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Integer constraints in a Cournot model - an application to electricity market modelling

Mel T. Devine[1] · **Muireann Á. Lynch**[2,3,4]

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Abstract

The costs associated with electricity generation include costs that are independent of their marginal output, such as the cost of starting their units, and constraints such as minimum generation levels. Modelling these costs and constraints requires integer formulation of the units, and so they have typically been ignored in electricity market modelling and simulation to date. We present a stochastic equilibrium model to include these costs and constraints in a Cournot game. We solve it using the Gauss-Seidel diagonalization algorithm and apply it to a model of the power system of the island of Ireland for varying levels of variable renewable power generation. We find that the impacts of integer modelling are non-trivial, and are heterogeneous across firms and wind levels. Furthermore, excluding integer modelling exaggerates the impact of price-making behaviour. We conclude that neglecting integer constraints in power system market models leads to inaccurate results, particularly at high penetrations of renewable energy sources.

Keywords Equilibrium modelling · Electricity market · Cournot modelling · Unit commitment

✉ Mel T. Devine
mel.devine@ucd.ie

Muireann Á. Lynch
muireann.lynch@esri.ie

[1] College of Business, University College Dublin, Dublin, Ireland

[2] Economic and Social Research Institute, Sir John Rogerson's Quay, Dublin, Ireland

[3] Department of Environmental and Public Health Engineering, Johns Hopkins University, Baltimore, MD, USA

[4] Department of Economics, Trinity College Dublin, Dublin, Ireland

1 Introduction

Electricity markets have been liberalised since the 1990 s, and are typically operated under an auction-based framework to efficiently dispatch and compensate electricity generators. Under these auction frameworks, firms submit price-quantity pairs (Wolak 2021), which can vary in their complexity from "plain vanilla" to more complex bids that allow firms to embed technical and economic constraints into their bids, such as limitations on how quickly a unit can change its output (ramp rates) as well as extra costs such as the cost of starting a unit. Market operators then dispatch generation and demand in a cost-minimising manner. In a competitive market, the dispatch equates to market clearing on the basis of short-run marginal costs.

In recent years, concerns over climate change have led to greatly increased levels of renewable generation in electricity systems worldwide (IEA 2023), while the Ukrainian war and concerns over energy affordability and security have prompted a further acceleration in renewable generation by European policy makers via the "Fit for 55" package (European Commission 2021). Furthermore, in an effort to reduce electricity bills to final consumers, attempts to reduce inframarginal rents to generators, including renewable generators, via instruments such as contracts for differences and forward contracting, have been proposed by the European Commission (European Commission 2023b, a).

This increase in renewable generation poses new challenges for electricity market design and modelling. These challenges include price cannibalisation, optimal location of renewables, and network design. Newbery et al (2018) and Lynch et al (2021) provide summaries of the market design challenges at high penetrations of variable Renewable Energy Sources (v-RES). In general, however, the interaction between v-RES penetration and non-marginal thermal generator costs (including, for example, start up and no load costs) has received less attention. This most likely stems from the fact that such costs make up a small proportion of the total costs, both of the power system and of individual firms' cost functions. As such, these costs have generally been ignored in market design discussions: the literature that focuses on the suitability of short-run marginal cost pricing generally restricts its attention to the price cannibalisation problem. Furthermore, electricity market operation itself does not always allow for the incorporation of these costs into firms' bids, particularly in EU markets. While most electricity markets in the USA include start and no load costs in the bids submitted by generators, EU markets generally require participants to internalise such costs in their bids, although fairly complex bidding patterns are available in at least some markets (Herrero et al 2020).

Increased levels of v-RES have been shown to substantially change the optimal dispatch when start costs are considered, compared to when such costs are ignored Shortt et al (2012). The incorporation of start costs and no-load costs into power system dispatch models makes generators' cost functions discontinuous and hence requires integer (binary) decision variables. This is known as the unit commitment problem, and has been widely studied in the engineering literature.

Tuohy et al (2009) presented an early examination of the impact of renewable generation on the unit commitment problem, and there is a vast literature on the topic, with many recent papers focusing on new efficient solution techniques for the problem itself; see e.g., Wu et al (2023); Ali et al (2023); Zhang et al (2023). However, the incorporation of start and no-load costs into electricity market models, particularly when the assumption of perfect competition is relaxed, has received little attention in the literature, and motivates this paper.

This paper considers the impact of these discontinuous costs (integer variables) in an imperfectly competitive market and furthermore, the extent to which the resulting equilibrium deviates from that which would prevail under continuous, non-integer, costs. This is, to the best of our knowledge, the first time this question has been explored in the literature. Furthermore, the impact of variability in the electricity supply via higher penetrations of v-RES is quantified. We thus provide insight in two main areas. The first is the consideration of market equilibria in a Cournot framework compared to a cost minimisation approach including unit commitment: we provide evidence on the impact of ignoring price-making behaviour modelled *à la* Cournot. The second main set of insights concerns the impacts of including discontinuous costs such as start costs and no-load costs, which require integer modelling, in firms' profit-maximising Cournot decisions, particularly as v-RES generation increases. Furthermore, we provide insight on whether, and to what extent, these two mechanisms compound: the impact of integer variables on market power exploitation, and vice versa, can be shown. Thus, we provide evidence for policy makers and market modellers alike on these impacts.

To the best our knowledge, this is the first work in the equilibrium with integers literature that solely uses the Gauss-Seidel diagonalization algorithm to find a Nash equilibrium. Previous works seek solution points via model reformulations and/or constraint relaxations. The Gauss-Seidel approach is commonly used to solve Equilibrium Problem with Equilibrium Constraints (Devine and Siddiqui 2023; Pozo et al 2017; Gabriel et al 2012). It involves solving the individual constrained optimisation problem of each player individually and iteratively. When one player's problem is being solved, the decision variables for all other players are assumed fixed. The algorithm iteratively converges to an equilibrium and stops once no player has an optimal deviation. An advantage of the algorithm is that it allows us to find a Nash equilibrium point in a computational tractable manner, without having to consider any reformulations or constraint relaxations. A disadvantage of the Gauss-Seidel algorithm is that the equilibrium point found is not guaranteed to be unique (Zerrahn and Huppmann 2017; Gabriel et al 2012).

To help improve computational efficiency for the models presented, we combine our solution techniques with a Rolling Horizon algorithm. In such an algorithm, instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problems whose time sets are overlapping subsets of the overall time set (Devine et al 2016) - see Sect. 3 for further details. Given the computational difficulties associated with solving equilibrium problem with integers (De Santis et al 2022; Gabriel et al 2021), this is a further contribution of the present work.

Furthermore, we present an advance on the literature by considering a real-world example: specifically, the all-island Irish electricity market comprising 58 generating units spread across 16 generating firms. This is due to the availability of a unique dataset produced and maintained by system operators on the island of Ireland. The dataset includes the technical and economic features of every generator in the market. In this manner, we avoid having to estimate the fixed and marginal costs of each generator in the market via their bids and are instead able to simulate their optimal generation decisions based on their technical characteristics.

We introduce a game-theoretic unit-commitment model which allows for price-making behaviour amongst the generating firms. We consider $|F|$ generating firms, who each hold multiple generating units with discontinuous costs. Each firm seeks to maximise its expected profits over several timesteps. The first set of timesteps is split into two stages. In the first stage, firms have perfect information on wind capacity factors. For the second stage, they have uncertain information. This uncertainty is modelled using $|S|$ scenarios, each of which have a probability associated with it. Thus, each firm's optimisation problem is a stochastic program. The overall model contains an inverse demand curve, and we assume each firm has price-making ability, modelled *à la* Cournot. Consequently, the solution technique seeks a Nash-Equilibrium.

To answer our research questions, we consider three further models, each of which are simplifications of the equilibrium with integers model described above. We consider an equilibrium model where all firms behave *à la* Cournot but none of them have discontinuous costs. This model does not require integers and hence can be solved as an MCP or Quadratic Program (QP). This allows us to examine the impact of removing unit commitment constraints from a Cournot model.

Next, we consider a cost minimisation model where all firms have unit commitment constraints and hence discontinuous costs, but none behave *à la* Cournot. Instead, for this model, all the firms are assumed to be price-takers and hence the market is perfectly competitive. This model is solved as a Mixed Integer Quadratically Constrained Program. By comparing this model to the game-theoretic unit-commitment model, we can examine the impact of removing Cournot behaviour from a unit-commitment model.

Finally, we consider a model where all firms are price-takers and only have continuous decision variables, i.e., there are no unit commitment constraints. This model is solved using Quadratic Programming. This allows us to examine the impact of removing unit commitment constraints and hence discontinuous costs, from a perfectly competitive model.

The results show considerable heterogeneity across firms and installed wind capacities, with integers and price-making behaviour giving rise to significantly different equilibrium solutions compared to the models that neglect either or both of these features. These findings generally, though not always, increase as wind generation increases. We therefore present evidence of the importance of considering market power and integer representations of generation constraints in order to ensure accurate results at high RES-E levels.

Furthermore, several works in the literature examine the impact of price-making behaviour by comparing the difference between solutions to models formulated with

and without market power (Devine and Bertsch 2022; Egging et al 2017; Koschker and Möst 2016). Our findings suggest that excluding integer variables from such studies may overestimate the impact of price-making behaviour.

The rest of this paper is structured as follows. Section 2 provides a review of the literature while Sect. 3 describes the methodology. Section 4 describes the dataset used. Section 5 outlines the results while Sect. 6 provides a discussion. Section 7 concludes our paper.

2 Literature review

Unit commitment models generally assume a cost minimization approach, which aligns with perfect competition, but which cannot account for strategic decisions by firms with price-making behaviour. Such strategic behaviour, also known as market power, aligns with imperfect competition, and occurs when market players alter their decisions so as to increase market prices. For example, a generating firm may withhold some generation in order to increase the market price and hence their overall profits. One recent exception is Kumar et al (2023), which uses a monarch butterly algorithm to determine a profit-based unit commitment, but this paper does not take an equilibrium-based approach - many of the results associated with the price-making behaviour in game theoretic models are not captured. Previous work on start costs in oligopolistic electricity markets was performed by Reguant (2014), based on an *ex post* analysis of bids made in the Spanish electricity market. The start costs, and their subsequent impacts on bids and prices, is examined via an econometric analysis of historical bids, and are found to limit the ability of firms to change output levels over time and thus to exacerbate price volatility.

In contrast to unit commitment models, imperfect competition resulting in price making behaviour in electricity market models is generally modelled *ex ante* via oligopolistic game-theoretic models. For instance, Devine and Bertsch (2022) use a Mixed Complementarity Problem (MCP) to examine the impact that demand response has on price-making behaviour. MCPs allow one to model the constrained optimisation problems of oligopolists as Cournot players. Other examples of MCPs being used to model electricity markets include Devine and Bertsch (2018), Koschker and Möst (2016), Lise and Kruseman (2008), and Liu et al (2007). Further examples of MCPs in the wider energy market modelling literature include Egging-Bratseth et al (2020), Baltensperger et al (2016), Egging (2013), and Gabriel et al (2009). For a more comprehensive list of such papers, we refer the reader to Egging-Bratseth et al (2020).

In addition, several other equilibrium models have been used to model price-making in power systems. Approaches include Bertrand equilibrium (Lee and Baldick 2003), Supply Function Equilibria (Ruiz et al 2011; Pozo and Contreras 2011), Stackelberg equilibrium (Fanzeres et al 2019), Multi-leader-multi-follower equilibrium (Devine and Siddiqui 2023; Wogrin et al 2013a, b), and Generalised Hierarchical equilibrium (Zerrahn and Huppmann 2017; Huppmann and Egerer 2015). For a comprehensive review of equilibrium models in electricity systems, we refer the reader to Pozo et al (2017). None of the equilibrium models

mentioned above incorporate discontinuous costs and hence unit-commitment constraints. Indeed, a recent review of European electricity market design in light of the Ukrainian crisis includes no mention of unit commitment or start costs Fabra (2023).

In recent years, equilibrium problems with integer variables have started to gain attention in the Operations Research and wider literature, particularly for applications in electricity systems. However, there are no tailored algorithms for solving such problems (De Santis et al 2022).

An early attempt to to include integer constraints in game-theoretic model is Ramos et al (1999) who consider a production cost model with equilibrium constraints.

One heuristic that has been deployed is to relax integrality constraints and solve the resulting problem using Karush-Kuhn-Tucker conditions in order to obtain initial solutions. Integrality constraints are then re-added. This approach has been applied to stylized electricity markets in Gabriel et al (2013b, 2013a) and Ruiz et al (2012). This approach leads to integer-value MCPs, but the solutions do not necessarily represent equilibria. In Weinhold and Gabriel (2020) and Huppmann and Siddiqui (2018) relaxation of integrality is first applied. Then solutions that ensure incentive compatibility for each player are obtained. A similar two-stage approach can be found in Fomeni et al (2019). They consider a Reformulation Linearization Technique (RLT) in the first phase and a Mixed-Integer-Linear-Program in the second phase.

Gabriel et al (2021) reformulate a Discretely Constrained MCP as a purely continuous problem. This allows them to utilise local Non-Linear Programming (NLP) solvers and hence to obtain locally optimal solutions. Looking to non-energy applications, De Santis et al (2022) introduce a branch-and-bound algorithm to solve a Mixed-Integer Linear Complementarity Problem (MILCP). Todd (2016) propose binary search and other algorithms to solve an integer constrained Cournot production problem. In contrast to the present work, Todd (2016) does not consider maximum production constraints.

This paper considers the impact of unit commitment constraints, and hence integer variables, in a Cournot equilibrium model of an electricity market. As mentioned in the Introduction, this is, to the best of our knowledge, the first time this has been explored in the literature.

3 Methodology

In this section we describe the four models used in this work:

1. Cournot equilibrium without integers model
2. Cournot equilibrium with integers model
3. Cost minimisation without integers model
4. Cost minimisation with integers model

For the four models we consider $|F|$ generating firms, each of which has multiple generating units. Each firm decides how much electricity to generate from their units over $|P|$ (hourly) time steps. In the two equilibrium models, each firm f behaves as a price-making Cournot player. Hence, these models represent an oligopolistic market. In the two cost minimisation models, each firm exhibits price-taking behaviour and hence these models represent a perfectly competitive market.

Furthermore, each of the models take the form of a stochastic program containing two stages. In the first-stage firms have perfect foresight of renewable energy capacity factors and make 'here-and-now' generation decisions that they commit to. In the subsequent second-stage, firms have uncertain information regarding renewable energy capacity factors and thus make 'wait-and-see' generation decisions that are scenario dependent. There are $|S|$ scenarios in total with each having a different time series of renewable energy capacity factors.

Each of the four models are also solved using a Rolling Horizon algorithm. In such an algorithm, instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problems whose time sets are overlapping subsets of the overall time set (Devine et al 2016). Each of these smaller problems is known as a 'roll', with $|R|$ rolls being solved in total for each model.

A rolling horizon has a number of advantages. Firstly, it can improve computational efficiency (Devine et al 2016). Secondly, it can reflect decision making in electricity markets more realistically as it allows parameters to be updated after each roll of the model (Devine et al 2016, 2014; Tuohy et al 2009). In this work, the uncertain information firms have about renewable energy capacity factors is updated after each roll of the optimisation/equilibrium model is solved. This reflects reality. For example, in the real-world, firms make generating decisions today with uncertain information for the next day's renewable capacity factors. However, as time goes on and the next day arrives, the firms have updated information for renewable capacity factors (for instance, through updated weather forecasts) and can thus adjust their generation decisions accordingly.

Tables 1, 2, 3 and 4 describe the indices and sets, binary variables, continuous variables, and parameters of the models, respectively, including units in parentheses where

Table 1 Indices and sets

$f \in F$	Generating firms
$p \in P$	Generating unit (plant)
$t \in T$	Time periods for entire model
$t \in T_r \subset T$	Time periods for roll r
$t \in T_r^{SS} \subset T_r$	Time periods in second stage for roll r
$t \in T_r^{FS} \subset T_r$	Time periods in first stage for roll r
$t \in T_r^{FIX} \subset T_r$	Time periods in fixed time period for roll r
$s \in S$	Scenarios
$r \in R$	Rolls
$k \in K$	Iterations for Gauss-Seidel diagonalization algorithm

Table 2 Binary decision variables

$z^{\text{fs, on}}_{f,p,t}$	Binary variable indicating if firm f's unit p is online or offline at time t of first-stage
$z^{\text{on}}_{f,p,t,s}$	Binary variable indicating if firm f's unit p is online or offline at time t and scenario s of second-stage
$z^{\text{fs, su}}_{f,p,t}$	Binary variable indicating if firm f's unit p is starting up at time t of first-stage
$z^{\text{su}}_{f,p,t,s}$	Binary variable indicating if firm f's unit p is starting up at time t and scenario s of second-stage

Table 3 Continuous variables

Decision variables	
$gen^{fs}_{f,p,t}$	Generation from firm f's unit p in period t of the first-stage (MWh)
$gen^{ss}_{f,p,t,s}$	Generation from firm f's unit p in period t and scenario s of the second-stage (MWh)
Δg^{fs}_{t}	Consumer load shedding for period t of the first-stage (cost minimisation models only) (MWh)
$\Delta g_{t,s}$	Consumer load shedding for period t and scenario s of the second-stage (cost minimisation models only) (MWh)
Other variables	
γ^{fs}_{t}	System price for time period t of first-stage (€/MWh)
$\gamma_{t,s}$	System price for time period t and scenario s of second-stage (€/MWh)
$\pi_{f,r}$	Firm f's expected profit in roll r (€)
$\pi^{fs}_{f,r}$	Firm f's first-stage profit in roll r (€)
$\pi^{ss}_{f,r,s}$	Firm f's profit in scenario s of second-stage in roll r (€)
π^{*}_{f}	Firm f's optimal expected profit (€)
$\pi_{f,r,k}$	Firm f's expected profit at iteration k of Gauss-Seidel diagonalization algorithm (roll r) (€)
ψ_{r}	Overall system costs for roll r (€)
ψ^{fs}_{r}	Overall system costs for first-stage of roll r (€)
$\psi^{ss}_{r,s}$	Overall system costs for scenario s of the second-stage of roll r (€)

appropriate. For each of the models, there are three time period subsets at each roll r: the fixed time period subset (T^{FIX}_{r}), followed by the first-stage time period subset (T^{FS}_{r}), followed by the second-stage time period subset (T^{SS}_{r}). The length of $|T^{FIX}_{r}|$ is LEN^{FS}. The decision variables at all time periods in this subset for roll r must equal the corresponding first-stage decision variables from the previous roll $r-1$. This ensures, if a firm decides to generate from a particular unit in the first-stage time periods of roll r then those decisions are accounted for in the subsequent roll $r+1$, again showing how the rolling horizon can reflect reality more accurately. Thus, the first-stage time period subset also has a cardinality of $|T^{FS}_{r}| = LEN^{FS}$. The length of the second-stage time period subset is $|T^{SS}_{r}| = LEN^{SS}$. The time period subsets change for each roll r and are defined using the following linkage constraints:

$$T^{FIX}_{r} = \{1 + (r-1) \times LEN^{FS}, 2 + (r-1) \times LEN^{FS}, ..., r \times LEN^{FS}\}, \ \forall\, r \in R, \tag{1a}$$

Table 4 Parameters

A_t	Demand curve intercept for timestep p (€/MWh)
B	Demand curve slope (€)
$C^{\mathrm{M}}_{f,p}$	Marginal generation cost curve intercept for firm f's unit p (€/MWh)
$C^{\mathrm{Q}}_{f,p}$	Marginal generation cost curve slope for firm f's unit p (€)
$C^{\mathrm{ON}}_{f,p}$	Online cost for firm f's unit p (€/MWh)
$C^{\mathrm{SU}}_{f,p}$	Start-up cost for firm f's unit p (€)
$GEN^{FS}_{t,-f}$	Combined generation for all firms except firm f at time t of first-stage (MWh)
$GEN^{SS}_{t,-f,s}$	Combined generation for all firms except firm f at time t and scenario s of second-stage (MWh)
LEN^{FS}	Length of fixed and first-stage time periods
LEN^{SS}	Length of second-stage time period
$MAX_CAP_{f,p,t}$	Maximum generating capacity for firm f's unit p at time t (MW)
$MIN_CAP_{f,p,t}$	Minimum generating capacity for firm f's unit p at time t (MW)
$GEN^{\mathrm{FIX}}_{f,p,t,r}$	Firm f's fixed generation for unit p in period t and roll r (MWh)
$Z^{\mathrm{ON,\ FIX}}_{f,p,t,r}$	Firm f's fixed online decision for unit p in period t and roll r (Binary)
$Z^{\mathrm{SU,\ FIX}}_{f,p,t,r}$	Firm f's fixed start-up decision for unit p in period t and roll r (Binary)
$NORM_{p,t,s}$	Generating profile for unit p at time t and scenario s (%)
$PROB_s$	Probability associated with scenario s (%)
TOL	Convergence tolerance for Gauss-Seidel diagonalization algorithm
$GEN^{FS,N}_{f,p,t}$	Naive first-stage generation decisions taken from without integers and used to calculate regret
$z^{\mathrm{FS,\ ON,\ N}}_{f,p,t}$	Naive first-stage online decisions derived from $GEN^{FS,N}_{f,p,t}$ and used to calculate regret
$z^{\mathrm{FS,\ SU,\ N}}_{f,p,t}$	Naive first-stage start-up decisions derived from $GEN^{FS,N}_{f,p,t}$ and used to calculate regret
$\gamma^{FS,N}_t$	Naive first-stage prices derived from $GEN^{FS,N}_{f,p,t}$ used to calculate regret

$$T^{FS}_r = \{1 + r \times LEN^{FS}, 2 + r \times LEN^{FS}, ..., (r+1) \times LEN^{FS}\},\ \forall\, r \in R, \tag{1b}$$

$$\begin{aligned} T^{SS}_r = \{&1 + (r+1) \times LEN^{FS}, 2 + (r+1) \times LEN^{FS}, ..., LEN^{SS} \\ &+ (r+1) \times LEN^{FS}\},\ \forall\, r \in R. \end{aligned} \tag{1c}$$

Thus, the time periods for roll r are $T_r = T^{FIX}_r \cup T^{FS}_r \cup T^{SS}_r$ with length $|T_r| = 2 \times LEN^{FS} + LEN^{SS}$.

Throughout this paper the following conventions are used: lowercase letters indicate indices or primal variables while upper-case letters represent parameters or sets. Sections 3.2 - 3.3 describe each of the models enumerated above, respectively, while Sect. 3.5 presents the Rolling Horizon algorithm.

3.1 Cournot equilibrium without integers model

In the Cournot Equilibrium with Integers Model, each firm f behaves as a Cournot player and chooses its generation levels for each unit p so as to maximise its profits over

$|T_r|$ timesteps. They have knowledge of the inverse demand curve and how their generation levels can affect it. Thus, firm f knows that as it reduces its generation levels the market price increases. Consequently, we consider each firm f to be *price-making*. This is in contrast to the cost minimisation models to be presented.

In contrast to the Cournot Equilibrium With Integers Model, start-up and online costs, and associated constraints, are excluded. Consequently, the Equilibrium Without Integers Model only contains continuous decision variables. As mentioned previously, it is solved using a Rolling Horizon Algorithm (Sect. 3.5).

For each roll r, firm f seeks to maximise its profits via a stochastic program. In both the first- and second-stage, its profits equal the revenue it gains from generation less the cost of generation. Its decision variables are $gen^{fs}_{f,p,t}$, $gen^{ss}_{f,p,t,s}$, are generation levels for each unit in the first- and second-stage, respectively.

Firm f's optimisation problem at roll r takes the following format:

$$\max \pi_{f,r} = \pi^{fs}_{f,r} + \sum_{s \in S} PROB_s \times \pi^{ss}_{f,s,r}, \tag{2a}$$

where $\pi^{fs}_{f,r}$ represents firm f's profits from the first stage:

$$\begin{aligned} \pi^{fs}_{f,r} = \sum_{t \in T^{FS}_r} \sum_{p \in P} \big((\gamma^{fs}_t - C^M_{f,p} \\ - C^Q_{f,p} \times gen^{fs}_{f,p,t}) \times gen^{fs}_{f,p,t} \big), \end{aligned} \tag{2b}$$

while $PROB_s$ is the probability associated with scenario s and $\pi^{ss}_{f,s,r}$ represents firm f's profit in the second stage for scenario s:

$$\begin{aligned} \pi^{ss}_{f,s,r} = \sum_{t \in T^{SS}_r} \sum_{p \in P} \big((\gamma_{t,s} - C^M_{f,p} \\ - C^Q_{f,p} \times gen_{f,p,t,s}) \times gen_{f,p,t,s} \big). \end{aligned} \tag{2c}$$

The constraints associated with the fixed time period (T^{FIX}_r) of firm f's problem for roll r are:

$$gen^{fs}_{f,p,t} = GEN^{\text{FIX}}_{f,p,t,r}, \quad \forall p \in P, t \in T^{FIX}_r, \tag{2d}$$

Constraint (2d) ensures that the first-stage generation decisions of roll $r-1$ are accounted for in roll r.

The constraints associated with the first-stage (T^{FS}_r) of firm f's problem at roll r are

$$\gamma^{fs}_t = A_t - B \times \Big(\sum_{f \in F} \sum_{p \in P} gen^{fs}_{f,p,t} \Big), \quad \forall t \in T^{FS}_r, \tag{2e}$$

$$\begin{aligned} gen^{fs}_{f,p,t} \le MAX_CAP_{f,p,t} \times \sum_{s \in S} PROB_s \\ \times NORM_{p,t,s}, \quad \forall p \in P, t \in T^{FS}_r, \end{aligned} \tag{2f}$$

Equation (2e) represents the inverse demand curve for the first-stage[1]. All firms' generation levels other than firm f are exogenous to firm f's optimisation problem. Equation (2f) provides maximum capacity constraints. The capacity factors in this constraint are expected capacity factors ($\sum_{s\in S} PROB_s \times NORM_{p,t,s}$).

The constraints associated with the second-stage (T_r^{SS}) of firm f's problem at roll r are

$$\gamma_{t,s} = A_t - B \times \Big(\sum_{f\in F}\sum_{p\in P} gen_{f,p,t,s}\Big), \quad \forall s \in S, t \in T_r^{SS}, \tag{2g}$$

$$gen_{f,p,t,s} \leq MAX_CAP_{f,p,t} \times NORM_{p,t,s}, \quad \forall p \in P, s \in S, t \in T_r^{SS}. \tag{2h}$$

Constraints (2g) - (2h) follow from constraints (2e) - (2f), respectively. In contrast to the first-stage constraints, they are scenario dependent and hence contain a subscript s. In particular, the capacity factors in equation (2h) are not expected values. For the data used in this work, capacity factors for renewable energy vary across scenarios.

Furthermore, firm f's generation levels are constrained to be non-negative. Because the Equilibrium without Integers model does not contain unit commitment constraints and hence integer variables, the Lagrangian of firm f's optimisation problem can be differentiated with respect to each of firm f's primal and dual variables. Thus, in contrast to Sect. 3.2, the KKT conditions for each firm can be derived.

In contrast to Sect. 3.2, the generation levels of all firms except firm f in the inverse demand curves (equations (2e) and (2g)) are represented by decision variables rather than parameters. This is because all $|F|$ optimisation problems are solved simultaneously and thus the Gauss-Seidel diagonalization algorithm is not required.

As the optimisation problem of each firm f is convex, the KKT conditions are both necessary and sufficient for optimality, and thus the solution obtained represents a (Generalised) Nash Equilibrium (Gabriel et al 2012). In Sect. 5, the model is solved by obtaining the KKT conditions of all firms. Using these conditions, the problem is then recast into a single-level Quadratic Program (QP), which is solved using a standard QP solver in GAMS. We refer the reader to Egging-Bratseth et al (2020) and Hashimoto (1985) on how to recast an equilibrium problem with continuous variables as a QP.

Note: when expected profits are analysed in Sect. 5, the objective function of (2) is not used. Instead, online and start-up costs are accounted for in the expected profits examined in Sect. 5 ex-post, by using the solution to the units' generation levels to determine when units are starting up and are online and hence incurred the corresponding start-up and online costs, respectively. This allows for a better comparison

[1] The variables γ_t^{fs} and $\gamma_{t,s}^{ss}$ refer to the system's marginal cost of electricity provision and can be interpreted as the wholesale electricity price in a uniform pricing auction. When we refer to prices throughout this paper, we are referring to these variables.

with the models with integers, where online and start-up are explicitly accounted for in the firms' objective/profits functions.

3.2 Cournot equilibrium with integers model

The Cournot Equilibrium with Integers Model follows from the model presented in Sect. 3.1. As before each firm f is a Cournot player and chooses its generation level for each unit p so as to maximise profits. In contrast to the Equilibrium without Integers Model, firm f also has unit commitment constraints and hence its optimisation problem contains binary variables in order to represent online and start-up costs.

The Cournot Equilibrium with Integers Model contains $|F|$ optimisation problems, one for each firm. The model is solved using the Gauss-Seidel diagonalization algorithm (Sect. 3.2.1), which solves each of the individual optimisation problems iteratively until a (Generalised) Nash Equilibrium is found. When firm f's optimisation is being solved the decision variables of all other firms are assumed fixed.

Because the firm f's problem contains binary variables, the Lagrangrian of its objective function cannot be differentiated. Hence, Karush-Kuhn-Tucker decisions cannot be obtained, and the model cannot be solved using a Mixed Complementarity Problem. As mentioned previously, the model is solved using a Rolling Horizon algorithm (Sect. 3.5).

For each roll r, firm f seeks to maximise its profits by choosing when to have each of its units start-up ($z^{\text{fs, su}}_{f,p,t}$, $z^{\text{ss, su}}_{f,p,t,s}$), be online ($z^{\text{fs, on}}_{f,p,t}$, $z^{\text{ss, on}}_{f,p,t,s}$), and the generation levels for each unit ($gen^{fs}_{f,p,t}$, $gen^{ss}_{f,p,t,s}$). Firm f's problem is a stochastic program. In both the first- and second-stage, its profits equal the revenue it gains from generation less the cost of generation and less the costs associated with starting-up and being online.

Its objective function at roll r is

$$\max \pi_{f,r} = \pi^{fs}_{f,r} + \sum_{s \in S} PROB_s \times \pi^{ss}_{f,s,r}, \tag{3a}$$

where $\pi^{fs}_{f,r}$ represents firm f's profits from the first stage:

$$\begin{aligned} \pi^{fs}_{f,r} = \sum_{t \in T^{FS}_r} \sum_{p \in P} \big(& (\gamma^{fs}_t - C^M_{f,p} - C^Q_{f,p} \times gen^{fs}_{f,p,t}) \times gen^{fs}_{f,p,t} \\ & - C^{\text{ON}}_t \times z^{\text{fs, on}}_{f,p,t} - C^{\text{SU}}_t \times z^{\text{fs, su}}_{f,p,t} \big), \end{aligned} \tag{3b}$$

while $PROB_s$ is the probability associated with scenario s and $\pi^{ss}_{f,s,r}$ represents firm f's profit in the second stage for scenario s:

$$\begin{aligned} \pi^{ss}_{f,s,r} = \sum_{t \in T^{SS}_r} \sum_{p \in P} \big(& (\gamma_{t,s} - C^M_{f,p} - C^Q_{f,p} \times gen_{f,p,t,s}) \times gen_{f,p,t,s} \\ & - C^{\text{ON}}_t \times z^{\text{on}}_{f,p,t,s} - C^{\text{SU}}_t \times z^{\text{su}}_{f,p,t,s} \big). \end{aligned} \tag{3c}$$

The constraints associated with the fixed time period (T_r^{FIX}) of firm f's problem for roll r are:

$$gen_{f,p,t}^{fs} = GEN_{f,p,t,r}^{\text{FIX}}, \ \ \forall p \in P, t \in T_r^{FIX}, \tag{3d}$$

$$z_{f,p,t}^{\text{fs,on}} = Z_{f,p,t,r}^{\text{FIX, ON}}, \ \ \forall p \in P, t \in T_r^{FIX}, \tag{3e}$$

$$z_{f,p,t}^{\text{fs,su}} = Z_{f,p,t,r}^{\text{FIX, SU}}, \ \ \forall p \in P, t \in T_r^{FIX}, \tag{3f}$$

Constraints (3d) - (3f) ensure that the first-stage decisions of roll $r-1$ are accounted for in roll r. For instance, if firm f commits a unit p to be online at the last time period of first-stage in roll $r-1$ and it is optimal for them to keep that unit online in the first hour of the first-stage of roll r, then firm f does not have to pay an online cost to do so. In the absence of constraints (3d) - (3f), firm f's objective function would include start-up costs for units that were already online from the previous roll.

The constraints associated with the first-stage (T_r^{FS}) of firm f's problem at roll r are

$$\gamma_t^{fs} = A_t - B \times (GEN_{t,-f}^{FS} + \sum_{p \in P} gen_{f,p,t}^{fs}), \ \ \forall t \in T_r^{FS}, \tag{3g}$$

$$\begin{aligned} gen_{f,p,t}^{fs} \leq\ & z_{f,p,t}^{\text{fs, on}} \times MAX_CAP_{f,p,t} \\ & \times \sum_{s \in S} PROB_s \times NORM_{p,t,s}, \ \ \forall p \in P, t \in T_r^{FS}, \end{aligned} \tag{3h}$$

$$gen_{f,p,t,s}^{fs} \geq z_{f,p,t}^{\text{fs, on}} \times MIN_CAP_{f,p,t}, \ \ \forall p \in P, t \in T_r^{FS}, \tag{3i}$$

$$z_{f,p,t}^{\text{fs, su}} \geq z_{f,p,t}^{\text{fs, on}} - z_{f,t,p-1}^{\text{fs, on}}, \ \ \forall p \in P, t \in T_r^{FS}. \tag{3j}$$

Equation (3g) represents the inverse demand curve for the first-stage. When solving firm f's optimisation problem, the generation levels of all other firms are assumed fixed. The parameter $GEN_{t,-f}^{FS}$ represents the combined generation levels of all firms, except firm f, for first-stage time periods. This parameter updates after each optimisation problem is solved in the Gauss-Seidel diagonalization algorithm (Section 3.2.1). Equation (3h) provides maximum capacity constraints. The capacity factors in this constraint are expected capacity factors ($\sum_{s \in S} PROB_s \times NORM_{p,t,s}$). Equation (3i) ensures that if unit p is online then it must generate at least to minimum capacity level. Equation (3j) represents unit commitment logic constraints for the first-stage.

The constraints associated with the second-stage (T_r^{SS}) of firm f's problem at roll r are

$$\gamma_{t,s} = A_t - B \times (GEN^{SS}_{t,-f,s} + \sum_{p \in P} gen_{f,p,t,s}), \quad \forall s \in S, t \in T_r^{SS}, \tag{3k}$$

$$gen_{f,p,t,s} \leq z^{\text{on}}_{f,p,t,s} \times MAX_CAP_{f,p,t} \times NORM_{p,t,s}, \quad \forall p \in P, s \in S, t \in T_r^{SS}, \tag{3l}$$

$$gen_{f,p,t,s} \geq z^{\text{on}}_{f,p,t,s} \times MIN_CAP_{f,p,t}, \quad \forall p \in P, t \in T_r^{SS}, \tag{3m}$$

$$z^{\text{su}}_{f,t,p+1,s} \geq z^{\text{on}}_{f,t,p+1,s} - z^{\text{on}}_{f,p,t,s}, \quad \forall p \in P, s \in S, t \in T_r^{SS} \tag{3n}$$

Constraints (3k) - (3n) follow from constraints (3g) - (3j), respectively. In contrast to the first-stage constraints, they are scenario dependent and hence contain a subscript s. The parameter $GEN^{SS}_{t,-f,s}$ represents the combined generation levels of all firms, except firm f, for scenario s of the second-stage. This parameter updates after each optimisation problem is solved in the Gauss-Seidel diagonalization algorithm (Section 3.2.1).

Constraint (3o) includes variables from both the first-stage and the second-stage. It ensures that if a unit p is committed to be on at the last time period of the first stage ($p = r \times LEN^{FS}$), then that must be accounted for in all scenarios of the first time period of the second stage.

$$z^{\text{su}}_{f,t,p+1,s} \geq z^{\text{on}}_{f,t,p+1,s} - z^{\text{fs, on}}_{f,p,t}, \quad \forall p \in P, s \in S, p = r \times LEN^{FS}. \tag{3o}$$

Firm f's optimisation problem is to maximise objective function (3a) subject to constraints (3g) - (3o). It is a Mixed Integer Quadratically Constrained Program (MIQCP) with a convex objective function and linear constraints. Hence, when firm f's optimisation is being solved in the Gauss-Seidel diagonalization algorithm (Sect. 3.2.1), it can be solved using a standard MIQCP solver.

3.2.1 Diagonalization algorithm for Equilibrium with Integers

As firm f's optimisation problem (3) contains integer variables we cannot derive Karush-Kuhn-Tucker optimality conditions. Instead, at each roll r, we solve the equilibrium with integers model by implementing the following Gauss-Seidel diagonalization algorithm (Gabriel et al 2012). The algorithm iteratively solves each firm f optimisation problem (3) by fixing all other firm decisions, until it converges to a point where no firm has an optimal deviation.

Algorithm 1 Gauss-Seidel diagonalization algorithm

while $\sum_f \frac{|\pi_{f,r,k} - \pi_{f,r,k-1}|}{\pi_{f,r,k-1}} > TOL$ and $k < |K|$ **do**
  for $f = 1, .., |F|$ **do**
    Assume firm$_{-f}$'s decision variables are fixed
    $GEN^{FS}_{t,-f} \leftarrow \sum_{\hat{f} \in F \setminus \{f\}} \sum_{p \in P} gen^{fs}_{\hat{f},t,p}, \quad \forall\, t \in T^{FS}_r$
    $GEN^{SS}_{t,-f,s} \leftarrow \sum_{\hat{f} \in F \setminus \{f\}} \sum_{p \in P} gen_{\hat{f},t,p,s}, \quad \forall\, s \in S, \forall\, t \in T^{SS}_r$
    Solve Firm f's MIQCP (3)
    $\pi_{f,r,k} \leftarrow \pi^*_f$
    $k \leftarrow k + 1$
  end for
end while

The parameter *TOL* and the set *K* represent a pre-defined convergence tolerance and the set of iterations, respectively. The variable $\pi_{f,k}$ represents firm *f*'s expected profits (objective function (3a)) at iteration *k*.

Algorithm 1 is based on Gauss-Seidel diagonalization, which is standard in the literature (Gabriel et al 2012). As described in the literature, if Algorithm 1 converges, the point it converges to is guaranteed to be a Generalised Nash Equilibrium if each optimisation is solved to optimality in each iteration.

As the Equilibrium with Integer Model is solved using a Rolling Horizon, Algorithm 1 is solved |*R*| times, once for each roll *r*.

The intuition behind the algorithm is that each firm iteratively keeps adjusting their decisions, based on all the other firms' updating decisions. The algorithm stops when no firm can improve their optimal profits beyond the convergence tolerance *TOL*. That is, firm *f* reaches its optimal decisions, given what all other firms' optimal decisions are, and hence a (Generalised) Nash equilibrium is reached.

3.3 Cost minimisation without integers model

The cost minimisation without integers model follows from the model presented in Sect. 3.1. In comparison to that model, it does not contain start-up and online costs, as well as associated constraints. Hence it does not contain binary variables. In contrast, however, the generating firms in the cost minimisation without integers model are not assumed to be price-making (Cournot) players. Instead, it is assumed that no firm has knowledge of the inverse demand curve, and all firms exhibit price-taking behaviour with all their generating units for all time periods. Consequently, the market model as described in this section is perfectly competitive.

Modelling a market via a cost minimisation problem, whereby a central planner (real or implicit) chooses the firms' generation levels so as to minimise overall system costs, the optimal results correspond to the perfectly competitive solution (Gabriel et al 2012; Devine 2012). Hence, there is no need to model each firm *f*'s optimisation problem individually, as Sects. 3.1 and 3.2.

As before, the cost minimisation without integers model is solved using a Rolling Horizon Algorithm (Sect. 3.5) and, for each roll r, it takes the form of a stochastic program. The (real or implicit) central planner seeks to minimise system costs by choosing the generation levels for each unit ($gen^{fs}_{f,p,t}$, $gen^{ss}_{f,p,t,s}$) and by choosing the amount of electricity consumers shed (Δg^{fs}_t, $\Delta g_{t,s}$).

The problem's objective function is

$$\min \psi_r = \psi^{fs}_r + \sum_{s \in S} PROB_s \times \psi^{ss}_{s,r}, \tag{4a}$$

where ψ^{fs}_r are overall system costs for the first stage:

$$\psi^{fs}_r = \sum_{t \in T^{FS}_r} \Bigg(\Big(\sum_{f \in F} \sum_{p \in P} C^M_{f,p} \times gen^{fs}_{f,p,t} + C^Q_{f,p} \times (gen^{fs}_{f,p,t})^2 \Big) + \frac{B}{2} \times (\Delta g^{fs}_t)^2 \Bigg), \tag{4b}$$

while $\psi^{ss}_{s,r}$ represents overall system costs for scenario s in the second stage :

$$\psi^{ss}_{f,s,r} = \sum_{t \in T^{SS}_r} \Bigg(\Big(\sum_{f \in F} \sum_{p \in P} C^M_{f,p} \times gen_{f,p,t,s} + C^Q_{f,p} \times (gen_{f,p,t,s})^2 \Big) + \frac{B}{2} \times (\Delta g_{t,s})^2 \Bigg). \tag{4c}$$

The constraints of the cost minimisation without integers model at roll r are

$$\Delta g^{fs}_t + \sum_{f \in F} \sum_{p \in P} gen^{fs}_{f,p,t} = \frac{A_t}{B}, t \in T^{FS}_r, \tag{4d}$$

$$\Delta g_{t,s} + \sum_{f \in F} \sum_{p \in P} gen_{f,p,t,s} = \frac{A_t}{B}, \quad s \in S, t \in T^{SS}_r, \tag{4e}$$

$$\text{Constraintnts (2d), (2f), (2h).} \tag{4f}$$

Constraints (4d) and (4e) represent the demand balancing equations in the cost minimisation with integers model. The parameter $\frac{A_t}{B}$ represents maximum demand, that is, demand if the system price were zero. Deviations from these maximum demands, at an appropriate cost, are represented by the load shedding variables (Δg^{fs}_t, $\Delta g_{t,s}$). These decision variables are also constrained to be non-negative and do not appear in either of the models presented in Sects. 3.1 and 3.2. The costs associated with these variables are $\frac{B}{2} \times (\Delta g^{fs}_t)^2$ and $\frac{B}{2} \times (\Delta g_{t,s})^2$, respectively, and represent the Value of Lost Load (VoLL) for consumers. These terms follow from Devine and Bertsch (2018, 2022) and ensure system prices in this model are consistent with those described in Sects. 3.1 and 3.2.

By assuming the system price for the first-stage (γ_r^{fs}) is the Lagrange Multiplier associated with constraint (4d) and by differentiating the Lagrangian of this model with respect to Δg_t^{fs}, leads us to the equation $\frac{\gamma_t^{fs}}{B} = \Delta g_t^{fs}$. Substituting this value for load shedding into constraint (4d) ensures that, at optimality,

$$\gamma_t^{fs} = A_t - B \times \Big(\sum_{f \in F} \sum_{p \in P} gen_{f,p,t}^{fs}\Big), \quad \forall \ t \in T_r^{FS}, \tag{5a}$$

which is consistent with equation (3g). Similar logic ensures

$$\gamma_t = A_t - B \times \Big(\sum_{f \in F} \sum_{p \in P} gen_{f,p,t,s}\Big), \quad \forall \ s \in S, t \in T_r^{FS}. \tag{5b}$$

The remaining constraints (equation (4f)) follow from those in the Cournot Equilibrium with and without Integers Models. In contrast to Sect. 3.2 however, at each roll r, the generation levels of all firms are determined by a single optimisation problem and hence the constraints of all firms are included in that problem.

The optimisation problem (6) has a strictly convex objective function and linear constraints. Thus, for each roll r, its optimal solutions can be obtained using a standard QP solver.

3.4 Cost minimisation with integers model

The cost minimisation with integers model follows from the models presented in Sects. 3.2 and 3.3. In comparison to the cost minimisation without integers model, the market considered is assumed to be perfectly competitive, and thus all firms are assumed to be price-taking. Consequently, generation levels can be obtained by assuming there is a central planner (real or implied) who chooses the optimal levels so as to minimise overall system costs. In contrast to cost minimisation without integers but in comparison to the equilibrium with integers model, the cost minimisation with integers model includes start-up and online costs, and associated constraints. Thus, it contains both continuous and binary variables.

As in previous subsections, the cost minimisation with integers model is solved using a Rolling Algorithm (Sect. 3.5) and, for each roll r, it takes the form of a stochastic program.

The (real or implicit) central planner seeks to minimise system costs by choosing when to have each generation unit start-up ($z_{f,p,t}^{\text{fs, su}}$, $z_{f,p,t,s}^{\text{ss, su}}$), be online ($z_{f,p,t}^{\text{fs, on}}$, $z_{f,p,t,s}^{\text{ss, on}}$), and the generation levels for each unit ($gen_{f,p,t}^{fs}$, $gen_{f,p,t,s}^{ss}$). In addition, the central planner also chooses the amount of electricity consumers shed (Δg_t^{fs}, $\Delta g_{t,s}$).

The problem's objective function at roll r is

$$\min \psi_r = \psi_r^{fs} + \sum_{s \in S} PROB_s \times \psi_{s,r}^{ss}, \tag{6a}$$

where ψ_r^{fs} are overall system costs for the first stage:

$$\psi_r^{fs} = \sum_{t \in T_r^{FS}} \Bigg(\Big(\sum_{f \in F} \sum_{p \in P} C_{f,p}^{M} \times gen_{f,p,t}^{fs} + C_{f,p}^{Q} \times (gen_{f,p,t}^{fs})^2 + C_t^{\mathrm{ON}} \times z_{f,p,t}^{\mathrm{fs,\,on}} + C_t^{\mathrm{SU}} \times z_{f,p,t}^{\mathrm{fs,\,su}} \Big) + \frac{B}{2} \times (\Delta g_t^{fs})^2 \Bigg), \tag{6b}$$

while $\psi_{s,r}^{ss}$ represents overall system costs for scenario s in the second stage :

$$\psi_{f,s,r}^{ss} = \sum_{t \in T_r^{SS}} \Bigg(\Big(\sum_{f \in F} \sum_{p \in P} C_{f,p}^{M} \times gen_{f,p,t,s} + C_{f,p}^{Q} \times (gen_{f,p,t,s})^2 + C_t^{\mathrm{ON}} \times z_{f,p,t,s}^{\mathrm{on}} + C_t^{\mathrm{SU}} \times z_{f,p,t,s}^{\mathrm{su}} \Big) + \frac{B}{2} \times (\Delta g_{t,s})^2 \Bigg). \tag{6c}$$

The constraints of the cost minimisation with integers model at roll r are:

$$\text{Constraints}(4d), (4e), \tag{6d}$$

and

$$\text{Constraints (3d) - (3f), (3h) - (3j), (3l) - (3n)} \ \ \forall f. \tag{6e}$$

In contrast to Sect. 3.2, at each roll r, the generation levels of all firms are determined by a single optimisation problem and hence the constraints of all firms are included in that problem.

The optimisation problem (6) has a strictly convex objective function and linear constraints. Thus its optimal solutions can be obtained using a standard Mixed Integer Quadratically Constrained Program (MIQCP) solver, from each roll r.

3.5 Rolling Horizon algorithm

We solve each of the four models presented in Sects. 3.1 - 3.4 using a Rolling Horizon algorithm. Instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problem whose time sets are overlapping subsets of the overall time set.

As mentioned previously, a Rolling Horizon algorithm improves computational efficiency and brings added realism to energy market modelling. Such algorithms are widely used throughout the energy market model literature (Devine et al 2016; Devine 2012; Tuohy et al 2009). Consequently, we omit a detailed description of the rolling horizon algorithm used in this work.

Finally, Table 5 displays the solution for roll r each of the models used in this work.

Table 5 Summary of solution techniques for each roll r

Model	Solution technique
Cournot Equilibrium without integers	QP (equations (3))
Cournot Equilibrium with integers	Algorithm 1
Cost minimisation without integers	QP (equations (4))
Cost minimisation with integers	MIQCP (equations (6))

4 Data

In this section, we discuss the input parameters we apply to the models presented in Sect. 3. In Sect. 4.1 we describe the data for $|F| = 16$ generation firms while in Sect. 4.2 we discuss the wind capacity factor scenarios. All parameter values not explicitly stated in this section can be found in the Online Appendix.[2]

For each roll r, we assume the lengths of the fixed-stage, first-stage, and second-stage are each 12 h. Thus, the number of hourly timesteps for each roll is 36. In total we consider $|r| = 15$ rolls. Hence, the model is solved over $|T| = 36 + 14 \times 12 = 204$ hourly time periods. The first roll considers 36 h and at each of the subsequent 14 rolls, the model moves forward 12 h.

While we choose 15 rolls, we do not present the results from the roll $r = 1$ in Sect. 5. This is to avoid start-of-model effects as the fixed decisions for the first roll are assumed to be zero. Hence, the results we present represent one week (168 h).

The demand curve intercept values (A_t) we assume are based on reference demand values taken from Devine and Bertsch (2022) and Lynch et al (2019), scaled to ensure a peak demand of 7500MW. These values can be found in our Online Appendix. The demand curve intercept value we choose is $B = 0.137$, which was obtained assuming an arc price elasticity of 0.11 from Di Cosmo and Hyland (2013), an average wholesale price from 2017-2021 of €67/MWh as reported by www.sem-o.com, and an average hourly demand over the same time period of 4456MW.

For the Gauss-Seidel diagonalisation algorithm, we set a convergence tolerance of $TOL = 10^{-4}$ with maximum number of iterations of $|K| = 10^2$.

4.1 Generator and power system data

The data for the thermal generation available are based on the electricity market of the island of Ireland. The regulatory bodies for the two power systems on the island of Ireland provide a validated set of technical and economic characteristics of every power plant on the system, and these data are publicly available (CRU and UREGNI 2021). The installed capacity, excluding wind and solar generation, is shown by firm and by fuel type in Table 6. Exact values for each unit's maximum and minimum generating capacities can be found in the Online Appendix.

[2] https://figshare.com/articles/dataset/Input_data_Devine_Lynch_2023_xlsx/24382018

Table 6 Installed capacities by firm and fuel type, excluding wind capacity (MW)

	Gas	Biomass / Peat	GasOil	Waste	Coal	Water	Oil	Waste wood	Total (MW)	Total (%)
Aughinish	162	-	-	-	-	-	-	-	162	2%
BGE	450	-	-	-	-	-	-	-	450	5%
BnM	-	118	116	-	-	-	-	-	234	3%
Contour	12	-	-	-	-	-	-	-	12	0%
Covanta	-	-	-	61	-	-	-	-	61	1%
Data and Power	116	-	-	-	-	-	-	-	116	1%
Energia Generation	745	-	-	-	-	-	-	-	745	9%
EP	350	-	258	-	476	-	-	-	1,084	12%
ESB	1,992	-	53	-	570	508	285	-	3,408	39%
Evermore Energy	-	-	-	-	-	-	-	18	18	0%
Grange	115	-	-	-	-	-	-	-	115	1%
Indaver	-	-	-	21	-	-	-	-	21	0%
iPower	-	-	58	-	-	-	-	-	58	1%
POWERNI	593	-	-	-	-	-	-	-	593	7%
SSE	464	-	208	-	-	-	592	-	1,264	14%
Tynagh	389	-	-	-	-	-	-	-	389	4%
Total (MW)	5,388	118	693	82	1,046	508	877	18		
Total (%)	62%	1%	8%	1%	12%	6%	10%	0%		

The validated dataset includes quantities of fuel required for start-up and no load, as well as heat rates. In order to convert these into generator costs, we utilise fuel and carbon prices as per the recommended parameters from the European Commission Directorate-General for Climate Action, European Commission ((2022)). Start costs, online costs, and marginal intercept costs we assume can be found in the Online Appendix. Following Devine and Bertsch (2022), we assume a marginal slope cost of $C^{Q}_{f,p} = 0.000213, \forall f, t$.

In the numerical results presented in Sect. 5 we consider three different test cases for total installed wind capacity; 0GW, 4GW, and 8GW. In the absence of wind ownership data, we assume ownership is split evenly amongst the six largest firms (ESB, EP, SSE, Energia, POWERNI, and BGE).

The Irish electricity market consists of a day-ahead, intraday and balancing market, much like many EU markets. Day ahead bids are formulated according to the EUPHEMIA algorithm, which clears the market, and balancing market bids are regulated. In this work, we do not attempt to model the various markets and clearing stages explicitly. Instead, we model the market as a single-stage settlement where generators face no restrictions or regulations on their bids. In this way, we align with much of the existing literature on Cournot modelling in electricity markets (Devine and Bertsch 2018; Newbery and Greve 2017), and we also abstract from market-specifics to generalisable conclusions. Thus, the aim of the paper is not an exact replication or prediction of the current Irish Single Electricity Market, but rather a demonstration of the impact of integers within a Cournot model, parameterised with Irish data.

4.2 Wind capacity factors

We consider $|S| = 3$ scenarios at each roll and obtain hourly wind capacity factor scenarios for $NORM_{p,t,s}$ by using the Simultaneous Backward Reduction algorithm (Dupačová et al 2003; Heitsch and Römisch 2003) to reduce down to three the six scenarios for south-west of Ireland used in Lynch et al (2019) and Bertsch et al (2018). We then choose the 8.5 days (204 h) that have the median weighted average capacity factor, which works out at 33%. The probabilities ($PROB_s$) associated with scenarios $s = 1 – 3$ are 0.485, 0.4 and 0.115, respectively. The values for $NORM_{p,t,s}$ can be found in the Online Appendix.

5 Results

In this section, we present the results obtained from the models. We solve the four models presented in Sect. 3 three times, with installed wind capacities of 0GW, 4GW, and 8GW. Thus, we present 12 test cases. The results are obtained from the first-stage decisions (that is, the committed-to decisions) from rolls $r = 2 – 15$. Consequently, the results represent 168 h (one week). As discussed previously, results for roll $r = 1$ are not included in order to avoid start-of-model effects.

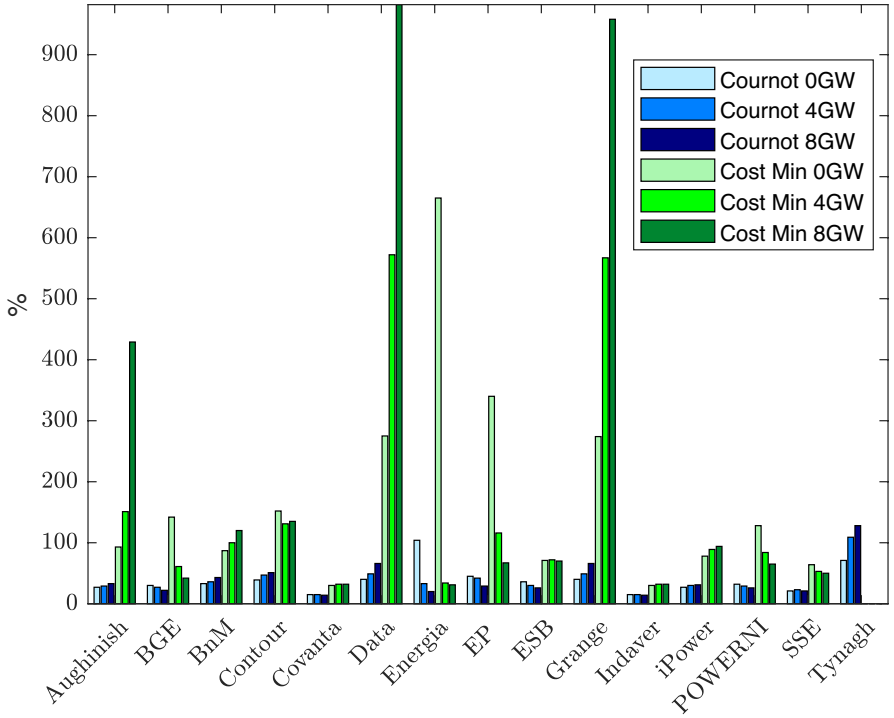


Fig. 1 % decrease in firm profits from the exclusion of integers

Throughout this section, for the two models without integers, expected profits are not calculated using the objective function of (2). Instead, online and start-up costs are accounted for in expected profits ex-post using generation levels to determine when units are starting up and online. This allows for a better comparison with the models with integers where online and start-up are explicitly accounted for in the firms' objective/profits functions.

5.1 The impact of including integers within a Cournot model

In this section, we analyse the impacts of excluding integer variables within a Cournot framework. Figure 1 shows the change in profits for each firm,[3] in percentage terms, from the exclusion of integers. The results are graphed as the profits of each firm when integers are modelled minus the profits of each firm when integers are excluded, divided by their profits with integers. In other words, Fig. 1 shows the percentage decrease in profits from the exclusion of integers. We model three

[3] We exclude Evermore as the percentage change in their profits was too large to graph; however, the relative values for all firms are given in the Appendix.

installed capacities of wind energy and assume the ownership of wind energy is distributed evenly amongst the firms. Table 14 in the Appendix gives the values that are graphed in Fig. 1.

Several points are of note. First, there is a substantial change in profits for many firms from the inclusion of integers, for at least some test cases. Thus, this suggests that market modelling that excludes integers will not yield accurate results, particularly from the firm's point of view. Furthermore, this result is particularly strong under perfectly competitive modelling. We conclude that while the impact of integers that was observed in Shortt et al (2012) is replicated here, this impact is dampened somewhat by the presence of price-making behaviour, modelled *à la* Cournot.

The second observation is the heterogeneity of results across firms. For firms with wind (ESB, EP, SSE, Energia, POWERNI, and BGE), as the amount of wind they have increases, the percentage decrease in their profits decreases. As these firms utilise more wind generation they reduce the amount of electricity they generate from conventional sources. As we assume wind does not have online or start-up costs, the effect of capacity decreases as wind capacity increases. Conversely, for the firms without wind, as the amount of wind on the system increases, the percentage decrease in their profits increases. As wind capacity increases, the generation from these firms reduces. Thus, the relative effect of excluding integers increases. This is particularly obvious in the cost minimisation cases.

For firms without wind, these results align with those of Shortt et al (2012), while the opposite holds for firms with wind. From a modelling perspective, the contrasting effect between the firms suggests that there is no obvious heuristic to use to capture the impacts of discontinuous costs without explicitly including integers in the modelling. From a regulatory and policy perspective, neglecting the impact of discontinuous costs in market design may therefore lead to inaccurate results and conclusions.

A third insight is the fact that the magnitude of the impact of the exclusion of integers is much higher under perfect competition than under Cournot modelling. This result stems from the fact that in general, Cournot modelling results in higher equilibrium prices, and thus higher profits, than perfect competition. The percentage decrease in profits as a result of the inclusion of integers is therefore smaller in the presence of Cournot modelling, as profits are much higher to begin with. The absolute change in profits, however, shows different patterns of variation within and between firms (see Fig. 2). In general, however, the impact of integers in absolute terms is proportional to the size of the firm, which is intuitive, but the impact varies by firm under different penetrations of wind energy. This underlines again the heterogeneity of the impact of integers at high RES-E.

Additionally, the percentage decrease in profits for several firms is greater than 100% in some of the cost minimisation test cases. This is because when online and start-up costs are accounted for ex-post in the cost minimisation without integers model, these firms have a negative profit. That is, they make a loss. This only occurs in the cost minimsation cases as prices are high enough in the Cournot cases to cover the discontinuous costs. These results further highlight the importance of including integer variables in cost minimisation unit commitment models as it is implausible for firms to commit to generating at a loss, unless uplift

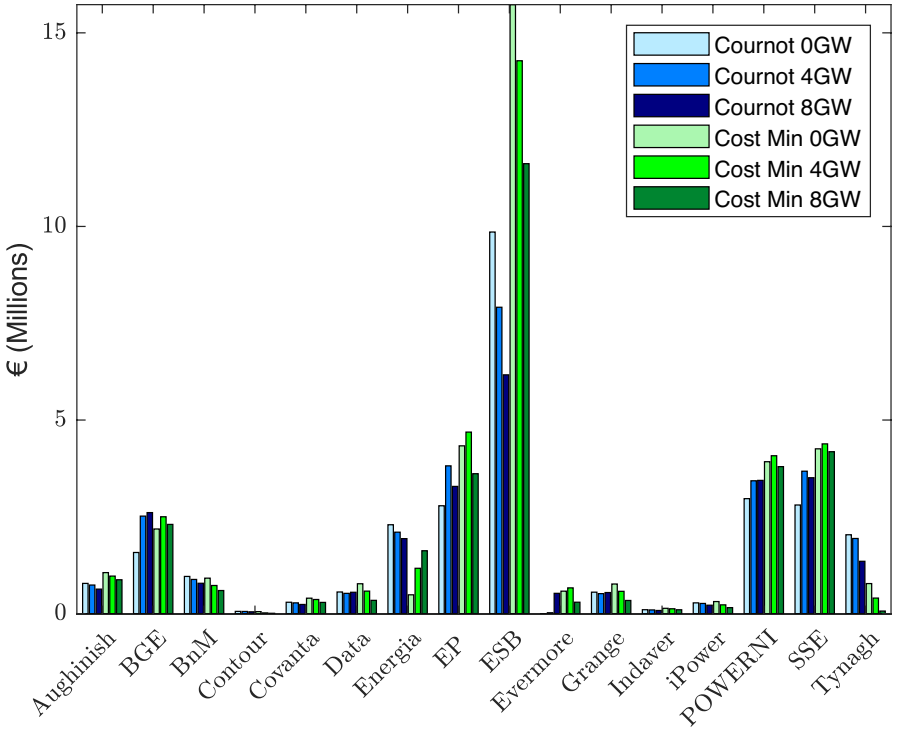


Fig. 2 Absolute decrease in firm profits from the exclusion of integers

Table 7 Percentage increase in consumer costs as a result of including integers

	Cournot	Perfect competition
0GW	11%	35%
4GW	12%	40%
8GW	12%	42%

payments are incorporated. However, accurate modelling of uplift payments also requires integer variables.

Table 7 shows the increase in consumer costs that arises as a result of including integers, and finds a similar result: consumer costs are lower under perfect competition, but including integer variables significantly increases consumer costs relative to a model that excludes integers. Under Cournot modelling, consumers already face high costs due to price mark-ups, and so there is a smaller proportional increase in consumer costs from the inclusion of integers.

In comparison to Table 7, Table 8 shows that the percentage decrease in producer surplus as a result of including integers is higher under perfect competition

Table 8 Percentage decrease in producer surplus as a result of including integers

	Cournot	Perfect competition
0GW	35%	92%
4GW	31%	75%
8GW	27%	61%

Table 9 The contribution of start, no load and incremental costs to total system costs

	Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW
% Incremental Costs	68.5%	66.6%	64.9%	66.3%	66.0%	65.1%
% Start Costs	0.9%	2.9%	4.7%	1.3%	2.7%	5.1%
% No load Costs	30.6%	30.5%	30.4%	32.4%	31.2%	29.8%

than Cournot. However, in contrast, it also shows that producer surplus decreases as wind capacity on the system increases.

In order to determine the drivers of these results, we consider the make-up of total costs that are accounted for by start costs, no load costs, and incremental costs. This is shown in Table 9.

The results under perfect competition are in line with those reported in Shortt et al (2012). We note at this point that the total costs decrease as installed wind capacity increases, due to wind displacing conventional generation (and due to the fact that we consider operational costs only and do not consider investment costs). Thus, discontinuous costs make up a greater proportion of total costs at higher wind levels, but a lower absolute level of cost.

There are several insights from Table 9. The first has been mentioned already, namely that start costs are higher at higher wind levels under perfect competition compared to Cournot modelling. In addition, incremental costs and no-load costs decrease as wind increases under the perfect competition test case.

To examine these results further, Table 10 shows the number of times each of the firms incurred a start cost, under each test case (Indaver and iPower incurred no starts and so are omitted).

Considering first the total number of starts, there are more starts in general under perfect competition versus Cournot modelling, and more starts with wind compared to no wind. There is no considerable increase in the total starts when wind moves from 4GW to 8GW, under either perfect competition or Cournot. Table 10 shows considerable variation in the number of starts both within and between firms, with ESB (the legacy monopolist and the firm with the greatest generation capacity) seeing considerably fewer starts under Cournot versus perfect competition, while the opposite pattern holds for most other firms. This could be because ESB starts their units once and is less likely to turn them off and restart them under Cournot, thereby avoiding incurring start costs, or it could

Table 10 Number of unit starts per firm

	Perfect Competition			Cournot		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	0	0	6	0	0	0
BGE	7	7	6	7	8	5
BnM	21	22	18	7	7	17
Contour	14	14	14	0	11	9
Covanta	0	0	0	0	0	0
Data	14	13	11	0	11	16
Energia	7	2	0	7	5	2
EP	49	48	31	56	53	45
ESB	42	40	40	14	11	7
Evermore	0	0	0	0	0	0
Grange	7	7	5	0	6	7
POWERNI	0	19	17	7	15	17
SSE	35	48	70	49	36	32
Tynagh	0	0	0	0	6	7
Total	196	220	218	147	169	164

be because ESB reduces starts because they are generally reducing their output under Cournot in an effort to increase prices.

In order to determine which case holds, Table 11 shows the number of hours each firm has a unit with non-zero output. Here we see that ESB has considerably fewer output hours under Cournot versus perfect competition, with other firms increasing their output. These results suggest that ESB is reducing power output in order to increase prices, with this reduction in output only partly compensated by other firms increasing their output. In other words, the reduction in starts by the monopolist is driven primarily by an attempt to reduce output and thus increase prices, rather than an attempt to avoid costly starts. Furthermore, wind curtailment does not vary significantly across the test cases (results not shown here), and so a reduction in thermal generation explains the reduction in total generation.

To further understand the impact of excluding integer variables, we consider the regret of not accounting for online and start-up costs for the two models without integers. We define regret (displayed in Fig. 3 and Table 15) as the online and start-up costs each firm incurs given their chosen optimal generation levels. These costs are not accounted for in the objective functions (2) and (4) and are calculated ex-post to the models being solved.

The steps involved in calculating regret are as follows:

Table 11 Number of hours that each firm has a generation unit online

	Perfect Competition			Cournot		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	336	336	185	336	336	336
BGE	119	248	214	224	257	224
BnM	413	288	223	504	423	350
Contour	168	60	37	168	153	121
Covanta	105	168	168	168	168	168
Data	126	102	49	257	287	226
Energia	14	172	168	28	190	173
EP	539	334	246	644	421	310
ESB	1414	1125	908	868	511	416
Evermore	168	0	0	133	0	0
Grange	168	49	19	168	140	115
Indaver	168	168	168	168	168	168
iPower	168	168	168	168	168	168
POWERNI	231	516	380	136	523	373
SSE	296	694	544	385	473	349
Tynagh	0	0	0	0	114	72

1. Solve the (naive) model without integers. For Cournot competition, this is problem (2) while for perfect competition, this is problem (4).
2. Fix the first-stage generation levels for each firm f at the values obtained at the previous step. We denote these as $GEN^{FS,N}_{f,t,f}$.
3. Use these generation levels to obtain when each unit starts and is online as follows:

$$z^{\text{FS, ON,N}}_{f,p,t} = \begin{cases} 1, \text{ if } GEN^{FS,N}_{f,p,t} > 0. \\ 0, \text{ otherwise.} \end{cases} \tag{7a}$$

$$z^{\text{FS, SU,N}}_{f,p,t} = \begin{cases} 1, \text{ if } GEN^{FS,N}_{f,p,t} > 0 \text{ and } GEN^{FS,N}_{f,p,t-1} = 0. \\ 0, \text{ otherwise.} \end{cases} \tag{7b}$$

4. Calculate the first-stage profit for each firm f at roll r accounting for these online and start-up costs:

$$\sum_{t \in T^{FS}_r} \sum_{p \in P} \Big((\gamma^{FS,N}_t - C^M_{f,p} - C^Q_{f,p} \times GEN^{FS,N}_{f,p,t}) \times GEN^{FS,N}_{f,p,t} - C^{\text{ON}}_t \times z^{\text{FS, ON, N}}_{f,p,t} - C^{\text{SU}}_t \times z^{\text{FS, SU, N}}_{f,p,t} \Big), \tag{7c}$$

where

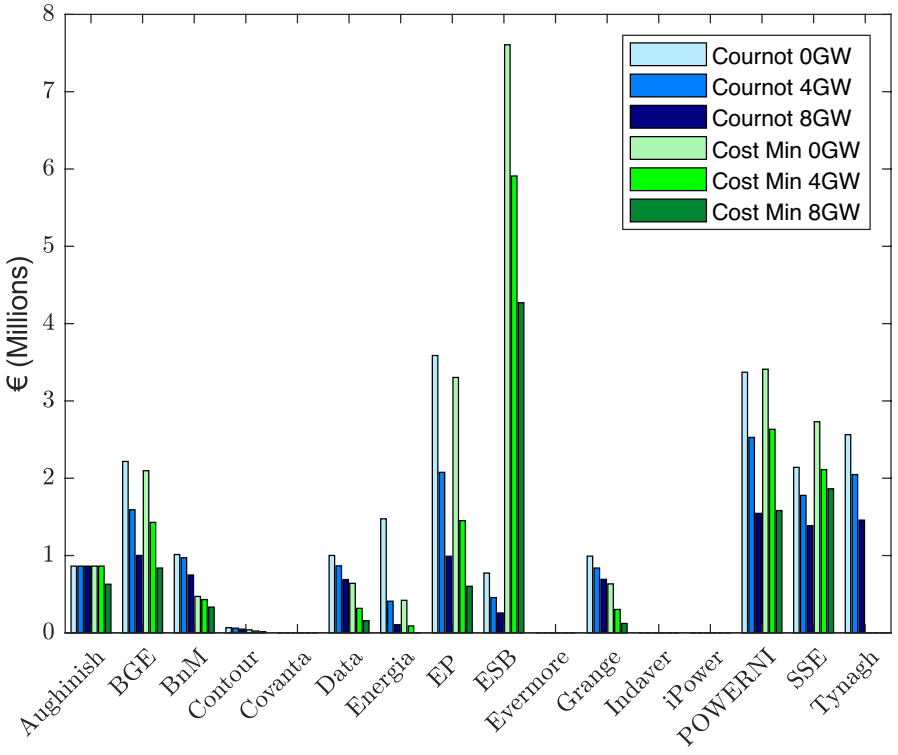


Fig. 3 Regret in €(millions) of not accounting for integer variables

$$\gamma_t^{FS,N} = A_t - B \times \Big(\sum_{f \in F} \sum_{p \in P} GEN_{f,p,t}^{FS,N}\Big), \quad \forall t \in T_r^{FS}. \tag{7d}$$

5. Using equation (7d), calculate the difference between the naive profits and the profits that account for the online and start-up costs ex-post:

$$REGRET_f = \sum_{r=2}^{15} \big(\text{Equation (2b) - Equation (7d)}\big) \tag{7e}$$

Equation (7e) captures the costs (regret) of using the generation levels from the models without integers. Note: as explained previously, when calculating expected profits, we account for first-stage decisions from rolls $r = 2 - 15$ only.

For further discussion on regret analyses, we refer the reader to Gonzalez-Romero et al (2021).

Regret is correlated with the size of the firm, with the largest firms having the highest levels of regret. Furthermore, regret levels decrease as the amount of wind on the system increases. Again, this is because wind displaces units that have online and start-up costs.

Table 12 Average regret in €(thousands) per unit by fuel type.

	Perfect Competition			Cournot		
	0GW	4GW	8GW	0GW	4GW	8GW
Coal	106	177	187	112	72	13
Gas	1007	766	546	1103	958	696
GasOil	153	109	73	133	66	47
Oil	60	0	0	66	23	0
Peat	160	0	0	592	557	541
Waste	205	225	100	75	75	227
Hydro	0	0	0	0	0	0
Wind	0	0	0	0	0	0

Generally speaking, regret levels are similar between the cost minimisation and Cournot models. An exception to this is the largest firm, ESB, whose regret is much lower in the Cournot model. As Tables 10 and 11 show, ESB has substantially fewer units of starts and hours online in the Cournot model. Thus, ESB has less regret about not accounting for online and start-up costs when firms are price-making.

Conversely, Tynagh's regret is higher in the Cournot model. This is because Tynagh, as a small firm, is not online as often in the cost minimisation model and thus has little regret as a consequence.

Previous works in the literature have suggested that the fuel type of a generating unit affects a firm's price-making ability, specifically hydro (Bushnell 2003; Moghimi et al 2023). In Table 12, we also consider the impact of average regret for units by fuel type. It shows that gas-powered units have the highest regret. This is because these units have the highest online but the cheapest marginal costs. Thus, in the models without integers, gas-powered units are used more often. Consequently, as firms do not account for online and start-up costs, gas-powered units have the highest regret on average. This suggests that firms with gas-powered units should be particularly conscious of including integer variables in their objective/profits functions.

Table 12 also shows that the regret associated with wind and Hydro are zero as we do not assume any online or start-up costs for these units.

Generally speaking, average regret by fuel type is similar to the cost minimisation without integers model and the Cournot without integers model. The exception is for peat-powered units, where the regret is substantially higher in the Cournot model. When firms are price-making, wholesale prices increase and thus this allows peat-fired units to enter the market. In the Cournot model these units are online for 119 h (0GW wind case). This reduces to 87 h in the cost minimisation without integer model. Thus, using Cournot modelling might more accurately capture increased revenue opportunities for peat-powered units. However, doing so in the absence of integers will overestimate these opportunities.

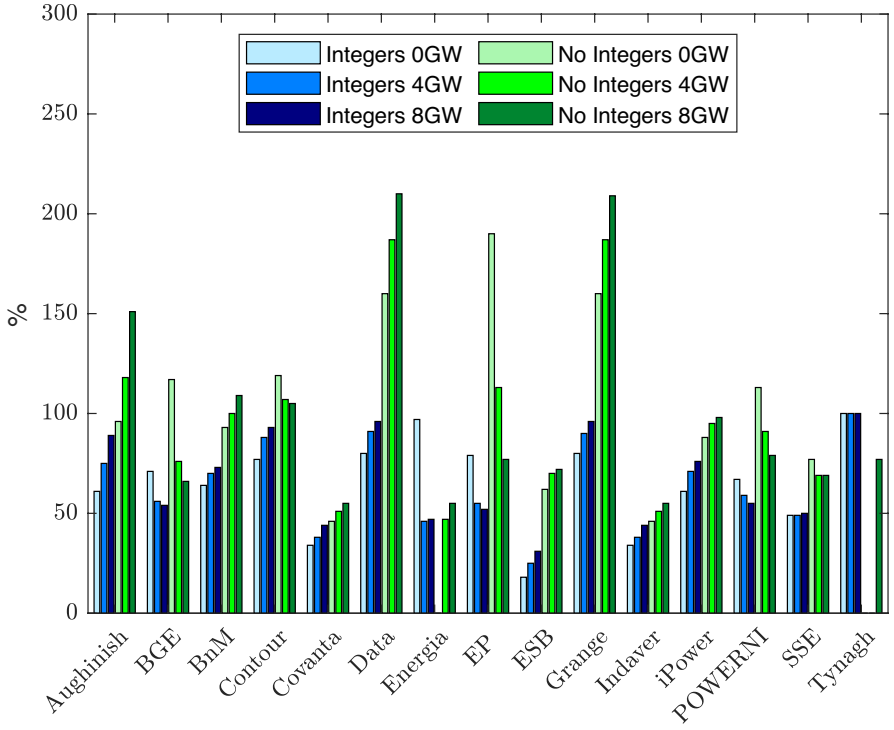


Fig. 4 % decrease in profit from removing Cournot behaviour

5.2 The impact of Cournot behaviour with integer variables

We now consider the impact of price-making ability, modelled here *à la* Cournot, on market outcomes, and on whether these impacts vary depending on whether integers are considered in the modelling. From Table 6, one can see that some generating firms in the Irish system hold a relatively small amount of generating capacity. Thus, it is questionable whether to model such firms as price-makers. However, when Cournot behaviour is introduced into the models, smaller firms (capacity $\leq 3\%$ of overall system capacity) tend to act as if they were price-taking. This is despite the fact they have price-making ability. That is, if the system price is greater than the marginal costs for these firms, their profit maximising position is to generate at their maximum capacity anyway.

Figure 4 shows the percentage decrease in profits, by firm, which arises from removing Cournot modelling.[4] For all firms, the percentage decrease of changing from price-making to price-taking behaviour is greater when no integers are explicitly considered. This is because integer variables serve as extra constraints and thus

[4] Evermore is excluded as they make very small or negative profits in all cases. Energia at 0GW of wind on the system and Tyangh at 4GW and 8GW of wind on the system are also excluded as they make negative profits in the Cournot and no integer cases.

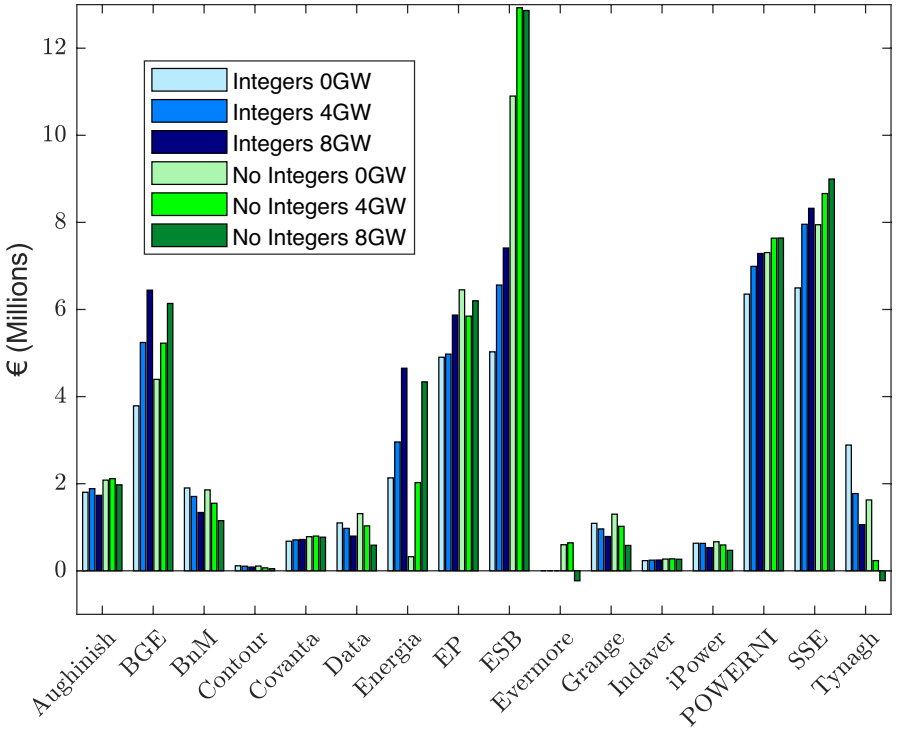


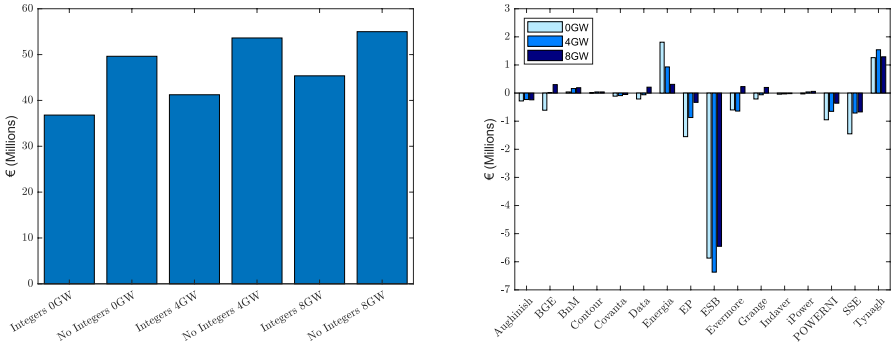
Fig. 5 Decrease in profit (absolute) from removing Cournot behaviour

restrict their price-making ability. Generally speaking, the size of the effect increases as wind increases. This broad pattern notwithstanding, there is some heterogeneity as the opposite is true for BGE, Energia, POWERNI, and SSE.

Looking at changes in profits in absolute terms (Fig. 5), there is further heterogeneity. Some firms see a greater decrease due to removing Cournot behaviour, while for other firms there is little difference between the integer and non-integer cases. Similarly, some firms see an increase as wind on the system increases (e.g., ESB and BGE) while this does not hold for other firms (e.g., BnM)

The main result is that the impact of Cournot modelling is considerably higher, in percentage terms, when integers are excluded: in general, removing Cournot modelling leads to a higher percentage decrease in profits when integers are excluded. For some firms, this result increases in wind generation, while for others the opposite holds. These results suggest again that ignoring the impact of integers can give misleading results, particularly under Cournot modelling. Price-making behaviour impacts on market outcomes, but the percentage difference between perfect competition and Cournot modelling is magnified in the absence of integer modelling.

Several works in the literature examine the impact of price-making behaviour by solving a model that assumes such behaviour is present, then solving assuming it is not present, and then comparing the difference (Devine and Bertsch 2022; Egging et al 2017; Koschker and Möst 2016). Figure 6a displays the increase in total costs



(a) Impact on costs (b) Impact on profits

Fig. 6 Impact of integers when moving from cost min to Cournot

to consumers as a result of moving from perfect competition to Cournot modelling, both in the presence and absence of integers.

Figure 6a indicates that the difference between consumer costs under perfect competition versus Cournot modelling is smaller when integers are considered compared to a case with no integers. This result holds both in absolute and percentage terms. This suggests that excluding integer modelling may overestimate the impacts of price-making behaviour on consumer costs, to some degree. However, the effect is not as strong as wind increases with little difference with 8GW of wind on the system.

Figure 6b shows the impact of integers on the increase in profits when moving from perfect competition to Cournot, given in equation (8):

$$\sum_{r=2}^{15} \left(\pi_{f,r}^{*,\text{Cournot with integers}} - \pi_{f,r}^{*,\text{Cost Min with integers}} \right) - \left(\pi_{f,r}^{*,\text{Cournot without integers}} - \pi_{f,r}^{*,\text{Cost Min without integers}} \right) \tag{8}$$

Figure 6b shows that in general, the increase in profits under Cournot modelling versus perfect competition is greater, in absolute terms, when integers are not modelled. In other words, excluding integer modelling exaggerates the impacts of price-making behaviour modelled *à la* Cournot. This is particularly true for the larger firms. Furthermore, there is no obvious pattern observed across firms under different wind levels.

5.3 Computational analysis

As mentioned in Sects. 1 and 2, this research is the first to consider integer constraints in Cournot electricity markets. Previous works in the literature avoided using integer variables for computational reasons. However, in this work, we implemented the Gauss-Seidel Diagonalisation algorithm along with a Rolling Horizon algorithm to solve such a model.

Table 13 Computational solve times (seconds)

Model	Solution Method	0GW	4GW	8GW
Cournot Equilibrium with Integers	Diagonalisation Algorithm	6942	4658	4333
Cournot Equilibrium without Integers	QP (Cplex)	6238	5896	6170
Cournot Equilibrium without Integers	Diagonalisation Algorithm	2483	2669	2536
Cost Minimisation with Integers	MIQCP (Cplex)	2101	4231	1689
Cost Minimisation without Integers	QP (Cplex)	8	7	6

Table 13 displays the solve times associated with each of the models considered in this paper. All the model runs were solved using GAMS on using a 3.1GHz i9-9900 processor with 32GB of RAM. The two models without integers are formulated as Quadratic Programs (QPs) and solved using Cplex. The Cost Minimisation with Integers is formulated as a Mixed Integer Quadratically Constrained Program (MIQCP) and was also solved using Cplex.

The Cournot with Integers was solved using the Gauss-Seidel diagonalisation algorithm (Sect. 3.2.1). As a comparison, the Cournot without Integers was solved using this algorithm. As the algorithm converges for each of the numerical experiments considered, the solutions found are guaranteed to be a Nash-Equilibrium (Su 2004; Gabriel et al 2012; Steffensen and Bittner 2014). However, we cannot guarantee that any of the solutions obtained are unique. The convergence tolerance we set was $TOL = 10^{-5}$.

A Rolling Horizon algorithm (Sect. 3.5) utilised for each of the models considered.

Table 13 shows that the cost minimisation models take less time to solve compared to the Cournot models. As expected, once integers are removed from the cost minimisation approach, the computational time is further reduced (substantially so) from a median of 2101 s to a median of 7 s.

However, when integers are removed from the Cournot model, the same reduction is not observed. In fact, the solve times for the Cournot with integers model are comparable to when the Cournot without integers is solved as a QP. This shows that the Gauss-Seidel algorithm is quite tractable for the Cournot with integers model considered in this model, where extensive data of the Irish power system is considered (86 generating units spread across 16 firms).

As a comparison, the Cournot without Integers model is also solved using the Gauss-Seidel diagonalisation algorithm. When the solutions from the two approaches are compared using the error metric in Algorithm 1, the highest error across the three wind capacity test cases was 10^{-4}. This further validates the use of the Gauss-Seidel algorithm approach.

Table 13 shows that when this algorithm is used the median solve time for the Cournot without Integers model is 59% faster than when the model is solved as a Quadratic Program. This suggests that there may be computational benefits to using the Gauss-Seidel algorithm instead of the Principle of Symmetry method outlined in Egging-Bratseth et al (2020). However, further analysis beyond the scope of this study

would be needed to conclude this. Particularly, one would need to experiment with increasing the number of timesteps in the horizons and the number of scenarios, etc.

Nonetheless, the solve times described in this section show that it is computationally tractable to solve a Cournot model with integer constraints.

6 Discussion

Electricity firms face both continuous (e.g., marginal) and discontinuous costs (e.g., online and start-up) costs. To model discontinuous costs integer variables are required. In addition, in recent years, climate change and the Ukrainian crisis have prompted electricity firms to accelerate the integration of variable Renewable Energy Sources (vRES) into their generating portfolios.

In general, there has been little attention in the literature paid to the interaction between the integration of vRES and discontinuous costs such as online and start-up costs. However, Shortt et al (2012) have shown that increased levels of vRES change optimal dispatch decisions when online and start-up are considered. However, Shortt et al (2012) only considered a cost minimisation (perfectly competitive) paradigm. This is unsuitable for liberalised energy markets such as the Irish market where price-making behaviour is possible. Thus, the advantage of considering integer variables in a Cournot electricity market is that more realistic results will be produced.

The results presented in this paper indicate that integer modelling has a material impact on the profits of firms competing in a Cournot electricity market model. This result holds when the game is modelled as perfectly competitive and *à la* Cournot alike. Furthermore, the impact of integer modelling varies under each test case and also varies between firms within test cases. Finally, for at least some results, more noticeably consumer costs, the impact of integers increases in the presence of wind. Thus, while integer modelling may not be required at low levels of wind penetration, the delta between results with and without integers increases at higher wind generation levels. This suggests that market designs and models that fail to account for discontinuous costs may not allocate resources efficiently at higher levels of renewable generation penetration. The importance of this result is compounded by the cannibalization effect of renewable generation (Cludius et al 2014; Brown and Reichenberg 2021; Prol et al 2020).

The heterogeneity of results across firms is of particular significance. For every metric considered here, the impact was heterogeneous across firms. This means that the impacts of integer modelling cannot be easily approximated by a general heuristic: the impacts are test case- and firm-specific.

These results also highlight the difference between market modelling, which considers firms as profit-maximising decision makers (imperfect competition), compared with power systems modelling, which typically takes a cost minimisation approach.

From the firms' perspective, firms that have units with online and start-up costs will experience inaccurate results if they fail to incorporate integer variables into their objective/profit functions. This effect is larger for units with higher online and start-up costs such as gas-powered and peat-powered units. However, the effects are

dampened for firms with a substantial amount of wind generation. Thus, for firms with relatively small generation portfolios made up entirely of conventional units, ignoring integer variables will overestimate their profit projections. For firms with large generating portfolios that include units that do not have online and start-up costs, the relative effect of not accounting for integer variables is smaller.

Cournot modelling led to far higher equilibrium prices than perfect competition, which is in line with the literature. Integer modelling imposes new constraints on generators, either directly, via minimum generation levels, or by allowing the consideration of extra costs resulting from starting and stopping units. These extra constraints limit the decisions available to generators and therefore reduce their opportunities to increase prices by varying their output. This means that ignoring discontinuous costs will lead to an overestimation of the impacts of price-making behaviour, modelled *à la* Cournot.

Thus, from a regulatory and/or a policy perspective, the results suggest that strong market power mitigation measures continue to be required in order to protect consumers from anti-competitive pricing by generation firms. However, these measures should be complemented by the consideration of discontinuous costs, particularly at higher levels of renewable generation. Furthermore, the impact of integer modelling often varied for the largest firm (ESB) compared to other, smaller, market players. This suggests that the impact of integer modelling depends on the price-making ability of each firm.

Previous energy market models in the literature would typically have ignored integer variables and hence online and start-up costs, most likely due for computational reasons. However, this paper shows that it is possible to use the Gauss-Seidel algorithm (combined with a Rolling Horizon Algorithm) to solve a unit commitment problem where price-making behaviour is present. Moreover, this was shown in a computationally tractable manner for a non-trivial real-world example. In particular, the all-island Irish power system assuming 86 generating units spread across 16 firms.

Modelling equilibrium investment decisions may also be impacted by the consideration (or lack thereof) of discontinuous costs, particularly as renewable generators have fixed costs only (while thermal generators have both fixed and variable costs). The impacts of forward contracting on the energy market, and of different renewable subsidy mechanisms, may also yield different equilibria with and without integer modelling. We leave these considerations for future work.

7 Conclusion

Modelling modern electricity markets requires the consideration of oligopolistic behaviour and technical aspects such as online and start-up costs. In this work, we study the application of game theory in an electricity market. In particular, we model output decisions by electricity generation firms, modelled under perfect competition and Cournot competition, with and without considering discontinuous costs such as start costs and no-load costs. The equilibrium prices and subsequent consumer costs are determined and compared at increasing levels of variable renewable generation.

Modelling Cournot competition with discontinuous costs requires integer variables. We solve such a model using the Gauss-Seidel diagonalization algorithm in a computationally tractable manner.

The results indicate that integer modelling yields different results for firms and consumers alike, and that the modelling impact of integers is heterogeneous across firms and under different levels of renewable generation. These results suggest that ignoring discontinuous costs, which is common in power and market modelling exercises, yields inaccurate results. While this will affect all firms, it will particularly to firms with a large portion of units with online and start-up costs, as the relative effect of ignoring integer variables is larger for them. The effect is larger for units with higher online and start-up costs such as gas-powered and peat-powered units.

In addition, this work will be of interest to policymakers and regulators as the results suggest that excluding integer variables may underestimate may underestimate consumer costs. However, policymakers and regulators will also be interested that excluding integer variables may overestimate the impact of price-making behaviour. This is because integer variables further constrain generating firms and thus dampen their price-making ability.

Market power mitigation remains a priority for policymakers, but accurate modelling of energy markets is also of increasing importance as renewable generation increases. Possible extensions of this work include the impacts of forward contracting, renewable subsidisation, and investment decisions.

Appendix A

See appendix tables 14, 15, 16 and 17.

Table 14 Percentage decrease in profits as a result of removing integers

	Perfect Competition			Cournot		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	93%	151%	429%	27%	29%	33%
BGE	142%	61%	42%	30%	27%	22%
BnM	87%	100%	120%	33%	36%	43%
Contour	152%	131%	135%	39%	47%	51%
Covanta	30%	32%	32%	15%	15%	14%
Data	275%	572%	982%	40%	49%	66%
Energia	665%	34%	31%	104%	33%	20%
EP	340%	116%	67%	45%	42%	29%
ESB	71%	72%	70%	36%	30%	26%
Grange	274%	567%	958%	40%	49%	66%
Indaver	30%	32%	32%	15%	15%	14%
iPower	78%	89%	94%	27%	30%	31%
POWERNI	128%	84%	65%	32%	29%	26%
SSE	64%	53%	50%	21%	23%	21%
Tynagh	100%	100%	100%	71%	109%	128%

Table 15 Regret (€(millions)) of not accounting for integers

	Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	0.86	0.86	0.86	0.86	0.86	0.63
BGE	2.22	1.59	1	2.1	1.43	0.84
BnM	1.01	0.97	0.75	0.47	0.43	0.33
Contour	0.07	0.06	0.05	0.04	0.02	0.01
Covanta	0	0	0	0	0	0
Data	1	0.87	0.69	0.64	0.32	0.16
Energia	1.48	0.41	0.1	0.42	0.09	0
EP	3.59	2.08	0.99	3.3	1.45	0.6
ESB	0.77	0.46	0.26	7.61	5.91	4.27
Evermore	0	0	0	0	0	0
Grange	0.99	0.84	0.69	0.63	0.3	0.12
Indaver	0	0	0	0	0	0
IPower	0	0	0	0	0	0
POWERNI	3.37	2.53	1.55	3.41	2.63	1.58
SSE	2.14	1.78	1.39	2.73	2.11	1.86
Tynagh	2.56	2.05	1.46	0	0	0

Table 16 % Decrease in profits by firm from the removal of Cournot modelling

	Integers			No Integers		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	61%	75%	89%	96%	118%	151%
BGE	71%	56%	54%	117%	76%	66%
BnM	64%	70%	73%	93%	100%	109%
Contour	77%	88%	93%	119%	107%	105%
Covanta	34%	38%	44%	46%	51%	55%
Data	80%	91%	96%	160%	187%	210%
Energia	97%	46%	47%	N/A	47%	55%
EP	79%	55%	52%	190%	113%	77%
ESB	18%	25%	31%	62%	70%	72%
Grange	80%	90%	96%	160%	187%	209%
Indaver	34%	38%	44%	46%	51%	55%
iPower	61%	71%	76%	88%	95%	98%
POWERNI	67%	59%	55%	113%	91%	79%
SSE	49%	49%	50%	77%	69%	69%
Tynagh	100%	100%	100%	N/A	N/A	77%

Table 17 Profits in €(millions) for each firm with and without Cournot modelling and integers

	Integers						No integers					
	Cournot			Perfect competition			Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	2.95	2.53	1.94	1.14	0.64	0.2	2.16	1.79	1.3	0.08	−0.33	−0.67
BGE	5.33	9.37	11.93	1.54	4.13	5.48	3.75	6.85	9.32	−0.65	1.62	3.18
BnM	2.96	2.44	1.84	1.05	0.73	0.5	1.99	1.55	1.05	0.14	0	−0.1
Contour	0.15	0.12	0.09	0.03	0.01	0.01	0.09	0.06	0.05	−0.02	0	0
Covanta	2.01	1.85	1.64	1.33	1.14	0.92	1.72	1.58	1.4	0.93	0.78	0.63
Data	1.38	1.08	0.83	0.28	0.1	0.03	0.82	0.55	0.28	−0.49	−0.48	−0.31
Energia	2.21	6.43	9.88	0.07	3.48	5.23	−0.09	4.33	7.94	−0.41	2.3	3.6
EP	6.18	9.01	11.29	1.27	4.04	5.42	3.39	5.2	8	−3.06	−0.65	1.8
ESB	27.33	26.3	24.12	22.3	19.74	16.71	17.47	18.39	17.95	6.57	5.46	5.09
Evermore	0	0	0	0	0	0	0.02	−0.02	−0.52	−0.58	−0.67	−0.29
Grange	1.37	1.06	0.82	0.28	0.1	0.04	0.81	0.55	0.28	−0.49	−0.47	−0.3
Indaver	0.69	0.64	0.56	0.46	0.39	0.32	0.59	0.54	0.48	0.32	0.27	0.22
IPower	1.03	0.89	0.7	0.4	0.26	0.16	0.76	0.62	0.48	0.09	0.03	0.01
POWERNI	9.42	11.86	13.16	3.07	4.87	5.88	6.45	8.43	9.72	−0.86	0.79	2.08
SSE	13.14	16.25	16.64	6.65	8.29	8.32	10.34	12.57	13.13	2.39	3.91	4.13
Tynagh	2.89	1.77	1.06	0	0	0	0.85	−0.17	−0.29	−0.78	−0.4	−0.07

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Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

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