A completely pivoted Hadamard matrix with a non-final pivot equal to n

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Abstract

We construct a Hadamard matrix of order $n = 64$ whose sixtieth pivot under complete pivoting is equal to n. This disproves a conjecture of Day and Peterson that every pivot except the last one is bounded by $n/2$. The same matrix provides a counterexample to two other conjectures concerning the fifth-last and sixth-last pivots. We also determine the possible pairs of fifth and sixth pivot magnitudes under complete pivoting for arbitrary Hadamard matrices.

A matrix $H \in \mathbb{R}^{n\times n}$ is a Hadamard matrix of order n if it has entries ± 1 and satisfies $HH^\top = nI$, where I is the identity matrix. Suppose Gaussian elimination (GE) on H proceeds without row or column interchanges and with nonzero pivots. Write $H_{1:k,1:k}$ for the submatrix in the first k rows and columns, and set $A(k) := |\det H_{1:k,1:k}|$, with $A(0) := 1$. The magnitude of the k-th pivot is then

$$p_k = \frac{A(k)}{A(k-1)}, \qquad k = 1, \dots, n.$$

The current Schur complement is the matrix remaining after the preceding elimination steps. We say that H is *completely pivoted* (CP) if, at every step of GE without interchanges, the top-left entry of this matrix has largest absolute value among all its entries (ties are allowed). The growth factor is the largest entry magnitude encountered during elimination divided by the largest entry magnitude of the original matrix. For a CP Hadamard matrix, it is therefore $g(H) = \max_k p_k$. For background on this problem, complete pivoting, and GE, see [1, 2], and for more recent results on growth under complete pivoting, see [3].

It is conjectured that $g(H) = n$ for every CP Hadamard matrix [1, 2]. This is known for $n = 1, 2, 4, 8$ [2], $n = 12$ [4], and $n = 16$ [5], but remains open in general. For a CP Hadamard matrix of order $n \geq 8$, the growth factor conjecture is stated alongside five assertions about individual pivots in [6]:

(i) The growth factor is $g(H) = n$.

(ii) The last four pivots satisfy $p_{n-3} \in \{n/4, n/2\}$ and $(p_{n-2}, p_{n-1}, p_n) = (n/2, n/2, n)$.

(iii) The fifth-last pivot belongs to $\{n/3, n/2\}$.

(iv) The sixth-last pivot belongs to $\{n/4, 3n/10, 3n/8\}$.

(v) Every pivot before the last satisfies $p_k \leq n/2$.

(vi) The first four pivots are $1, 2, 2, 4$, followed by $p_5 \in \{2, 3\}$ and $p_6 \in \{8/3, 10/3, 4\}$.

Day and Peterson prove (ii) in [2, Proposition 5.5]. The bound in (v) was posed as a conjecture by Day and Peterson [2] and was later reiterated by Edelman and Friedman [7]. Day and Peterson also prove the pattern in (vi) for the first five pivots [2, Proposition 5.8] and Lardas and Mitrouli [8] study the first six pivots for Hadamard matrices of order 20. In Appendix A, we prove a sharper version of the sixth pivot assertion.

In this note, we give a counterexample to (iii), (iv), and (v). The matrix in Figure 1 is a CP Hadamard matrix of order $n = 64$ with pivot magnitudes

$$\begin{aligned}(p_1, \dots, p_{64}) = \big(&1, 2, 2, 4, 2, 4, 4, 6, 4, 4, 4, \tfrac{16}{3}, 6, \tfrac{16}{3}, 8, 8,\\ &4, 6, 6, 8, 4, \tfrac{16}{3}, \tfrac{16}{3}, 8, 4, 4, 4, 8, 8, 8, 8, 8,\\ &8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 12, \tfrac{32}{3}, 8, 8, 8, 16,\\ &12, \tfrac{32}{3}, 8, 16, 16, 16, 16, 16, 16, 32, 32, \mathbf{64}, 16, 32, 32, 64\big).\end{aligned}$$

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In particular, $p_{60} = p_{n-4} = 64 = n$ and $p_{59} = p_{n-5} = 32 = n/2$. The former contradicts (iii) and (v), while the latter contradicts (iv). Thus a pivot can reach n before the final step. The growth-factor conjecture in (i) remains open.

Construction. The idea is to reach a Schur complement containing a scaled 4×4 Hadamard block whose elimination produces a pivot of magnitude 64 before the final step. We use the Sylvester Hadamard matrix W_6 of order 64, defined recursively by

$$W_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad W_{k+1} = \begin{pmatrix} W_k & W_k \\ W_k & -W_k \end{pmatrix}.$$

A computational search chose among tied maximal entries at each elimination step. After 56 steps, the remaining 8×8 Schur complement, written in the order used for the final eight pivots, is $S = 16K$, where

$$K = \left(\begin{array}{cccc|cccc} -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\ \hline 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \end{array}\right).$$

The leading 4×4 block of K is Hadamard and has pivot magnitudes $1, 2, 2, 4$. Hence the next four pivot magnitudes for S are

$$(p_{57}, p_{58}, p_{59}, p_{60}) = 16(1, 2, 2, 4) = (16, 32, 32, 64).$$

The matrix H is obtained by permuting the rows and columns of W_6 into this elimination order.

Verification. To make the matrix in Figure 1 easier to reproduce, the following Python program constructs H from W_6 using explicit row and column permutations, indexed from zero. Thus H is Hadamard by construction. The program then performs Gaussian elimination in exact rational arithmetic, verifying complete pivoting at every step and the final eight pivot magnitudes.

```
from fractions import Fraction

n = 64
rows = [int(x) for x in """
    56 45 43 62 59 50 14 40 58 47 48 37 60 35 22 57
    33 31 12 29 49  7 20 61 10 52 28 53 15 38  3 41
    55  5 23 11 44 21 27 17  9 24  6 30 13 26 51 36
    54 25 39 19 42 18 46 34  8 16 32 63  0  1  2  4
""".split()]
cols = [int(x) for x in """
    43 34 49 33 29 40 55 52 50 30 36 45 27 46 24 39
    54 53 51 48 42 47 44 41 18 23 35  7 38  2  1 37
     4 32 16 13 21  8  5  6 20 28 19 31 63 15 12 58
    14 10 56  3 26 11 60 22  9 25 17 57  0 59 61 62
""".split()]

# H is the Sylvester matrix W_6 with its rows and columns permuted.
assert sorted(rows) == sorted(cols) == list(range(n))
H = [[(-1) ** (x & y).bit_count() for y in cols] for x in rows]

# Gaussian elimination without interchanges, in exact arithmetic.
S = [[Fraction(h) for h in row] for row in H]
pivots = []
for k in range(n):
    pivot = S[0][0]
    assert abs(pivot) == max(abs(s) for row in S for s in row)  # complete pivoting
    pivots.append(abs(pivot))
    S = [[S[i][j] - S[i][0] * S[0][j] / pivot for j in range(1, len(S))]
         for i in range(1, len(S))]

assert pivots[-8:] == [16, 32, 32, 64, 16, 32, 32, 64]
print("H is CP, with p_59 = 32 and p_60 = 64")
```

The code is made available at `https://github.com/matias-ma/hadamard_cp`.

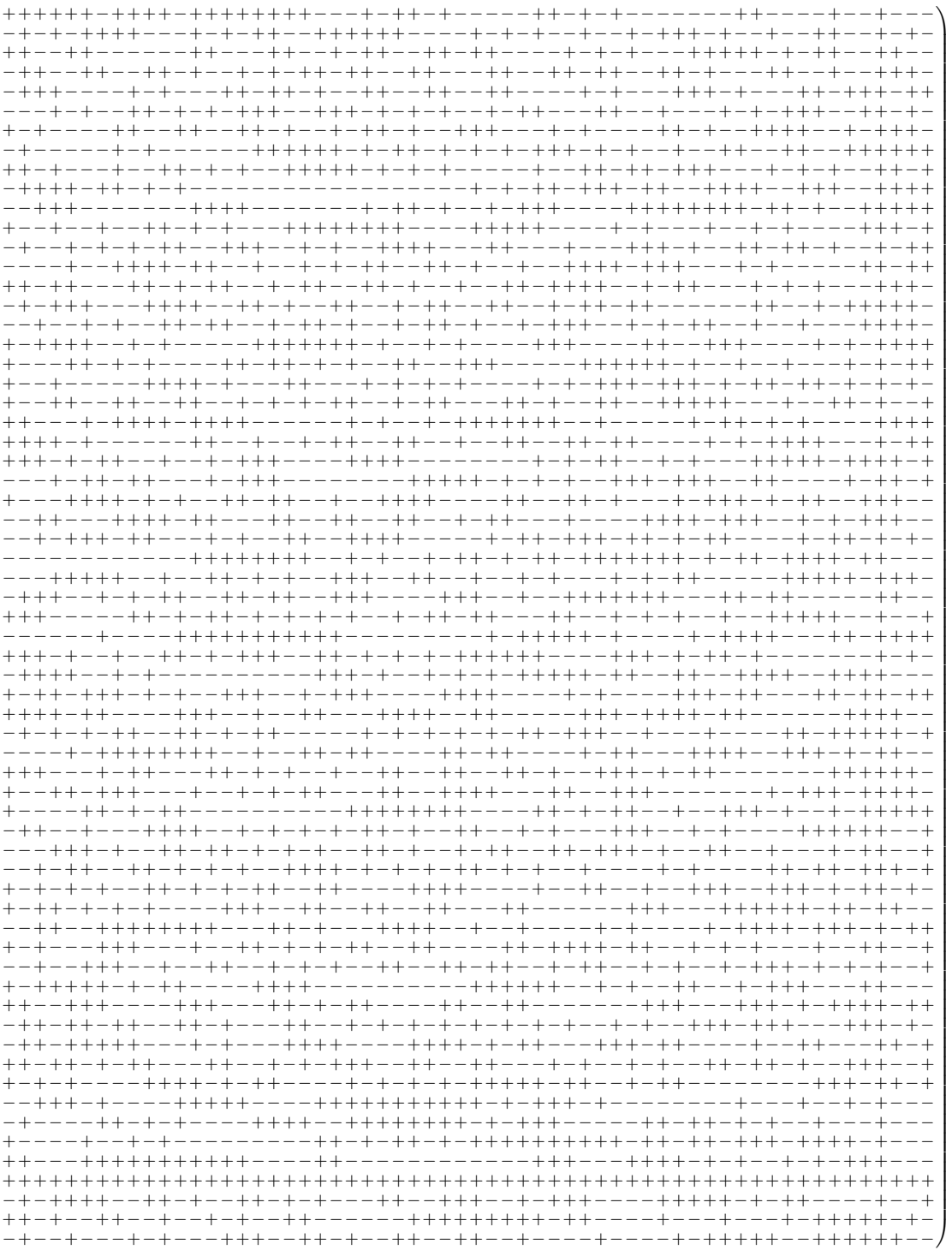


Figure 1: The completely pivoted Hadamard matrix H of order 64. Each sign + or − denotes +1 or −1, respectively.

Acknowledgements. AI tools were used extensively in constructing this counterexample and developing the proof in the appendix. In particular, GPT-6 Astra Pro produced the order 64 matrix and an initial version of the proof during a more general investigation of complete pivoting for Hadamard matrices. The author organized the final presentation and takes full responsibility for all content.

A The sixth pivot

The first five pivots of a CP Hadamard matrix H of order $n \geq 8$ are known to be $1, 2, 2, 4, p_5$ with $p_5 \in \{2, 3\}$ [2, Proposition 5.8]. With $A(k) = |\det H_{1:k,1:k}|$, Kravvaritis and Mitrouli [9, Table 1 and Corollary 3] give

$$A(4) = 16, \qquad A(5) \in \{32, 48\}, \qquad A(6) \in \{64, 96, 128, 160\}, \qquad p_6 \leq 4.$$

Since $p_k = A(k)/A(k-1)$, the remaining candidates are

p_5	possible values of p_6
2	2, 3, 4
3	4/3, 2, 8/3, 10/3

Below we will write $H_{i,:}$ and $H_{:,j}$ for the i-th row and j-th column of H, respectively.

Proposition A.1. *For every real Hadamard matrix H of order $n \geq 8$, complete pivoting, with arbitrary choices among maximizing ties, gives pivot magnitudes satisfying*

$$(p_5, p_6) \in \{(2, 4), (3, 8/3), (3, 10/3)\}.$$

Remark. The conclusion is sharp, since all three pairs (p_5, p_6) are attained by Hadamard matrices of order 16 under complete pivoting [6, Table 4].

Proof. By the preceding table, it suffices to exclude $p_6 \leq 3$ when $p_5 = 2$, and $p_6 \leq 2$ when $p_5 = 3$. Fix an arbitrary complete pivoting run, with pivot magnitudes $p_1, \ldots, p_n$, and order the rows and columns of H according to this run. In this ordering, H is CP. Let

$$B = H_{1:4,1:4}, \qquad F = H_{1:5,1:5} = \begin{pmatrix} B & c \\ r^\top & \varepsilon \end{pmatrix}.$$

Equality in Hadamard's determinant bound, $|\det B| = 1 \cdot 2 \cdot 2 \cdot 4 = 16 = 2^4$, gives $BB^\top = 4I$ and $B^{-1} = B^\top/4$. Thus B is Hadamard.

We use three observations throughout this proof.

1. *Operations preserving Schur complement bounds.* For $k = 4, 5$, write the Schur complements

$$S_k = H_{k+1:n,k+1:n} - H_{k+1:n,1:k}(H_{1:k,1:k})^{-1}H_{1:k,k+1:n}, \qquad \|S_k\|_{\max} = \max_{i,j} |(S_k)_{ij}|.$$

   Complete pivoting gives

$$\|S_4\|_{\max} = p_5 = |\varepsilon - r^\top B^{-1} c|, \qquad \|S_5\|_{\max} = p_6.$$

   We may negate any full row or column of H, permute its first four rows or columns, or transpose H without changing its Hadamard property or the above equalities. The transformed matrix need not remain completely pivoted, but this will not cause problems because we will only use the above Schur complement identities.

   Below, we will use these equalities to study p_5, p_6. These numbers remain fixed as we simplify H using the operations above, so we retain the notation H, B, F for the resulting matrices.

2. *Eliminated rows.* For $i > 4$, let $h = H_{i,:}$ be a row of H, and set

$$u_h = h - h_{1:4}B^{-1}H_{1:4,:}, \qquad \|u_h\|_\infty = \max_j |(u_h)_j|.$$

   The second term agrees with h in its first four entries, so u_h has four leading zeros followed by row $i-4$ of S_4. Further, letting $z = H_{5,:}$ and $h = H_{i,:}$ with $i > 5$, we have that

$$u_h - \frac{(u_h)_5}{(u_z)_5} u_z$$

   has zeros in its first five entries and its $6, \ldots, n$ entries form row $i-5$ of S_5.

3. *Splitting a sum by signs.* If $w \in \mathbb{R}^n$ is orthogonal to the all-ones vector $\mathbf{1}$ and to a vector $s \in \{-1, 1\}^n$, then

$$\sum_{j:s_j=1} w_j = 0, \qquad \sum_{j:s_j=-1} w_j = 0.$$

Indeed, $w^\top \mathbf{1} = 0$ says that these two sums add to zero, while $w^\top s = 0$ says that they are equal.

Using Observation 1, we can make the first row and column of H positive by sign changes. Each remaining row of B then has one additional $+1$, in a different column for each row. Reordering rows $2, 3, 4$ gives

$$B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = B^\top, \qquad H_{1,:} = \mathbf{1}^\top, \quad H_{:,1} = \mathbf{1}.$$

Case 1: $p_5 = 2$.

Suppose, for contradiction, $p_6 \le 3$. We first make $c = \mathbf{1}$ while retaining the displayed form of B.

We call a sign vector even or odd according to its number of minus signs. The rows and columns of this B are precisely the even sign vectors starting with $+1$, and are closed under entrywise multiplication.

If c is odd, its inner product with each even row of B is ± 2. Thus $Bc/2$ is a sign vector whose entries sum to $2c_1 = 2$, so it too is odd. If r were also odd, then $r^\top(Bc/2) \in \{0, \pm 4\}$, giving $r^\top B^{-1} c = r^\top (Bc/2)/2 \in \{0, \pm 2\}$ and $p_5 \in \{1, 3\}$, a contradiction. Transposing H if necessary, take c to be even so that it equals a column t of B. Multiply each full row i by H_{it}, making c and column t the ones vectors. Further, this simply permutes the columns of B, so we can reorder them to recover the displayed B, giving

$$c = \mathbf{1}, \qquad H_{1,:} = \mathbf{1}^\top, \quad H_{:,1} = \mathbf{1}, \qquad B^{-1}c = e_1 = (1, 0, 0, 0)^\top.$$

Consequently $(u_h)_5 = h_5 - h_{1:4} B^{-1} c = h_5 - 1$. For row $z = H_{5,:}$, $|(u_z)_5| = p_5 = 2$, so $z_5 = -1$.

Let $R = \{i : H_{i5} = -1\}$ and note that since H is Hadamard, columns $2, 3, 4$ are all orthogonal to both column 1, which is all ones, and column 5. Consequently, by Observation 3,

$$\sum_{i \in R} H_{i2} = \sum_{i \in R} H_{i3} = \sum_{i \in R} H_{i4} = 0 \implies \sum_{i \in R} (H_{i2} z_2 + H_{i3} z_3 + H_{i4} z_4) = 0.$$

Because $5 \in R$, noting $H_{5k} = z_k$ for $k = 2, 3, 4$, the sum includes the term $z_2^2 + z_3^2 + z_4^2 = 3$. Therefore, some other row $x = H_{i,:}$ with $i \in R$ must satisfy $x_2 z_2 + x_3 z_3 + x_4 z_4 < 0$. Each product is 1 when the entries agree and -1 when they differ, so x and z differ in $d = 2$ or 3 of the columns $2, 3, 4$.

The first four entries of column 5 are all 1, and $x \ne z$, so $i > 5$. Since $x_5 = H_{i5} = -1 = z_5$, we have $(u_x)_5 = (u_z)_5 = -2$. Eliminating column 5 therefore replaces u_x by $u_x - u_z$. Here u_x, u_z are rows of S_4 preceded by four zeros, while $u_x - u_z$ is a row of S_5 preceded by five zeros. Observation 1 therefore gives

$$\|u_x\|_\infty,\ \|u_z\|_\infty \le \|S_4\|_{\max} = p_5 = 2, \qquad \|u_x - u_z\|_\infty \le \|S_5\|_{\max} = p_6 \le 3. \tag{1}$$

If $d = 2$, the vector $x_{1:4} - z_{1:4}$ has exactly two nonzero entries, each ± 2. Since $B^{-1} = B/4$, multiplying by B^{-1} gives the average of two distinct rows of B, each with a possible sign change. These two signed rows are orthogonal, so they agree in two positions and differ in the other two. Their average therefore has two zero entries and two entries equal to ± 1. Consequently, for some $\alpha, \beta \in \{-1, 1\}$ and distinct $k, \ell \in \{1, 2, 3, 4\}$,

$$u_x - u_z = x - z - (x_{1:4} - z_{1:4}) B^{-1} H_{1:4,:} = x - z - \alpha H_{k,:} - \beta H_{\ell,:}.$$

Since x and z lie outside the first four rows of H, the four rows on the right are distinct and hence orthogonal. Each has squared Euclidean norm n, so $\|u_x - u_z\|_2^2 = 4n$.

Each entry of this four-row sum belongs to $\{-4, -2, 0, 2, 4\}$. The bound $\|u_x - u_z\|_\infty \le 3$ in (1) excludes ± 4, so every entry has magnitude at most 2. Since the first five entries vanish,

$$4n = \|u_x - u_z\|_2^2 \le 4(n - 5),$$

a contradiction.

If $d = 3$, exchange the names x, z if necessary so that $z_{1:4}$ is even. Choose $k \le 4$ with $H_{k,1:4} = z_{1:4}$, multiply each column j by H_{kj}, and reorder the first four rows to recover B. This changes signs in the same positions in u_x and u_z, preserving their bounds, and gives

$$z_{1:4} = (1,1,1,1), \qquad x_{1:4} = (1,-1,-1,-1), \qquad H_{1,:} = \mathbf{1}^\top, \quad H_{:,1} = \mathbf{1}.$$

With $L_j = H_{2j} + H_{3j} + H_{4j}$, multiplication by $B^{-1} = B/4$ then gives

$$(u_z)_j = z_j - 1, \qquad (u_x)_j = x_j + \frac{1 - L_j}{2}.$$

For $x_j = 1$, (1) implies $L_j \ge -1$ and $L_j + 2z_j \ge -1$. Averaging gives $L_j + z_j \ge -1$. Since L_j and z_j are odd, their sum is even and hence $L_j + z_j \ge 0$.

Each of the rows $H_{2,:}, H_{3,:}, H_{4,:}, z$ is distinct from both $H_{1,:} = \mathbf{1}^\top$ and x, so it is orthogonal to both. Applying Observation 3 to these four rows with $s = x$ and adding gives

$$\sum_{j:x_j=1} (L_j + z_j) = 0 \implies L_1 + z_1 = 0,$$

since $x_1 = 1$ and every summand is nonnegative. But the first column of H is all ones, so

$$L_1 + z_1 = H_{21} + H_{31} + H_{41} + z_1 = 4 \neq 0,$$

a contradiction. Thus $p_6 = 4$.

Case 2: $p_5 = 3$.

Suppose, for contradiction, $p_6 \le 2$. We now change the form of B by further sign changes in the first four full rows and columns of H, making $r = c = \mathbf{1}$ as permitted by Observation 1. If $\varepsilon = 1$, negate the first four rows and column 5 to make $\varepsilon = -1$ while keeping $r = c = \mathbf{1}$.

Since $\|B^{-1}c\|_2^2 = c^\top B B^\top c/16 = 1$, the fifth-pivot identity and Cauchy–Schwarz give

$$p_5 = 3 = |-1 - r^\top B^{-1} c| \le 1 + |r^\top B^{-1} c| \le 1 + \|r\|_2 \|B^{-1}c\|_2 = 3.$$

Thus $r^\top B^{-1} c = 2$, and equality in Cauchy–Schwarz gives

$$\left\| B^{-1}c - \frac{r}{2} \right\|_2^2 = \|B^{-1}c\|_2^2 - r^\top B^{-1} c + \frac{\|r\|_2^2}{4} = 1 - 2 + 1 = 0 \implies B^{-1}c = \frac{r}{2}.$$

Hence, using $r = c = \mathbf{1}$ and $B^{-1} = B^\top/4$, we obtain $B\mathbf{1} = B^\top \mathbf{1} = 2\mathbf{1}$.

Every row and column of B therefore sums to 2 and contains exactly one minus sign. Permuting the first four full columns of H puts these minuses on the diagonal, giving

$$F = J_5 - 2I_5, \qquad F^{-1} = \frac{1}{6} J_5 - \frac{1}{2} I_5, \tag{2}$$

where J_5 is the all-ones matrix and the inverse follows by multiplication using $J_5^2 = 5J_5$.

Negate rows $i > 5$ and columns $j > 5$ of H with more than two minus signs in their first five entries. Afterwards, every row and column has at most two such minus signs.

Let N_t count rows with t minus signs in their first five entries for $t = 0, 1, 2$. Then $N_0 + N_1 + N_2 = n$. These entries sum to $5, 3, 1$, respectively, so

$$25N_0 + 9N_1 + N_2 = \sum_{i=1}^{n} \left(\sum_{j=1}^{5} H_{ij} \right)^2 = \left\| \sum_{j=1}^{5} H_{:,j} \right\|_2^2 = 5n,$$

where the last equality uses column orthogonality. Together with $N_0 + N_1 + N_2 = n$, this gives $N_2 = n/2 + 2N_0 \ge n/2 > 0$.

We will show that each pair of minus positions occurs in at most one row, and that any such row requires $n \ge 36$. Since there are only $\binom{5}{2} = 10$ pairs, this will give $n \le 20$, a contradiction.

Let $x = H_{i,:}$ and $y = H_{\ell,:}$, not necessarily distinct, have the same two minus positions $A \subset \{1, \dots, 5\}$ in their first five entries. Each of the first five rows has exactly one minus sign in its first five entries, so $i, \ell > 5$. Set

$$a_j = \sum_{t \in A} H_{tj}, \qquad b_j = \sum_{t \in \{1,\dots,5\} \setminus A} H_{tj}.$$

The entries of $x_{1:5}$ sum to 1, and $x_{1:5}$ is negative on A and positive elsewhere. Hence

$$x_{1:5}F^{-1}H_{1:5,j} = \frac{1}{6}\mathbf{1}^\top H_{1:5,j} - \frac{1}{2}x_{1:5}H_{1:5,j} = \frac{a_j+b_j}{6} - \frac{b_j-a_j}{2} = \frac{2a_j-b_j}{3} = y_{1:5}F^{-1}H_{1:5,j},$$

where the final equality follows from $x_{1:5} = y_{1:5}$. For $j > 5$, as in Case 1, $\|S_5\|_{\max} = p_6 \le 2$ gives

$$\underbrace{\left|x_j - \frac{2a_j-b_j}{3}\right|}_{|(S_5)_{i-5,j-5}|}, \quad \underbrace{\left|y_j - \frac{2a_j-b_j}{3}\right|}_{|(S_5)_{\ell-5,j-5}|} \le \|S_5\|_{\max} = p_6 \le 2. \tag{3}$$

Now, we bound $xy^\top$ from below by defining

$$P_j = (x_j + a_j - b_j)(y_j + a_j - b_j) - 2a_j^2 + 4a_jb_j - \frac{b_j^2-1}{2} = x_jy_j + (x_j+y_j)(a_j-b_j) - a_j^2 + 2a_jb_j + \frac{b_j^2+1}{2},$$

whose sum is $xy^\top$. Indeed, since x, y are orthogonal to the first five rows, which are mutually orthogonal and have squared Euclidean norm n, we have

$$\sum_j (x_j+y_j)(a_j-b_j) = \sum_j a_jb_j = 0, \qquad \sum_j a_j^2 = 2n, \quad \sum_j b_j^2 = 3n.$$

Consequently,

$$\sum_{j=1}^n P_j = xy^\top - 2n + \frac{3n+n}{2} = xy^\top.$$

Because each column has at most two minus signs in its first five entries, the following lists all possible (a_j, b_j), the signs allowed by (3), and the minimum of P_j:

(a_j, b_j)	$(2,3)$	$(0,3)$	$(2,1)$	$(-2,3)$	$(0,1)$	$(2,-1)$
allowed x_j, y_j	±1	±1	±1	-1	±1	1
$\min P_j$	12	0	0	0	0	0

Thus $P_j \ge 0$ for $j > 5$.

For $j \in A$, the expression for F given in (2) gives that two of the columns satisfy $(a_j, b_j) = (0, 3)$ and $x_j = y_j = -1$, so $P_j = 12$. For the other three columns $j \le 5$, we have $(a_j, b_j) = (2, 1)$ and $x_j = y_j = 1$, so $P_j = 4$. Hence

$$xy^\top = \sum_{j=1}^n P_j \ge 2 \cdot 12 + 3 \cdot 4 = 36.$$

Distinct rows would give $xy^\top = 0$, so $x = y$, implying $xy^\top = n \ge 36$.

Further, since $N_2 > 0$, such a row exists. On the other hand, $n/2 \le N_2 \le \binom{5}{2} = 10$, so $n \le 20$, a contradiction. This excludes $p_6 \le 2$, leaving $p_6 \in \{8/3, 10/3\}$. □

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