit [81, 132]

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathcal{A}(s,0)\bigg|_{s=0} \geq 0\,. \tag{5.2}$$

For example, with regard to the operators ${}^8Q^{(1)}_{\phi^2H^2}$ and ${}^8Q^{(2)}_{\phi^2H^2}$, if we take

$$|u\rangle = |\phi\rangle + x\,|\pi_a\rangle\,, \qquad |v\rangle = |\phi\rangle + |\pi_a\rangle\,, \tag{5.3}$$

where π_a is any of the real components of the Higgs doublet

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_1 + i\,\pi_3\\ \pi_2 + i\,\pi_4\end{pmatrix}\,, \tag{5.4}$$

the amplitude is

$$\begin{aligned}\mathcal{A}(s,\theta) &= \mathcal{A}(u,v \to u,v)\\ &= \frac{s^2}{32}\Big[{}^8C^{(1)}_{\phi^2H^2}\,(6+44x+6x^2) + {}^8C^{(2)}_{\phi^2H^2}\,(11+34x+11x^2)\\ &\quad - 4\left(2\,{}^8C^{(1)}_{\phi^2H^2} - {}^8C^{(2)}_{\phi^2H^2}\right)(1-x)^2\cos\theta\\ &\quad + \left(2\,{}^8C^{(1)}_{\phi^2H^2}\,(1+x)^2 + {}^8C^{(2)}_{\phi^2H^2}\,(1+6x+x^2)\right)\cos(2\theta)\Big]\end{aligned} \tag{5.5}$$

and in the forward limit reduces to

$$\mathcal{A}(s,0) = \frac{s^2}{2}\left[4\,{}^8C^{(1)}_{\phi^2H^2}\,x + {}^8C^{(2)}_{\phi^2H^2}\,(1+x)^2\right]\,, \tag{5.6}$$

leading to the bound in eq. (5.13) by imposing $4\,{}^8C^{(1)}_{\phi^2H^2}\,x + {}^8C^{(2)}_{\phi^2H^2}\,(1+x)^2 \geq 0$ for every $x \in \mathbb{R}$.

Another example is provided by the operators ${}^8Q^{(1)}_{\phi^2B^2}$, ${}^8Q^{(2)}_{\phi^2B^2}$, and ${}^8Q^{(3)}_{\phi^2B^2}$. By parameterizing $|u\rangle$ and $|v\rangle$ as

$$|u\rangle = |\phi\rangle + x_-\,|B^-\rangle + x_+\,|B^+\rangle\,, \qquad |v\rangle = |\phi\rangle + y_-\,|B^-\rangle + y_+\,|B^+\rangle\,, \tag{5.7}$$

and by using the amplitudes reported in eqs. (B.43) and (B.44), the second derivative of $\mathcal{A}(s,\theta) = \mathcal{A}(u,v \to u,v)$ with respect to s in the forward limit can be written as

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\mathcal{A}(s,0)\bigg|_{s=0} = \mathbf{V}\,\mathbf{A}\,\mathbf{V}^T\,, \tag{5.8}$$

JHEP07(2026)172

with

$$\mathbf{V} = \left(\mathrm{Re}\, x_- \;\; \mathrm{Im}\, x_- \;\; \mathrm{Re}\, x_+ \;\; \mathrm{Im}\, x_+ \;\; \mathrm{Re}\, y_- \;\; \mathrm{Im}\, y_- \;\; \mathrm{Re}\, y_+ \;\; \mathrm{Im}\, y_+\right) , \tag{5.9}$$

$$\mathbf{A} = \begin{pmatrix} -2\,{}^8C^{(1)}_{\phi^2B^2}\,\mathbb{I}_4 & \mathbf{B} \\ \mathbf{B} & -2\,{}^8C^{(1)}_{\phi^2B^2}\,\mathbb{I}_4 \end{pmatrix} , \tag{5.10}$$

and

$$\mathbf{B} = \begin{pmatrix} {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} & -4\,{}^8C^{(3)}_{\phi^2B^2} & {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} & 4\,{}^8C^{(3)}_{\phi^2B^2} \\ -4\,{}^8C^{(3)}_{\phi^2B^2} & -{}^8C^{(1)}_{\phi^2B^2} - 4\,{}^8C^{(2)}_{\phi^2B^2} & -4\,{}^8C^{(3)}_{\phi^2B^2} & {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} \\ {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} & -4\,{}^8C^{(3)}_{\phi^2B^2} & {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} & 4\,{}^8C^{(3)}_{\phi^2B^2} \\ 4\,{}^8C^{(3)}_{\phi^2B^2} & {}^8C^{(1)}_{\phi^2B^2} + 4\,{}^8C^{(2)}_{\phi^2B^2} & 4\,{}^8C^{(3)}_{\phi^2B^2} & -{}^8C^{(1)}_{\phi^2B^2} - 4\,{}^8C^{(2)}_{\phi^2B^2} \end{pmatrix} . \tag{5.11}$$

Finally, imposing $\mathbf{A} \succeq 0$, one obtains the constraint in eq. (5.15).

In the following, we present the complete set of positivity bounds associated with the dimension-8 ALP operators listed in table 3.

$\phi^4 D^4$ class. The positivity bound related to this operator reads [81, 133]

$${}^8C_{\phi^4} \geq 0\,. \tag{5.12}$$

$\phi^2 H^2 D^4$ class. The positivity bound for this class of Wilson coefficients, obtained by considering all possible superpositions of scattering states, is shown in figure 7 and reads

$${}^8C^{(2)}_{\phi^2H^2} - \left|2\,{}^8C^{(1)}_{\phi^2H^2} + {}^8C^{(2)}_{\phi^2H^2}\right| \geq 0\,, \tag{5.13}$$

whereas the bound obtained without including such superpositions is

$${}^8C^{(2)}_{\phi^2H^2} \geq 0\,. \tag{5.14}$$

They are consistent with those of [134].

$\phi^2 X^2 D^2$ class. The positivity bounds for this class of Wilson coefficients, obtained by considering all possible superpositions of scattering states, are

$${}^8C^{(1)}_{\phi^2X^2} + \sqrt{\left({}^8C^{(1)}_{\phi^2X^2} + 4\,{}^8C^{(2)}_{\phi^2X^2}\right)^2 + \left(4\,{}^8C^{(3)}_{\phi^2X^2}\right)^2} \leq 0 \tag{5.15}$$

for every $X = B, W, G$ and are shown in figure 8, whereas the bounds obtained without including such superpositions are

$${}^8C^{(1)}_{\phi^2X^2} \leq 0\,. \tag{5.16}$$

To highlight the impact of positivity bounds, we compare the volume of the parameter space allowed by unitarity (see eqs. (4.41)–(4.46)), $\mathrm{Vol}(\mathcal{U})$, with the one obtained after

JHEP07(2026)172

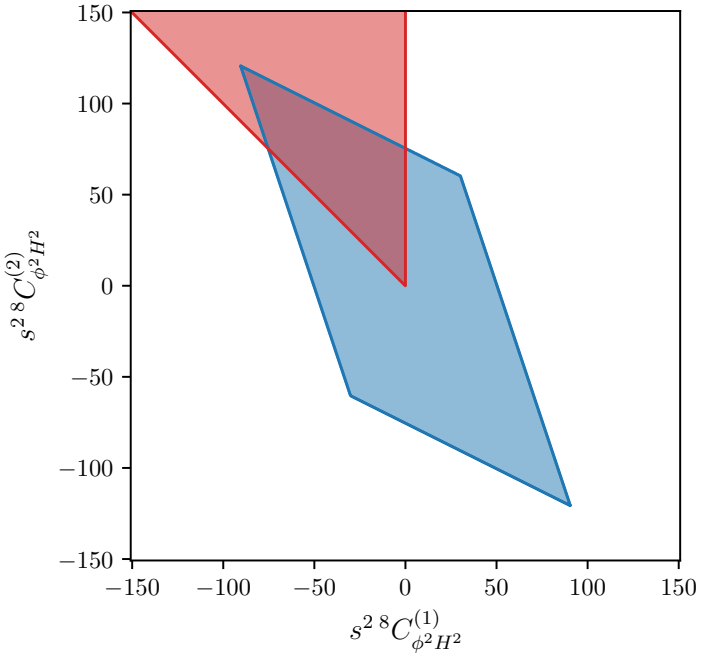


Figure 7. Parameter space allowed by partial wave unitarity bounds (eq. (4.40), blue) and positivity bounds (eq. (5.13), red) associated with the operators ${}^8Q^{(1)}_{\phi^2H^2}$ and ${}^8Q^{(2)}_{\phi^2H^2}$. The intersection area constitutes 1/4 of the area of the blue region.

imposing the positivity constraints in eq. (5.15), $\mathrm{Vol}(\mathcal{U}\cap\mathcal{P})$. In particular, for fixed values of the center-of-mass energy, we find

$$\frac{\mathrm{Vol}(\mathcal{U}\cap\mathcal{P})}{\mathrm{Vol}(\mathcal{U})}=\begin{cases}\dfrac{36\left(6+\sqrt{2}\right)}{2057}\approx 0.13 & \text{if } d(G)=1,\\[2ex] \dfrac{18\sqrt{d(G)}-11\sqrt{2}}{36\sqrt{d(G)}-6\sqrt{2}} & \text{if } d(G)\geq 2,\end{cases} \tag{5.17}$$

depending on the dimensionality $d(G)$ of the associated gauge group G.

These bounds also apply if we substitute the ALP field with a Higgs doublet. If one performs this substitution, one finds the dimension-8 SMEFT operators

$$Q^{(1)}_{X^2H^2D^2}=(D^\mu H^\dagger D^\nu H)X^{\mathcal{A}}_{\mu\rho}X^{\mathcal{A}\rho}_{\nu}\,, \tag{5.18}$$

$$Q^{(2)}_{X^2H^2D^2}=(D^\mu H^\dagger D_\mu H)X^{\mathcal{A}}_{\nu\rho}X^{\mathcal{A}\nu\rho}\,, \tag{5.19}$$

$$Q^{(3)}_{X^2H^2D^2}=(D^\mu H^\dagger D_\mu H)X^{\mathcal{A}}_{\nu\rho}\widetilde{X}^{\mathcal{A}\nu\rho}\,, \tag{5.20}$$

with $X^{\mathcal{A}}_{\mu\nu}=B_{\mu\nu},W^I_{\mu\nu},G^A_{\mu\nu}$, of the basis in [135]. It follows that the associated Wilson coefficients satisfy the constraints

$$C^{(1)}_{X^2H^2D^2}+\sqrt{\left(C^{(1)}_{X^2H^2D^2}+4\,C^{(2)}_{X^2H^2D^2}\right)^2+\left(4\,C^{(3)}_{X^2H^2D^2}\right)^2}\leq 0 \tag{5.21}$$

for every $X=B,W,G$, which represent a generalization of the results presented in [86, 96] that accounts for a non-vanishing CP-odd $C^{(3)}_{X^2H^2D^2}$.

JHEP07(2026)172

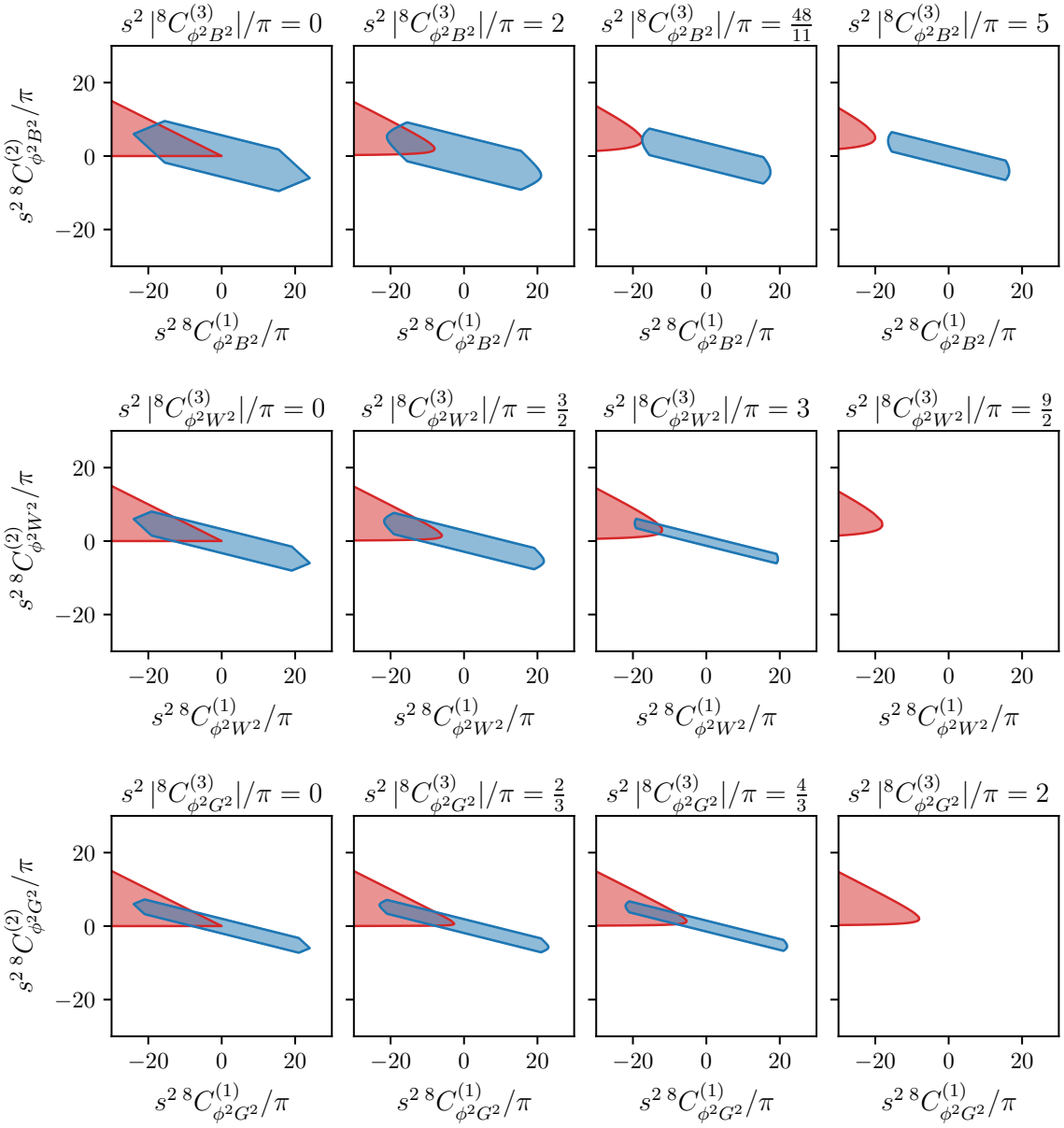


Figure 8. Parameter space allowed by the partial wave unitarity bounds (eqs. (4.41)–(4.46), blue) and the positivity constraints (eq. (5.15), red) regarding the Wilson coefficients belonging to the class $\phi^2X^2D^2$ in the ${}^8C^{(1)}_{\phi^2X^2}$–${}^8C^{(2)}_{\phi^2X^2}$ plane and computed for different values of $\left|{}^8C^{(3)}_{\phi^2X^2}\right|$.

$\phi^2\psi^2D^3$ class. The positivity bounds for this class of Wilson coefficients, obtained by considering all possible superpositions of scattering states, are

$$ {}^8C_{\phi^2\psi^2} \succeq 0\,, \tag{5.22} $$

or, equivalently,

$$ 0 \leq \mathrm{Tr}\left({}^8C_{\phi^2\psi^2}\right) = {}^8C^{11}_{\phi^2\psi^2} + {}^8C^{22}_{\phi^2\psi^2} + {}^8C^{33}_{\phi^2\psi^2}\,, \tag{5.23} $$

JHEP07(2026)172

$$0 \leq \frac{1}{2}\left[\left(\mathrm{Tr}\left({}^{8}C_{\phi^2\psi^2}\right)\right)^2 - \mathrm{Tr}\left({}^{8}C^2_{\phi^2\psi^2}\right)\right] = {}^{8}C^{11}_{\phi^2\psi^2}\,{}^{8}C^{22}_{\phi^2\psi^2} + {}^{8}C^{11}_{\phi^2\psi^2}\,{}^{8}C^{33}_{\phi^2\psi^2} + {}^{8}C^{22}_{\phi^2\psi^2}\,{}^{8}C^{33}_{\phi^2\psi^2} - \left|{}^{8}C^{12}_{\phi^2\psi^2}\right|^2 - \left|{}^{8}C^{13}_{\phi^2\psi^2}\right|^2 - \left|{}^{8}C^{23}_{\phi^2\psi^2}\right|^2 , \tag{5.24}$$

$$0 \leq \det\left({}^{8}C_{\phi^2\psi^2}\right) = {}^{8}C^{11}_{\phi^2\psi^2}\,{}^{8}C^{22}_{\phi^2\psi^2}\,{}^{8}C^{33}_{\phi^2\psi^2} + {}^{8}C^{12}_{\phi^2\psi^2}\,{}^{8}C^{23}_{\phi^2\psi^2}\,{}^{8}C^{31}_{\phi^2\psi^2} + {}^{8}C^{13}_{\phi^2\psi^2}\,{}^{8}C^{32}_{\phi^2\psi^2}\,{}^{8}C^{21}_{\phi^2\psi^2} - {}^{8}C^{11}_{\phi^2\psi^2}\left|{}^{8}C^{23}_{\phi^2\psi^2}\right|^2 - {}^{8}C^{22}_{\phi^2\psi^2}\left|{}^{8}C^{13}_{\phi^2\psi^2}\right|^2 - {}^{8}C^{33}_{\phi^2\psi^2}\left|{}^{8}C^{12}_{\phi^2\psi^2}\right|^2 , \tag{5.25}$$

for every $\psi = \ell, e, q, u, d$. This means that:

- Every diagonal entry of ${}^{8}C_{\phi^2\psi^2}$ must be positive;
- The modulus of the off-diagonal entries of ${}^{8}C_{\phi^2\psi^2}$ is bounded from above by combinations of its diagonal entries.

Instead, if one did not consider all possible superpositions of scattering states, the resulting bound would just involve the diagonal entries:

$${}^{8}C^{pp}_{\phi^2\psi^2} \geq 0\,. \tag{5.26}$$

Also in this case, by substituting the ALP field with a Higgs doublet, one recovers the dimension-8 SMEFT operators

$$Q^{(1)pr}_{\psi^2H^2D^3} = i(\overline{\psi}_p\gamma^\mu D^\nu\psi_r)(D_{(\mu}D_{\nu)}H^\dagger H)\,, \tag{5.27}$$

$$Q^{(2)pr}_{\psi^2H^2D^3} = i(\overline{\psi}_p\gamma^\mu D^\nu\psi_r)(H^\dagger D_{(\mu}D_{\nu)}H)\,, \tag{5.28}$$

with $\psi = \ell, e, q, u, d$, of the basis in [135]. Identifying ${}^{8}C^{pr}_{\phi^2\psi^2}$ with $-\frac{1}{2}C^{(1)pr}_{\psi^2H^2D^3} - \frac{1}{2}C^{(2)pr}_{\psi^2H^2D^3}$, the positivity bounds in eqs. (5.23), (5.24), and (5.25) are translated in

$$C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3} \preceq 0\,, \tag{5.29}$$

or, equivalently,

$$0 \geq \mathrm{Tr}\left(C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3}\right), \tag{5.30}$$

$$0 \leq \left[\mathrm{Tr}\left(C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3}\right)\right]^2 - \mathrm{Tr}\left[\left(C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3}\right)^2\right], \tag{5.31}$$

$$0 \geq \det\left(C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3}\right), \tag{5.32}$$

for every $\psi = \ell, e, q, u, d$, which represent a generalization of the results presented in [96] that accounts for non-vanishing off-diagonal entries of $C^{(1)}_{\psi^2H^2D^3}$ and $C^{(2)}_{\psi^2H^2D^3}$.[4]

[4] According to the results presented in [94–96, 136], one can notice that the positivity bound in eq. (5.29) is consistent with the fact that the one-loop renormalization group equation for the combination $C^{(1)}_{\psi^2H^2D^3} + C^{(2)}_{\psi^2H^2D^3}$, if restricted to the terms stemming from double insertions of dimension-6 operators, is positive semidefinite. In fact, from [137], we have that, e.g., $\frac{\mathrm{d}}{\mathrm{d}\ln\mu}(C^{(1)pr}_{e^2H^2D^3} + C^{(2)pr}_{e^2H^2D^3}) \supset \frac{1}{4\pi^2}C^{pq}_{He}C^{qr}_{He}$, which is positive semidefinite since C_{He} is a Hermitian matrix, being the Wilson coefficient associated with the operator $Q_{He} = i(H^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e)$ [138]. One can check that the same is true for all the other ψ.

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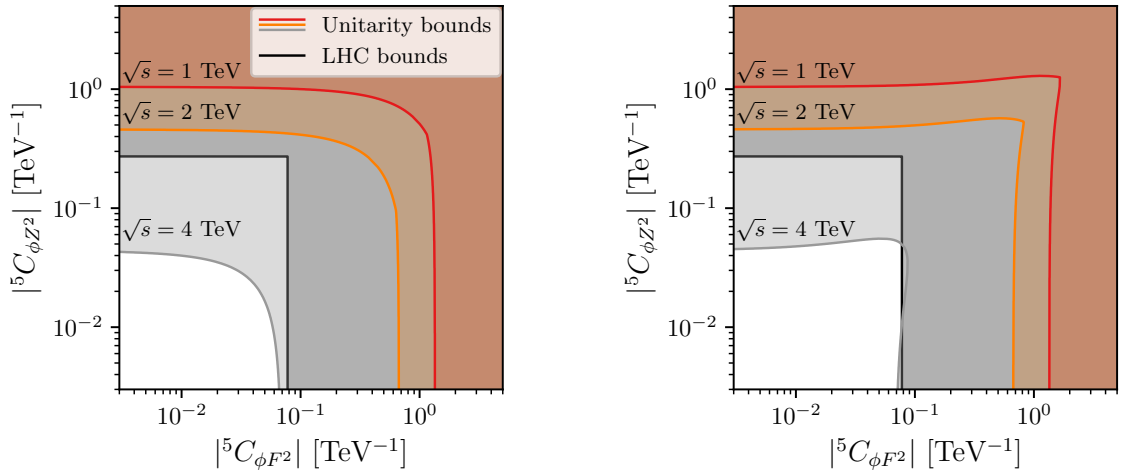


Figure 9. Parameter space excluded by the partial wave unitarity bounds in eq. (4.15) for the dimension-5 ALP couplings to photons (${}^5C_{\phi F^2} = c_W^2\,{}^5C_{\phi B^2} + s_W^2\,{}^5C_{\phi W^2}$) and Z-bosons (${}^5C_{\phi Z^2} = s_W^2\,{}^5C_{\phi B^2} + c_W^2\,{}^5C_{\phi W^2}$), in the cases where ${}^5C_{\phi F^2}\,{}^5C_{\phi Z^2} < 0$ (*left panel*) and ${}^5C_{\phi F^2}\,{}^5C_{\phi Z^2} > 0$ (*right panel*). In both cases the coupling to gluons have been fixed to $\left|{}^5C_{\phi G^2}\right| = 0.225\,\text{TeV}^{-1}$. The black region corresponds to the parameter space excluded by ref. [141] via non-resonant ALP searches at the LHC, where the limits $\left|{}^5C_{\phi F^2}\,{}^5C_{\phi G^2}\right| < 0.018\,\text{TeV}^{-2}$ and $\left|{}^5C_{\phi Z^2}\,{}^5C_{\phi G^2}\right| < 0.061\,\text{TeV}^{-2}$ are found for $m_\phi \lesssim 200\,\text{GeV}$.

We note that the positivity bounds derived above are obtained at tree level. In the SMEFT context, it is known that one-loop corrections can quantitatively modify positivity bounds [94, 99, 102, 139, 140]. In particular, renormalization-group running of the Wilson coefficients induces an energy-scale dependence of the bounds, while loop contributions from light states entering the dispersive integral can shift the resulting constraints. A systematic inclusion of these effects in the ALP-SMEFT framework is beyond the scope of the present work. Nevertheless, the tree-level bounds derived here provide the leading-order constraints implied by analyticity, unitarity, and crossing symmetry.

6 Phenomenological applications

In this section, we provide a few illustrative examples to emphasize the importance of imposing partial wave unitarity bounds. As we will show, the size of physical effects in several observables can be drastically reduced once we assume perturbative unitarity to hold at a given energy scale.

6.1 Dimension-5 couplings to gauge bosons

As a first example, we consider the partial wave unitarity bounds for the dimension-5 ALP couplings to gauge bosons. As discussed in ref. [141], non-resonant ALP searches at the LHC are particularly effective in this case, leading to the excluded black regions of figure 9. Depending on the assumed center-of-mass energy $\sqrt{s}$, unitarity bounds can be competitive or superior to the experimental bounds to probe the ALP parameter space.

JHEP07(2026)172

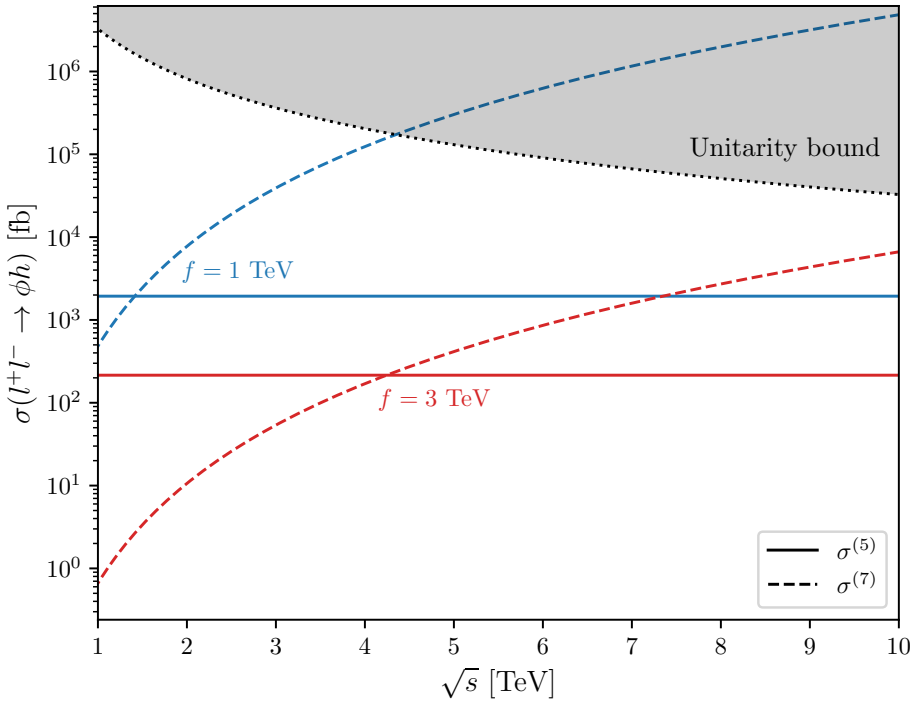


Figure 10. Cross section for the process $l^+l^- \to \phi h$, with $l = e, \mu$, in the high-energy limit. The relevant Wilson coefficients are parameterized as ${}^5C^{ll}_{\phi \ell e H} = i\, y_l/f$ and ${}^7C^{(1)ll}_{\phi \ell e H} = {}^7C^{(2)ll}_{\phi \ell e H} = 1/f^3$, where f can be interpreted as the ALP decay constant.

6.2 Dimension-7 couplings in $l^+l^- \to \phi h$

The relevance of higher-dimensional ALP operators can be nicely illustrated through the process $l^+l^- \to \phi h$ [80], which receives contributions from both dimension-5 and -7 operators. In figure 10, we show the cross section for this process in the high-energy limit. The relevant Wilson coefficients are parameterized as ${}^5C^{ll}_{\phi \ell e H} = i\, y_l/f$ and ${}^7C^{(1)ll}_{\phi \ell e H} = {}^7C^{(2)ll}_{\phi \ell e H} = 1/f^3$, where f can be interpreted as the ALP decay constant.

In the high-energy limit, the cross section of $l^+l^- \to \phi h$ is $\sigma = \sigma^{(5)} + \sigma^{(7)}$, where

$$\sigma^{(7)} = \frac{s^2}{768\pi}\left[\left|{}^7C^{(1)ll}_{\phi \ell e H}\right|^2 + \left|{}^7C^{(2)ll}_{\phi \ell e H}\right|^2 + \mathrm{Re}\left({}^7C^{(1)ll}_{\phi \ell e H}\, {}^7C^{(2)ll\,*}_{\phi \ell e H}\right)\right], \quad \sigma^{(5)} = \frac{1}{64\pi}\left|{}^5C^{ll}_{\phi \ell e H}\right|^2. \tag{6.1}$$

On the other hand, the partial wave unitarity bounds lead to the following conditions

$$\sigma^{(7)} \leq \frac{5\pi}{3s}, \qquad \sigma^{(5)} \leq \frac{\pi}{s}, \tag{6.2}$$

that are illustrated in figure 10. As a result, large effects to the $l^+l^- \to \phi h$ cross section can be significantly reduced depending on the ALP decay constant and the center-of-mass energy.

6.3 Weak-violating ALPs and rare decays

In the case of weak-violating ALP interacting with electrons (with $\overline{g}_{ee} = g_{\nu_e} = 0$), the partial wave unitarity bound reads

$$|g_{ee}| \lesssim 10^{-5}\left(\frac{\mathrm{TeV}}{\sqrt{s}}\right)^2. \tag{6.3}$$

JHEP07(2026)172

Instead, in the weak-preserving case (with $\overline{g}_{ee} = g_{ee}$ and $g_{\nu_e} = 0$), we have

$$|g_{ee}| \lesssim 4 \left(\frac{\text{TeV}}{\sqrt{s}}\right) . \tag{6.4}$$

In order to monitor the impact of the above bounds on physical observables, we consider the branching fractions of $\pi^+ \to e^+\nu_e\phi$, $K^+ \to e^+\nu_e\phi$, and $W^+ \to e^+\nu_e\phi$ decays [78] in the weak-violating case. Neglecting ALP mass effects and taking into account partial wave unitarity bounds, we find that

$$\mathcal{B}(\pi^+ \to e^+\nu_e\phi) \lesssim 4 \times 10^{-9} \left(\frac{\text{TeV}}{\sqrt{s}}\right)^4 , \tag{6.5}$$

$$\mathcal{B}(K^+ \to e^+\nu_e\phi) \lesssim 7 \times 10^{-8} \left(\frac{\text{TeV}}{\sqrt{s}}\right)^4 , \tag{6.6}$$

$$\mathcal{B}(W^+ \to e^+\nu_e\phi) \lesssim 6 \times 10^{-5} \left(\frac{\text{TeV}}{\sqrt{s}}\right)^4 . \tag{6.7}$$

The current bound for the $\pi^+ \to e^+\nu_e\phi$ process arises from the SINDRUM experiment which looked for e^+e^- resonances in $\pi^+ \to e^+\nu_e\phi$ followed by $\phi \to e^+e^-$ with sensitivity to branching ratios of $\mathcal{O}(10^{-10})$ [142]. In the future, the PIONEER experiment aims to reach the sensitivity of $\mathcal{O}(10^{-11})$ [143]. Concerning the $K^+ \to e^+\nu_e\phi$ decay, we exploit the E865 search for the SM process $K^+ \to e^+\nu_e e^+e^-$ [144]. Assuming that the ALP channel does not exceed twice the uncertainty in this measurement, we impose the bound $\mathcal{B}(K^+ \to e^+\nu_e\phi) \leq 4 \times 10^{-9}$ [78]. As a future projection, we assume a reference bound of 10^{-10} [78]. Moreover, following the analysis of ref. [78], we also impose the constraints from leptonic charged meson decays in the CHARM experiment [145]. As for the $W^+ \to e^+\nu_e\phi$ decay, we require this mode not to contribute at a visible level to the total width of the W boson $\Gamma_W = 2.085 \pm 0.042\,\text{GeV}$ [146]. Dedicated analyses for other rare W decays by CMS could improve the bound on the branching ratios of $W^+ \to e^+\nu_e\phi$ at the level of 10^{-6} [78].

As shown in figure 11, unitarity bounds already exclude large regions of the parameter space for $\sqrt{s} = 1\,\text{TeV}$ and are typically stronger than the current experimental bounds on charged meson as well as W boson decays.

7 Conclusions

Axion-like particles (ALPs) are generic pseudo-Nambu-Goldstone bosons (pNGBs) emerging from the spontaneous breaking of some global symmetry above the electroweak scale. The pNGB nature of ALPs forces their interactions with Standard Model fields to be derivative. Therefore, ALP interactions exhibit an inherent growth with the energy, possibly leading to the breakdown of perturbative unitarity at some large energy scale.

In this work, we have systematically evaluated partial wave unitarity bounds for ALP Effective Field Theories (EFTs) including higher-dimensional operators up to dimension 8. The adopted methodology is based on a recently developed formalism, based on on-shell methods, which has proven particularly efficient for studying unitarity bounds of EFTs at high energies [72].

JHEP07(2026)172

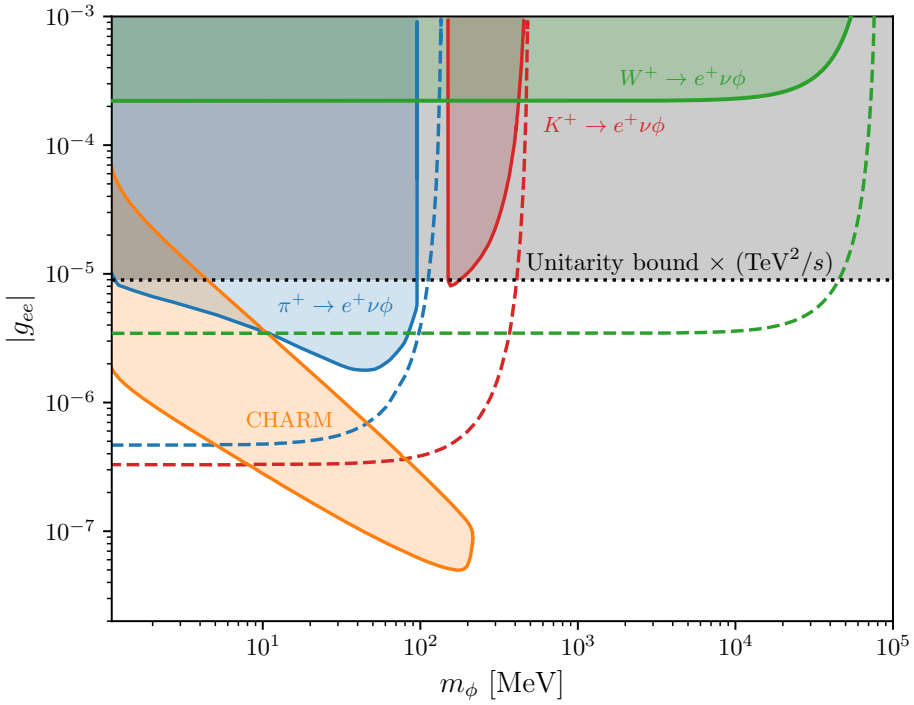


Figure 11. Theoretical and experimental bounds on the coupling g_{ee} of a weak-violating ALP [78]. The partial wave unitarity bound is shown in grey and scales as $1/s$. Searches for leptonic charged meson decays are shown in blue (pions) [143] and red (kaons) [78, 144], while searches for rare W boson decays are shown in green [78, 146]. Bounds from leptonic charged meson decays in the CHARM proton beam dump [145] are shown in orange [78]. Dashed lines refer to the potential sensitivities with dedicated searches at the PIONEER experiment [143] and kaon factories [78].

In particular, we first revisited and improved the bounds of ref. [57] relative to dimension-5 and -6 operators, finding that the processes $XX \to XX$ and $XX \to \phi\phi$ (with $X = B, W, G$) set the most stringent bounds at any energy scale. The summary of the partial wave unitarity bounds on the individual Wilson coefficients associated with these operators is reported in figure 2. Stronger bounds can be obtained through a coupled-channel analysis, where several scattering processes such as $BB \to WW$, $WW \to GG$, and $BB \to GG$ are generated by the simultaneous presence of different Wilson coefficients; see figure 1. As a relevant phenomenological application, we discussed the impact of the above bounds on non-resonant ALP searches at the LHC [141]. Unitarity bounds can be competitive or even more stringent than the experimental bounds already for center-of-mass energies at the TeV scale; see figure 9.

Moreover, we discussed partial wave unitarity bounds for dimension-5 weak-violating ALP interactions, see figure 6, as in this case there is a striking energy enhancement (energy$/m_l$) in several processes such as charged meson and W boson decays [78]. As shown in figure 11, unitarity bounds exclude large regions of the parameter space already for $\sqrt{s} = 1\,\text{TeV}$ and are typically stronger than the current experimental bounds.

Finally, we analyzed the partial wave unitarity bounds for dimension-7 and -8 operators which were recently classified in [41, 79, 80]; see figures 3, 4, and 5. As a phenomenological application, we considered the process $l^+l^- \to \phi h$, which receives contributions from both

JHEP07(2026)172

dimension-5 and -7 operators, and which can be looked for at future high-energy lepton colliders [80]. As shown in figure 10, potentially large effects by dimension-7 operators are severely reduced depending on the ALP decay constant and the center-of-mass energy.

The last part of this work was devoted to the derivation of positivity bounds, stemming from the analyticity and causality of scattering amplitudes, for dimension-8 operators. These were derived from elastic $2 \to 2$ scattering processes that grow with energy as s^2. As shown in figures 7 and 8, positivity and partial wave unitarity bounds are highly complementary in constraining the parameter space of ALP EFTs. Interestingly, our results can also be used to derive new positivity constraints in the SMEFT; see section 5.

To conclude, the positivity and partial wave unitarity bounds derived in our study can be applied in several directions, such as ALP searches at colliders or in a variety of rare decays or even as constraints for the event generation which can impact the shapes of the expected distributions of experimental searches. Our work highlights the synergy and interplay of first-principle-based theoretical bounds and experimental limits in our adventure to discover new physics phenomena.

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A Conventions

Concerning the convention for the Minkowski metric, which is relevant for setting and interpreting the positivity bounds, we adopt the mostly-minus signature.

As for the helicity spinors λ_α and $\widetilde{\lambda}^{\dot{\alpha}}$, they transform in the $(1/2, 0)$ and $(0, 1/2)$ representations of $\mathrm{SL}(2, \mathbb{C})$, respectively. Spinor indices are raised and lowered according to $\lambda^\alpha = \epsilon^{\alpha\beta}\lambda_\beta$ and $\widetilde{\lambda}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\widetilde{\lambda}^{\dot{\beta}}$, where we use the convention $\epsilon^{12} = \epsilon^{\dot{1}\dot{2}} = -\epsilon_{12} = -\epsilon_{\dot{1}\dot{2}} = 1$. The spinor decomposition of a light-like four-momentum p_μ of an outgoing particle is given by

$$p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \widetilde{\lambda}_{\dot{\alpha}} \,, \tag{A.1}$$

where $\sigma^\mu = (1, \vec{\sigma})$ and σ^I are the Pauli matrices. The Lorentz invariant angle and square inner products are then defined as

$$\langle ij \rangle = \lambda_i^\alpha \lambda_{j\,\alpha} = \epsilon_{\alpha\beta}\lambda_i^\alpha \lambda_j^\beta \,, \qquad [ij] = \widetilde{\lambda}_{i\,\dot{\alpha}} \widetilde{\lambda}_j^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}} \widetilde{\lambda}_i^{\dot{\alpha}} \widetilde{\lambda}_j^{\dot{\beta}} \,, \tag{A.2}$$

in such a way that Mandelstam invariants can be written as $s_{ij} = 2p_i \cdot p_j = \langle ij \rangle [ji]$. Within this formalism, polarization vectors can be represented as

$$\varepsilon_-^\mu(p) = \frac{\langle p\, \sigma^\mu\, q]}{\sqrt{2}[pq]} \,, \qquad \varepsilon_+^\mu(p) = \frac{\langle q\, \sigma^\mu\, p]}{\sqrt{2}\langle qp \rangle} \,, \tag{A.3}$$

JHEP07(2026)172

where q is a reference momentum such that $[pq], \langle qp\rangle \neq 0$, while spinors associated with Dirac fermions are

$$u_+(p) = v_-(p) = \begin{pmatrix} \lambda_\alpha \\ 0 \end{pmatrix}, \qquad u_-(p) = v_+(p) = \begin{pmatrix} 0 \\ \widetilde{\lambda}^{\dot\alpha} \end{pmatrix}, \tag{A.4}$$

$$\overline{u}_+(p) = \overline{v}_-(p) = \begin{pmatrix} 0 & \widetilde{\lambda}_{\dot\alpha} \end{pmatrix}, \qquad \overline{u}_-(p) = \overline{v}_+(p) = \begin{pmatrix} \lambda^\alpha & 0 \end{pmatrix}. \tag{A.5}$$

B Amplitudes

In this appendix, we list the relevant amplitudes considered for obtaining the most stringent partial wave unitarity bounds reported in the main text.

All particles are considered outgoing and massless, since we are interested in the high-energy limit, and the superscripts $\pm$ denote the signs of helicities. The following notation is used:

- $p, r, \ldots \in \{1, 2, 3\}$ are weak-eigenstate indices;
- $i, j, \ldots \in \{1, 2\}$ are SU(2) fundamental indices;
- $I, J, \ldots \in \{1, 2, 3\}$ are SU(2) adjoint indices;
- $\alpha, \beta, \ldots \in \{1, 2, 3\}$ are SU(3) fundamental indices;
- $A, B, \ldots \in \{1, \ldots, 8\}$ are SU(3) adjoint indices.

B.1 Dimension-5 and -6 interactions

The relevant amplitudes generated by one or two insertions of dimension-5 operators (see table 1) are

$$\mathcal{A}(\phi, \ell^-_{ip}, \overline{e}^-_r, \overline{H}_j) = {}^5C^{pr}_{\phi\ell eH} \langle 23\rangle \delta_{ij}, \tag{B.1}$$

$$\mathcal{A}(\phi, q^-_{\alpha ip}, \overline{d}^-_{\beta r}, \overline{H}_j) = {}^5C^{pr}_{\phi qdH} \langle 23\rangle \delta_{\alpha\beta}\delta_{ij}, \tag{B.2}$$

$$\mathcal{A}(\phi, q^-_{\alpha ip}, \overline{u}^-_{\beta r}, H_j) = {}^5C^{pr}_{\phi quH} \langle 23\rangle \delta_{\alpha\beta}\epsilon_{ij}, \tag{B.3}$$

$$\mathcal{A}(B^+, B^+, B^-, B^-) = -4\, {}^5C^2_{\phi B^2} \frac{\langle 34\rangle^2 [21]}{\langle 12\rangle}, \tag{B.4}$$

$$\mathcal{A}(\phi, \phi, B^-, B^-) = 8\, {}^5C^2_{\phi B^2} \langle 34\rangle^2, \tag{B.5}$$

$$\mathcal{A}(W^+_I, W^+_J, W^-_K, W^-_L) = -4\, {}^5C^2_{\phi W^2} \frac{\langle 34\rangle^2 [21]}{\langle 12\rangle} \delta^{IJ}\delta^{KL}, \tag{B.6}$$

$$\begin{aligned}\mathcal{A}(W^+_I, W^+_J, W^+_K, W^+_L) = 4\, {}^5C^2_{\phi W^2} \Bigg[&\frac{[21][43]^2}{\langle 12\rangle} \delta^{IJ}\delta^{KL} + \frac{[31][42]^2}{\langle 13\rangle} \delta^{IK}\delta^{JL} \\ &+ \frac{[41][32]^2}{\langle 14\rangle} \delta^{IL}\delta^{JK} \Bigg],\end{aligned} \tag{B.7}$$

$$\mathcal{A}(\phi, \phi, W^-_I, W^-_J) = 8\, {}^5C^2_{\phi W^2} \langle 34\rangle^2 \delta^{IJ}, \tag{B.8}$$

$$\mathcal{A}(G^+_A, G^+_B, G^-_C, G^-_D) = -4\, {}^5C^2_{\phi G^2} \frac{\langle 34\rangle^2 [21]}{\langle 12\rangle} \delta^{AB}\delta^{CD}, \tag{B.9}$$

JHEP07(2026)172

$$\mathcal{A}(G_A^+,G_B^+,G_C^+,G_D^+) = 4\,{}^5C^2_{\phi G^2}\Big[\frac{[21][43]^2}{\langle 12\rangle}\delta^{AB}\delta^{CD}+\frac{[31][42]^2}{\langle 13\rangle}\delta^{AC}\delta^{BD} + \frac{[41][32]^2}{\langle 14\rangle}\delta^{AD}\delta^{BC}\Big]\,, \tag{B.10}$$

$$\mathcal{A}(\phi,\phi,G_A^-,G_B^-) = 8\,{}^5C^2_{\phi G^2}\,\langle 34\rangle^2\delta^{AB}\,, \tag{B.11}$$

$$\mathcal{A}(B^+,B^+,W_I^-,W_J^-) = -4\,{}^5C_{\phi B^2}\,{}^5C_{\phi W^2}\,\frac{\langle 34\rangle^2[21]}{\langle 12\rangle}\delta^{IJ}\,, \tag{B.12}$$

$$\mathcal{A}(B^+,B^+,W_I^+,W_J^+) = 4\,{}^5C_{\phi B^2}\,{}^5C_{\phi W^2}\,\frac{[43]^2[21]}{\langle 12\rangle}\delta^{IJ}\,, \tag{B.13}$$

$$\mathcal{A}(B^+,B^+,G_A^-,G_B^-) = -4\,{}^5C_{\phi B^2}\,{}^5C_{\phi G^2}\,\frac{\langle 34\rangle^2[21]}{\langle 12\rangle}\delta^{AB}\,, \tag{B.14}$$

$$\mathcal{A}(B^+,B^+,G_A^+,G_B^+) = 4\,{}^5C_{\phi B^2}\,{}^5C_{\phi G^2}\,\frac{[43]^2[21]}{\langle 12\rangle}\delta^{AB}\,, \tag{B.15}$$

$$\mathcal{A}(W_I^+,W_J^+,G_A^-,G_B^-) = -4\,{}^5C_{\phi W^2}\,{}^5C_{\phi G^2}\,\frac{\langle 34\rangle^2[21]}{\langle 12\rangle}\delta^{IJ}\delta^{AB}\,, \tag{B.16}$$

$$\mathcal{A}(W_I^+,W_J^+,G_A^+,G_B^+) = 4\,{}^5C_{\phi W^2}\,{}^5C_{\phi G^2}\,\frac{[43]^2[21]}{\langle 12\rangle}\delta^{IJ}\delta^{AB}\,. \tag{B.17}$$

The relevant amplitude generated by the only dimension-6 operator (see table 1) is

$$\mathcal{A}(\phi,\phi,H_i,\overline{H}_j) = -{}^6C_{\phi^2H^2}\,\langle 12\rangle[21]\delta_{ij}\,. \tag{B.18}$$

B.2 Dimension-7 interactions

The relevant amplitudes generated by one insertion of dimension-7 operators (see table 2) are

$$\mathcal{A}(\phi,\ell_{ip}^-,\overline{e}_r^-,\overline{H}_j) = -\frac{1}{2}\left({}^7C^{(1)pr}_{\phi\ell eH}\,\langle 13\rangle[31]+{}^7C^{(2)pr}_{\phi\ell eH}\,\langle 12\rangle[21]\right)\langle 23\rangle\delta_{ij}\,, \tag{B.19}$$

$$\mathcal{A}(\phi,q_{\alpha ip}^-,\overline{d}_{\beta r}^-,\overline{H}_j) = -\frac{1}{2}\left({}^7C^{(1)pr}_{\phi qdH}\,\langle 13\rangle[31]+{}^7C^{(2)pr}_{\phi qdH}\,\langle 12\rangle[21]\right)\langle 23\rangle\delta_{\alpha\beta}\delta_{ij}\,, \tag{B.20}$$

$$\mathcal{A}(\phi,q_{\alpha ip}^-,\overline{u}_{\beta r}^-,H_j) = -\frac{1}{2}\left({}^7C^{(1)pr}_{\phi quH}\,\langle 13\rangle[31]+{}^7C^{(2)pr}_{\phi quH}\,\langle 12\rangle[21]\right)\langle 23\rangle\delta_{\alpha\beta}\epsilon_{ij}\,, \tag{B.21}$$

$$\mathcal{A}(\phi,\ell_{ip}^-,\overline{\ell}_{jr}^+,B^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi\ell^2B}+i\,{}^7C^{(2)pr}_{\phi\ell^2B}\right)\langle 14\rangle\langle 24\rangle[31]\delta_{ij}\,, \tag{B.22}$$

$$\mathcal{A}(\phi,e_p^+,\overline{e}_r^-,B^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi e^2B}+i\,{}^7C^{(2)pr}_{\phi e^2B}\right)\langle 14\rangle\langle 34\rangle[21]\,, \tag{B.23}$$

$$\mathcal{A}(\phi,q_{\alpha ip}^-,\overline{q}_{\beta jr}^+,B^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi q^2B}+i\,{}^7C^{(2)pr}_{\phi q^2B}\right)\langle 14\rangle\langle 24\rangle[31]\delta_{\alpha\beta}\delta_{ij}\,, \tag{B.24}$$

$$\mathcal{A}(\phi,u_{\alpha p}^+,\overline{u}_{\beta r}^-,B^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi u^2B}+i\,{}^7C^{(2)pr}_{\phi u^2B}\right)\langle 14\rangle\langle 34\rangle[21]\delta_{\alpha\beta}\,, \tag{B.25}$$

$$\mathcal{A}(\phi,d_{\alpha p}^+,\overline{d}_{\beta r}^-,B^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi d^2B}+i\,{}^7C^{(2)pr}_{\phi d^2B}\right)\langle 14\rangle\langle 34\rangle[21]\delta_{\alpha\beta}\,, \tag{B.26}$$

$$\mathcal{A}(\phi,\ell_{ip}^-,\overline{\ell}_{jr}^+,W_I^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi\ell^2W}+i\,{}^7C^{(2)pr}_{\phi\ell^2W}\right)\langle 14\rangle\langle 24\rangle[31]\sigma^I_{ij}\,, \tag{B.27}$$

$$\mathcal{A}(\phi,q_{\alpha ip}^-,\overline{q}_{\beta jr}^+,W_I^-) = \frac{1}{\sqrt{2}}\left({}^7C^{(1)pr}_{\phi q^2W}+i\,{}^7C^{(2)pr}_{\phi q^2W}\right)\langle 14\rangle\langle 24\rangle[31]\delta_{\alpha\beta}\sigma^I_{ij}\,, \tag{B.28}$$

JHEP07(2026)172

$$\mathcal{A}(\phi, q^-_{\alpha ip}, \overline{q}^+_{\beta jr}, G^-_A) = \frac{1}{\sqrt{2}} \left({}^7C^{(1)pr}_{\phi q^2 G} + i\, {}^7C^{(2)pr}_{\phi q^2 G} \right) \langle 14\rangle \langle 24\rangle [31] \lambda^A_{\alpha\beta} \delta_{ij} \,, \tag{B.29}$$

$$\mathcal{A}(\phi, u^+_{\alpha p}, \overline{u}^-_{\beta r}, G^-_A) = \frac{1}{\sqrt{2}} \left({}^7C^{(1)pr}_{\phi u^2 G} + i\, {}^7C^{(2)pr}_{\phi u^2 G} \right) \langle 14\rangle \langle 34\rangle [21] \lambda^A_{\alpha\beta} \,, \tag{B.30}$$

$$\mathcal{A}(\phi, d^+_{\alpha p}, \overline{d}^-_{\beta r}, G^-_A) = \frac{1}{\sqrt{2}} \left({}^7C^{(1)pr}_{\phi d^2 G} + i\, {}^7C^{(2)pr}_{\phi d^2 G} \right) \langle 14\rangle \langle 34\rangle [21] \lambda^A_{\alpha\beta} \,, \tag{B.31}$$

$$\mathcal{A}(\phi, H_i, \overline{H}_j, B^-) = \frac{1}{2\sqrt{2}} \left({}^7C^{(1)}_{\phi B H^2} + i\, {}^7C^{(2)}_{\phi B H^2} \right) \langle 14\rangle (\langle 24\rangle [21] - \langle 34\rangle [31]) \delta_{ij} \,, \tag{B.32}$$

$$\mathcal{A}(\phi, H_i, \overline{H}_j, W^-_I) = \frac{1}{2\sqrt{2}} \left({}^7C^{(1)}_{\phi W H^2} + i\, {}^7C^{(2)}_{\phi W H^2} \right) \langle 14\rangle (\langle 24\rangle [21] - \langle 34\rangle [31]) \sigma^I_{ij} \,, \tag{B.33}$$

$$\begin{aligned} \mathcal{A}(\phi, H_i, \overline{H}_j, H_k, \overline{H}_l) = {} & \frac{i}{2}\, {}^7C_{\phi H^4} \left(-\langle 12\rangle [21] + \langle 13\rangle [31] - \langle 14\rangle [41] + \langle 15\rangle [51] \right) \\ & \times (\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}) \,, \end{aligned} \tag{B.34}$$

$$\mathcal{A}(\phi, e^+_p, \overline{e}^-_r, H_i, \overline{H}_j) = i\, {}^7C^{pr}_{\phi e^2 H^2} \langle 13\rangle [21] \delta_{ij} \,, \tag{B.35}$$

$$\mathcal{A}(\phi, u^+_{\alpha p}, \overline{u}^-_{\beta r}, H_i, \overline{H}_j) = i\, {}^7C^{pr}_{\phi u^2 H^2} \langle 13\rangle [21] \delta_{\alpha\beta} \delta_{ij} \,, \tag{B.36}$$

$$\mathcal{A}(\phi, d^+_{\alpha p}, \overline{d}^-_{\beta r}, H_i, \overline{H}_j) = i\, {}^7C^{pr}_{\phi d^2 H^2} \langle 13\rangle [21] \delta_{\alpha\beta} \delta_{ij} \,, \tag{B.37}$$

$$\mathcal{A}(\phi, \ell^-_{ip}, \overline{\ell}^+_{jr}, H_k, \overline{H}_l) = i \left[\left({}^7C^{(1)pr}_{\phi \ell^2 H^2} - {}^7C^{(2)pr}_{\phi \ell^2 H^2} \right) \delta_{ij}\delta_{kl} + 2\, {}^7C^{(2)pr}_{\phi \ell^2 H^2}\, \delta_{il}\delta_{kj} \right] \langle 12\rangle [31] \,, \tag{B.38}$$

$$\mathcal{A}(\phi, q^-_{\alpha ip}, \overline{q}^+_{\beta jr}, H_k, \overline{H}_l) = i \left[\left({}^7C^{(1)pr}_{\phi q^2 H^2} - {}^7C^{(2)pr}_{\phi q^2 H^2} \right) \delta_{ij}\delta_{kl} + 2\, {}^7C^{(2)pr}_{\phi q^2 H^2}\, \delta_{il}\delta_{kj} \right] \langle 12\rangle [31] \delta_{\alpha\beta} \,. \tag{B.39}$$

B.3 Dimension-8 interactions

The relevant amplitudes generated by one insertion of dimension-8 operators (see table 3) are

$$\mathcal{A}(\phi, \phi, \phi, \phi) = 2\, {}^8C_{\phi^4} \left(\langle 12\rangle^2 [21]^2 + \langle 13\rangle^2 [31]^2 + \langle 14\rangle^2 [41]^2 \right) , \tag{B.40}$$

$$\mathcal{A}(\phi, \phi, H_i, \overline{H}_j, H_k, \overline{H}_l) = -2\, {}^8C_{\phi^2 H^4} \langle 12\rangle [21] (\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}) \,, \tag{B.41}$$

$$\mathcal{A}(\phi, \phi, H_i, \overline{H}_j) = \frac{1}{2}\, {}^8C^{(1)}_{\phi^2 H^2} \langle 12\rangle^2 [21]^2 + \frac{1}{4}\, {}^8C^{(2)}_{\phi^2 H^2} \left(\langle 13\rangle^2 [31]^2 + \langle 14\rangle^2 [41]^2 \right) , \tag{B.42}$$

$$\mathcal{A}(\phi, \phi, B^-, B^-) = 2 \left(\frac{1}{4}\, {}^8C^{(1)}_{\phi^2 B^2} + {}^8C^{(2)}_{\phi^2 B^2} + i\, {}^8C^{(3)}_{\phi^2 B^2} \right) \langle 34\rangle^3 [43] \,, \tag{B.43}$$

$$\mathcal{A}(\phi, \phi, B^-, B^+) = {}^8C^{(1)}_{\phi^2 B^2} \langle 13\rangle^2 [41]^2 \,, \tag{B.44}$$

$$\mathcal{A}(\phi, \phi, W^-_I, W^-_J) = 2 \left(\frac{1}{4}\, {}^8C^{(1)}_{\phi^2 W^2} + {}^8C^{(2)}_{\phi^2 W^2} + i\, {}^8C^{(3)}_{\phi^2 W^2} \right) \langle 34\rangle^3 [43] \delta^{IJ} \,, \tag{B.45}$$

$$\mathcal{A}(\phi, \phi, W^-_I, W^+_J) = {}^8C^{(1)}_{\phi^2 W^2} \langle 13\rangle^2 [41]^2 \delta^{IJ} \,, \tag{B.46}$$

$$\mathcal{A}(\phi, \phi, G^-_A, G^-_B) = 2 \left(\frac{1}{4}\, {}^8C^{(1)}_{\phi^2 G^2} + {}^8C^{(2)}_{\phi^2 G^2} + i\, {}^8C^{(3)}_{\phi^2 G^2} \right) \langle 34\rangle^3 [43] \delta^{AB} \,, \tag{B.47}$$

$$\mathcal{A}(\phi, \phi, G^-_A, G^+_B) = {}^8C^{(1)}_{\phi^2 G^2} \langle 13\rangle^2 [41]^2 \delta^{AB} \,, \tag{B.48}$$

$$\mathcal{A}(\phi, \phi, \ell^-_{ip}, \overline{\ell}^+_{jr}) = \frac{1}{2}\, {}^8C^{pr}_{\phi^2 \ell^2} \left(\langle 13\rangle \langle 14\rangle [41]^2 + \langle 23\rangle \langle 24\rangle [42]^2 \right) \delta_{ij} \,, \tag{B.49}$$

$$\mathcal{A}(\phi, \phi, e^+_p, \overline{e}^-_r) = \frac{1}{2}\, {}^8C^{pr}_{\phi^2 e^2} \left(\langle 13\rangle \langle 14\rangle [31]^2 + \langle 23\rangle \langle 24\rangle [32]^2 \right) , \tag{B.50}$$

$$\mathcal{A}(\phi, \phi, q^-_{\alpha ip}, \overline{q}^+_{\beta jr}) = \frac{1}{2}\, {}^8C^{pr}_{\phi^2 q^2} \left(\langle 13\rangle \langle 14\rangle [41]^2 + \langle 23\rangle \langle 24\rangle [42]^2 \right) \delta_{\alpha\beta} \delta_{ij} \,, \tag{B.51}$$

JHEP07(2026)172

$$\mathcal{A}(\phi,\phi,u^+_{\alpha p},\overline{u}^-_{\beta r}) = \frac{1}{2}\,{}^8C^{pr}_{\phi^2u^2}\left(\langle 13\rangle\langle 14\rangle[31]^2+\langle 23\rangle\langle 24\rangle[32]^2\right)\delta_{\alpha\beta}\,, \tag{B.52}$$

$$\mathcal{A}(\phi,\phi,d^+_{\alpha p},\overline{d}^-_{\beta r}) = \frac{1}{2}\,{}^8C^{pr}_{\phi^2d^2}\left(\langle 13\rangle\langle 14\rangle[31]^2+\langle 23\rangle\langle 24\rangle[32]^2\right)\delta_{\alpha\beta}\,, \tag{B.53}$$

$$\mathcal{A}(\phi,\phi,\ell^-_{ip},\overline{e}^-_r,\overline{H}_j) = -\,{}^8C^{pr}_{\phi^2\ell eH}\,\langle 12\rangle\langle 34\rangle[21]\delta_{ij}\,, \tag{B.54}$$

$$\mathcal{A}(\phi,\phi,q^-_{\alpha ip},\overline{d}^-_{\beta r},\overline{H}_j) = -\,{}^8C^{pr}_{\phi^2qdH}\,\langle 12\rangle\langle 34\rangle[21]\delta_{\alpha\beta}\delta_{ij}\,, \tag{B.55}$$

$$\mathcal{A}(\phi,\phi,q^-_{\alpha ip},\overline{u}^-_{\beta r},H_j) = -\,{}^8C^{pr}_{\phi^2quH}\,\langle 12\rangle\langle 34\rangle[21]\delta_{\alpha\beta}\epsilon_{ij}\,. \tag{B.56}$$

C Positivity bounds in a UV extension

To test the validity of the positivity bounds presented in section 5, we match a UV theory to the ALP EFT. A simple way to generate at tree level the operators subject to the positivity constraints consists in integrating out a massive spin-2 field $h_{\mu\nu}$ *minimally* coupled to the stress-energy tensor $T_{\mu\nu}$ [147, 148]:

$$\mathcal{L} = \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} + \partial_\mu h^{\mu\nu}\partial_\nu h - \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{M^2}{2}(h_{\mu\nu}h^{\mu\nu}-h^2) + \frac{c}{\Lambda}h^{\mu\nu}T_{\mu\nu} \tag{C.1}$$

where $h = h^\mu_\mu$ is the trace of the field, c is a real dimensionless coefficient, and Λ is an energy scale much higher than the ALP EFT cutoff scale. Regarding the stress-energy tensor, we can consider

$$T_{\mu\nu} = T^{(\phi)}_{\mu\nu} + T^{(H)}_{\mu\nu} + \sum_{\psi=\ell,e,q,u,d} T^{(\psi)}_{\mu\nu} + \sum_{X=B,W,G} T^{(X)}_{\mu\nu} \tag{C.2}$$

with

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\,\partial^\rho\phi\,, \tag{C.3}$$

$$T^{(H)}_{\mu\nu} = D_\mu H^\dagger D_\nu H + D_\nu H^\dagger D_\mu H - g_{\mu\nu}D_\rho H^\dagger D^\rho H\,, \tag{C.4}$$

$$T^{(\psi)}_{\mu\nu} = \kappa^{(\psi)}_{pr}\left[\frac{i}{4}\left(\overline{\psi}_p\gamma_\mu\overleftrightarrow{D}_\nu\psi_r + \overline{\psi}_p\gamma_\nu\overleftrightarrow{D}_\mu\psi_r\right) - g_{\mu\nu}\overline{\psi}_p i\not{D}\psi_r\right]\,, \tag{C.5}$$

$$T^{(X)}_{\mu\nu} = -X^{\mathcal{A}}_{\mu\rho}X^{\mathcal{A}\,\rho}_{\nu} + \frac{1}{4}g_{\mu\nu}X^{\mathcal{A}}_{\rho\sigma}X^{\mathcal{A}\rho\sigma}\,. \tag{C.6}$$

The matrices $\kappa^{(\psi)}$ must be Hermitian and positive definite ($\kappa^{(\psi)} \succ 0$) to ensure that unitarity is not violated.

Integrating out at tree level $h_{\mu\nu}$, we generate dimension-8 operators that belong both to the SMEFT and to the ALP EFT. Concerning the latter, the associated Wilson coefficients take the following values:

$${}^8C_{\phi^4} = \frac{1}{3}\left(\frac{c}{\Lambda M}\right)^2\,, \tag{C.7}$$

$${}^8C^{(1)}_{\phi^2H^2} = -\frac{2}{3}\left(\frac{c}{\Lambda M}\right)^2\,, \qquad {}^8C^{(2)}_{\phi^2H^2} = 2\left(\frac{c}{\Lambda M}\right)^2\,, \tag{C.8}$$

$${}^8C^{(1)}_{\phi^2X^2} = -\left(\frac{c}{\Lambda M}\right)^2\,, \qquad {}^8C^{(2)}_{\phi^2X^2} = \frac{1}{4}\left(\frac{c}{\Lambda M}\right)^2\,, \quad {}^8C^{(3)}_{\phi^2X^2} = 0\,, \quad X = B,W,G\,, \tag{C.9}$$

$${}^8C^{pr}_{\phi^2\psi^2} = \kappa^{(\psi)}_{pr}\left(\frac{c}{\Lambda M}\right)^2\,, \qquad \psi = \ell,e,q,u,d\,. \tag{C.10}$$

All of them are consistent with the positivity constraints of section 5.

JHEP07(2026)172

The same conclusion holds if $h_{\mu\nu}$ is *conformally* coupled, which consists in substituting the expressions for $T^{(\phi)}_{\mu\nu}$ and $T^{(H)}_{\mu\nu}$ in eqs. (C.3) and (C.4) with the improved traceless combinations [149]

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\,\partial^\rho\phi - \frac{1}{6}(\partial_\mu\partial_\nu - g_{\mu\nu}\partial_\rho\partial^\rho)\phi^2\,, \tag{C.11}$$

$$T^{(H)}_{\mu\nu} = D_\mu H^\dagger D_\nu H + D_\nu H^\dagger D_\mu H - g_{\mu\nu}D_\rho H^\dagger D^\rho H - \frac{1}{6}(D_\mu D_\nu + D_\nu D_\mu - 2g_{\mu\nu}D_\rho D^\rho)(H^\dagger H)\,, \tag{C.12}$$

respectively. With respect to the previous case, none of the Wilson coefficients change value, and the effective Lagrangian remains consistent with the positivity bounds.

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Code Availability Statement. This article has no associated code or the code will not be deposited.

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JHEP07(2026)172

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JHEP07(2026)172